# Fourier Transform Practice (Exercises due Wed 9/26)

Use the definition of Fourier transform given in the text:

$$\hat{f}(\gamma) = \int_{\mathbf{R}} f(x) e^{-2\pi i \gamma x} dx.$$

(Other mathematicians may define it without the $2\pi$ in the exponential.)

**Exercise 1** Prove that if $f(x) = \chi_{[-a,a]}(x)$ and $a > 0$, then $\hat{f}(\gamma) = \frac{\sin(2\pi a\gamma)}{\pi\gamma}$.

**Exercise 2** Prove that if $f(x) = e^{-a|x|}$ and $a > 0$, then $\hat{f}(\gamma) = \frac{2a}{a^2+(2\pi\gamma)^2}$.

**Exercise 3** Prove that if $f(x)$ has Fourier transform $F(\gamma)$, then the Fourier transform of $e^{2\pi i\alpha x} f(x)$ is $F(\gamma - \alpha)$.

**Exercise 4** Prove that if $f(x)$ has Fourier transform $F(\gamma)$, then the Fourier transform of $f(ax)$ is $\frac{1}{|a|}F(\gamma/a)$.

**Exercise 5** Prove that if $f(x)$ has Fourier transform $F(\gamma)$, then the Fourier transform of $f(x-b)$ is $e^{-2\pi i\gamma b}F(\gamma)$.

**Exercise 6** Use the preceding exercises to find the Fourier transforms of the following functions (**don't** evaluate them directly from the definition):

**a.** $e^{-3|x-1|}$

**b.** $e^{-|2x+4|}$

**c.** $\chi_{[0,2]}(x)$