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# Price Discrimination in Competitive Markets

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We present models in which price discrimination in the context of a two-part price can occur in some competitive markets. Purchases take place in groups, which choose which firms to patronize. While firms are perfectly competitive with respect to groups, they have some market power over individual consumers, who are constrained by their groups' choices. We find that firms will charge an entry fee that is below marginal cost, and the second part of the price is marked up above marginal cost. The markup not only is positive but increases with the quality of the product.

## I. Introduction

A standard textbook example of monopoly pricing policies is that of the monopolist that charges a two-part price.<sup>1</sup> One part of the price consists of an entrance fee. Entrance fees may or may not entitle the consumers to a certain product they directly consume. At an amusement park, for example, the entrance fee may simply give the consumers the right to purchase rides. The second part of the two-part price is the price of the rides. In the case of a movie theater, on the other hand, the entrance fee entitles the consumers to watch a movie and gives them the right to purchase other goods, such as popcorn, to consume along with the movie. The price of popcorn in this case is the second part of the two-part price.

The sale of popcorn in movie theaters provides a useful specific context to describe two-part pricing. Suppose that all moviegoers are identical. The optimal strategy for a movie theater is to set the price

<sup>1</sup> Landsburg (1989) and Friedman (1990) are two intermediate textbooks that give a good treatment of this question.

of popcorn equal to its marginal cost. By doing so the movie theater maximizes the sum of consumers' and producer's surplus, which it can then capture through the appropriate entrance fee.<sup>2</sup> If moviegoers differ but the movie theater can charge different consumers different entrance fees, the optimal popcorn pricing policy remains the same.

When moviegoers differ and price discrimination with respect to the entrance fee is not possible, the movie theater's optimal price for popcorn will no longer be its marginal cost. Since everyone is charged the same entrance fee, some individuals pay less than what they are willing to pay for the combination of movie and popcorn. By raising the price of popcorn above its marginal cost, the movie theater will capture more from those individuals with a large consumer surplus than it will lose by having to lower its entrance fee. It will therefore usually be optimal for the movie theater to price popcorn above its marginal cost.<sup>3</sup>

This explanation is commonly used to account for the high price of popcorn in movie theaters. The explanation seems to be equally applicable to candy and soft drinks at movie theaters, to wine and other extras at restaurants, to food and drinks at sporting events, and to a myriad of other less similar cases. The wide applicability of the explanation of price discrimination via two-part pricing actually seems to us to be a problem. Current models of price discrimination, whether involving two-part pricing or not, require market power on the part of the firm. Could so many firms in so many markets have sufficient market power? We are skeptical, and we wish to offer in this paper an explanation of how price discrimination may be practiced in a certain (admittedly limited) type of competitive market.

As long as the price of popcorn (or in general the second part of the price) is above marginal cost, competitive firms would find it attractive to lure away those customers with a high willingness to pay for popcorn by lowering the price of popcorn and raising the entrance fee. Price discrimination would thus unravel under competition. For it to be possible for competitive firms to price-discriminate, customers with a high willingness to pay for popcorn must be constrained to some extent from switching to other firms, without violating the competitive nature of the market. We believe that this happens to some extent in markets in which purchases take place in

<sup>2</sup> This is not exactly correct since consumer surplus, defined as the area underneath the Marshallian demand curve, will not normally be exactly equal to consumers' willingness to pay.

<sup>3</sup> It is optimal for the movie theater to price popcorn above marginal cost as long as the willingness to pay for the movie is not too strongly negatively correlated with the willingness to pay for popcorn.

groups composed of different types of persons. While firms compete for the patronage of groups, individuals within each group are constrained by their group's choice of which firm to patronize. Firms that are perfectly competitive across groups may still have some power to price-discriminate between members of a group.

Once group purchasing is assumed, the analysis is similar to that of the monopoly case. One way our model differs from the monopoly case is that we must specify how groups make their decisions of where to purchase. We present a model for the pricing of popcorn in movie theaters, where each group goes to the movie theater that gives the highest value of a Bergson-Samuelson group's welfare function, which is increasing and concave in each group member's individual utility function. We also consider the case in which group members vote to go to the theater of their choice, and the group goes to the movie theater that gets the majority of the votes. In Section III we provide a model of the entire price schedule of the quality of wine at restaurants, where again group decisions are based on the maximization of a group welfare function.

## II. The Pricing of Popcorn

The firm in our first model is a movie theater, which sells two goods. One of them, the movie, comes only in a fixed quantity and must be consumed by everyone entering the theater. The second good, popcorn, can be purchased in different quantities.

Each consumer has a utility function  $U(X, M, q, \theta)$ , where  $X$  is a composite commodity other than the ones sold at the movie theater,  $M$  stands for consumption of the movie,  $q$  is the quantity of popcorn, and  $\theta$  is an index of the consumer's appreciation of popcorn. Following Mussa and Rosen (1978), we disregard all income effects and assume that  $U$  is given by

$$U = X + \gamma M + \theta h(q), \quad (1)$$

where  $h$  is an increasing and concave function of  $q$ . All consumers value the movie itself equally, but they differ in their valuation of popcorn.

All consumers have the same money income,  $Y$ . The price of the movie is  $P_m$ , and  $P_q$  is the price of popcorn. The consumer's budget constraint is given by

$$X = Y - P_m - P_q q, \quad (2)$$

where units have been chosen in such a way that  $M = 1$ .

On going to the movies, the consumer chooses the amount of popcorn (and of the composite commodity) that maximizes (1), subject

to (2). The amount of popcorn purchased will be the solution to the following first-order condition:

$$\theta h'(q) = P_q. \quad (3)$$

The solution to (3) is  $q^*(P_q, \theta)$ , where  $q^*$  is decreasing in  $P_q$  and increasing in  $\theta$ . Substituting the solution for  $q$  and  $X$  into (1) yields the indirect utility function,  $U^*(P_m, P_q, \theta)$ . Its partial derivatives are given by

$$\frac{\partial U^*(P_m, P_q, \theta)}{\partial P_m} = -1, \quad (4)$$

$$\frac{\partial U^*(P_m, P_q, \theta)}{\partial P_q} = -q^*, \quad (5)$$

and

$$\frac{\partial U^*(P_m, P_q, \theta)}{\partial \theta} = h[q^*(P_q, \theta)]. \quad (6)$$

Consumers go to the movie theaters in groups. The groups are identical, and within each group the value of  $\theta$  is distributed according to a density function  $f(\theta)$  defined in the interval  $[\underline{\theta}, \bar{\theta}]$ . We are therefore assuming that moviegoers do not form groups on the basis of the affinity of their members' tastes for popcorn, but rather that groups are organized according to some other principles, such as the personal relationships of the members. If groups could form according to members' tastes for popcorn, it would not be possible for theaters to charge above marginal cost for popcorn. A movie theater charging the marginal cost of popcorn would then attract all groups consisting of consumers with high values of  $\theta$ . The assumption that groups consist of individuals that differ in their tastes for popcorn is essential, but it is also reasonable.

When confronted with the option of choosing between two different theaters, each (heterogeneous) group must have a decision rule. We consider now the case in which a theater is chosen if it gives the highest value of a Bergson-Samuelson group's welfare function given by

$$\int_{\underline{\theta}}^{\bar{\theta}} G[U(\theta)] f(\theta) d\theta = \int_{\underline{\theta}}^{\bar{\theta}} G[U^*(P_m, P_q, \theta)] f(\theta) d\theta. \quad (7)$$

The function  $G$  is assumed to be concave, reflecting the fact that the group is more sensitive to the wishes of those with lower utility. The assumption seems to receive support from the behavior of real-world groups, which often give individual members the right to veto group

choices. Moreover, without the concavity of  $G$  the maximization problem may lead to corner solutions.<sup>4</sup>

Since we assume a competitive market, in order to attract customers a movie theater must provide a collective utility equal to that offered by other movie theaters. Let  $G^a$  be this level. The theater maximizes profits subject to the condition that the value of (7) be equal to  $G^a$ . The Lagrangean for this problem is given by

$$L = (P_q - C_q) \int_{\underline{\theta}}^{\bar{\theta}} q(P_q, \theta) f(\theta) d\theta + (P_m - C_m) + \lambda \left\{ \int_{\underline{\theta}}^{\bar{\theta}} G[U^*(P_m, P_q, \theta)] f(\theta) d\theta - G^a \right\}. \quad (8)$$

Maximizing with respect to  $P_m$  and  $P_q$ , one obtains

$$\int_{\underline{\theta}}^{\bar{\theta}} q(P_q, \theta) f(\theta) d\theta = -(P_q - C_q) \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial q(P_q, \theta)}{\partial P_q} f(\theta) d\theta + \int_{\underline{\theta}}^{\bar{\theta}} q(P_q, \theta) s(\theta) d\theta, \quad (9)$$

where  $s(\theta)$  is a density function defined in the interval  $[\underline{\theta}, \bar{\theta}]$  as

$$s(\theta) = \frac{G'[U^*(P_m, P_q, \theta)] f(\theta)}{\int_{\underline{\theta}}^{\bar{\theta}} G'[U^*(P_m, P_q, \theta)] f(\theta) d\theta}. \quad (10)$$

The left-hand side of (9) is the increase in revenue resulting from a one-unit increase in the price of popcorn, and the right-hand side is the sum of the loss in profit (if  $P_q > C_q$ ) from the decline in popcorn sales (the first term) and the loss from having to reduce the entrance fee (the second term) to keep the group from going elsewhere.

Let us define  $q^s$  as the mean value of  $q$  when one uses  $s(\theta)$  instead of  $f(\theta)$  as the density function and  $q_d$  as the mean value of  $q$  when the density function  $f(\theta)$  is used. The concavity of the function  $G$  and the fact that  $U^*$  increases with  $\theta$  (recall eq. [6]) imply that the function  $s(\theta)$  has more weight in the lower values of  $\theta$  than the function  $f(\theta)$ , so  $q^s$  is smaller than  $q_d$ . We can now rewrite (9) to obtain the following expression for the markup on popcorn:

$$\frac{P_q - C_q}{P_q} = \frac{q^s - q_d}{q_d \cdot E_q}, \quad (11)$$

<sup>4</sup> In a group with two individuals the concavity of  $G$  is necessary in order for there to be convex indifference curves.

where  $E_q$  is the average elasticity of consumption:

$$E_q = \left\{ \int_{\theta}^{\bar{\theta}} \frac{\partial q[P_q, \theta]}{\partial P_q} f(\theta) d\theta \right\} \frac{P_q}{q_d} < 0. \quad (12)$$

Since  $q^s < q_d$ , it follows that the movie theater will charge for popcorn a price that exceeds its marginal cost. The result is quite general and does not depend on the properties of  $f(\theta)$ . We expect, however, that the more disperse the distribution of  $\theta$  the larger will be the markup, because there is more variability in  $G'(U)$  and greater differences in the distribution functions  $s(\theta)$  and  $f(\theta)$ .

Once  $P_q$  has been determined, the firm sets the price of the movie using the constraint on the utility of the group. For the market as a whole, both  $P_m$  and  $G^a$  are determined by the zero-profit condition

$$P_m = C_m - \int_{\theta}^{\bar{\theta}} (P_q - C_q) q(P_q, \theta) d\theta. \quad (13)$$

The market equilibrium can be more succinctly described as the package that maximizes (7) subject to the restriction that profits are zero. Any other "pseudoequilibrium" satisfying the first-order condition (9) will not prevail since all moviegoers will prefer the theater that offers the higher value of (7).

We have also developed a model in which group members vote to go to the theater of their choice, and the group goes to the theater that gets the majority of the votes. In such a case the median voter theorem applies and the theater offering the highest utility to the median consumer will be the one chosen.<sup>5</sup> Although the firm apparently chooses two prices, equation (13) implies that the price of a movie is a function of the popcorn price, so the firms can be analyzed considering the price of popcorn only. When the group has to vote between two theaters, the one that offers the highest utility to the median voter will always be the one chosen. For example, if the alternative theater charges a lower price, equation (5) implies that as  $\theta$  decreases, utility decreases faster when the group goes to the alternative theater ( $q$  is higher for all values of  $\theta$ ). In consequence the median voter and all the voters with lower values of  $\theta$  will vote against the alternative theater.

The market will choose the prices that maximize the utility of the

<sup>5</sup> The median voter is the consumer whose value of  $\theta$  is equal to the median of the distribution of  $\theta$ . Since the distribution is given, the median is uniquely defined.

median voter subject to the zero-profit condition. Let  $\theta^m$  be the median of the distribution of  $\theta$ . The Lagrangean for this problem is

$$L = U^*(P_m, P_q, \theta^m) + \lambda \left[ P_m - C_m + (P_q - C_q) \int_0^{\bar{\theta}} q(P_q, \theta) d\theta \right]. \quad (14)$$

Maximizing with respect to  $P_q$  and  $P_m$ , one obtains

$$\frac{P_q - C_q}{P_q} = \frac{q^m - q_d}{q_d \cdot E_q}, \quad (15)$$

where  $q_d$  is the mean consumption of popcorn,  $q^m$  is the median, and  $E_q$  was defined by (12).

According to (15), the firm will price popcorn above marginal cost only if the consumption of the median member is less than the mean consumption of the group. This seems likely to be the case, if for no other reason than that a significant percentage of consumers often do not buy any popcorn, skewing the distribution of consumption of popcorn to the right. In movie theaters in which the consumption of popcorn is more uniform, the markup should be smaller.

### III. Pricing Wine by Its Quality

In this section we analyze a higher degree of price discrimination in competitive markets. The firm, a restaurant, sells two goods. The meal, denoted by  $M$ , is homogeneous and is consumed by everyone entering the restaurant. The second good is the wine, and it is assumed that everyone buys a bottle of wine. There are, however, different qualities of wine, denoted by  $q$ , and consumers differ in their appreciation of quality. The restaurant's problem is to find an entire schedule relating price and quality that maximizes profits, as opposed to simply a single price as in the previous section. We assume that there is perfect competition in the restaurant industry and that all firms have identical cost curves.

The consumer's utility function is identical to the one considered previously:

$$U = X + \gamma M + \theta h(q), \quad (16)$$

where  $X$  is a composite commodity other than the ones sold at the restaurant, and  $q$  is now the quality of the wine. The function  $h$ , besides being increasing and concave in  $q$ , also satisfies the property that as  $q$  approaches zero,  $h'(q)$  tends to infinity. This new condition ensures that for all consumers with positive  $\theta$  it is optimal to consume



wine.<sup>6</sup> For convenience, we again make  $M$  equal to one. The schedule relating the price of a bottle of wine to its quality that confronts consumers is denoted by  $P(q)$ .

Let  $P_m$  be the price of the meal and  $Y$  be each consumer's income. The problem faced by each consumer is to choose the quality of wine,  $q$ , that maximizes (16), subject to the budget constraint

$$X = Y - P_m - P(q). \quad (17)$$

The optimal  $q$  must satisfy

$$\theta h'(q) = P'(q), \quad \theta h(q) \geq P(q). \quad (18)$$

The first condition gives the optimal choice of quality, and the inequality states that the consumer is better off consuming wine than not consuming.

As in the previous section, we also assume that consumers go to restaurants in groups, that all groups are identical, and that the distribution of  $\theta$  within each group is described by a density function  $f(\theta)$  defined in the interval  $[\underline{\theta}, \bar{\theta}]$ . Once again we look at the case in which the group chooses the restaurant that offers the higher value of the group's welfare function given by the left-hand side of equation (7).

Competition among restaurants ensures that each of them must offer the same collective utility. Moreover, free entry forces long-run profits to be zero. The market finds the prices that maximize

$$\int_{\underline{\theta}}^{\bar{\theta}} G[U(\theta)] f(\theta) d\theta, \quad (19)$$

subject to the condition that profits must be zero.

The optimization problem can be solved using the techniques introduced by Mirrlees (1971).<sup>7</sup> Invoking the envelope theorem, one can differentiate (13) and obtain

$$\frac{dU}{d\theta} = h(q(\theta)). \quad (20)$$

This equation is known as the "self-selection constraint." It implies that utility cannot decrease with  $\theta$  since consumers with greater appreciation for wine can always hide and choose the selection chosen by individuals with lower  $\theta$ , thereby enjoying higher utility. If the restaurant could observe  $\theta$  directly, it could extract a higher surplus from individuals with higher  $\theta$ , and constraint (20) will not hold.

<sup>6</sup> This assumption makes the analysis easier and does not appear to change any of the important implications.

<sup>7</sup> Mussa and Rosen (1978) provide an intuitive approach to these problems.

The Hamiltonian for the problem considered here is given by

$$H = G(U)f(\theta) + \lambda h(q) + \phi \{[-U(\theta) + Y - C_m + \gamma + \theta h(q)] - C_q q\} f(\theta), \quad (21)$$

where there are two state variables,  $U$  and the firm's profits  $\pi(\theta)$ , which is given by

$$\begin{aligned} \pi(\theta) &= \int_{\underline{\theta}}^{\theta} [P(q(v)) - C_q q(v) + P_m - C_m] f(v) dv \\ &= \int_{\underline{\theta}}^{\theta} \{[-U(v) + Y - C_m + \gamma + \theta h(q)] - C_q\} f(v) dv, \end{aligned} \quad (22)$$

where  $C_m$  and  $C_q$  are the cost of the meal and the cost of wine quality, which are assumed to be constant.<sup>8</sup> In this equation, we used both (16) and (17) to express the price of quality,  $P(q)$ , as a function of the state and control variables. The zero-profit condition requires that the variable  $\pi$  must satisfy  $\pi(\underline{\theta}) = 0$  and  $\pi(\bar{\theta}) = 0$ . In the Hamiltonian the variable  $\lambda$  is the costate variable associated with  $U$  and  $\phi$  is the costate variable associated with  $\pi(\theta)$ . The control variable is  $q$ . The first-order conditions for this problem are

$$\frac{\partial H}{\partial q} = 0, \quad -\frac{\partial H}{\partial U} = \frac{d\lambda}{d\theta}, \quad \frac{d\phi}{d\theta} = -\frac{\partial H}{\partial \pi}. \quad (23)$$

In addition to (23), the following transversality conditions must be satisfied:<sup>9</sup>

$$\lambda(\underline{\theta}) = 0, \quad \lambda(\bar{\theta}) = 0. \quad (24)$$

Together, (21), (23), and (18) imply the following expression for the derivative of the markup with respect to  $q$ :

$$\frac{d[P(q) - C_q q]}{dq} = P'(q) - C_q = \frac{\lambda(\theta)P'(q)}{(-\phi)\theta f(\theta)}. \quad (25)$$

It can be shown that the markup on wine increases with quality. Since  $-\phi$  is constant and negative, one needs only to prove that  $\lambda$  is never positive. The proof is sketched as follows. One can show that once  $\partial\lambda/\partial\theta$  becomes positive, it will not change sign. Thus if there is

<sup>8</sup> Even in the case of quality this is an innocuous assumption, since one can always choose the units of measurement in such a way that costs are linear.

<sup>9</sup> The condition regarding the value of  $\lambda$  when  $\theta$  reaches its minimum value holds only if there is no binding constraint on  $U$ . In this case, for all  $\theta$ ,  $U$  must exceed  $Y$ , the value of the utility function if one does not go to the restaurant.

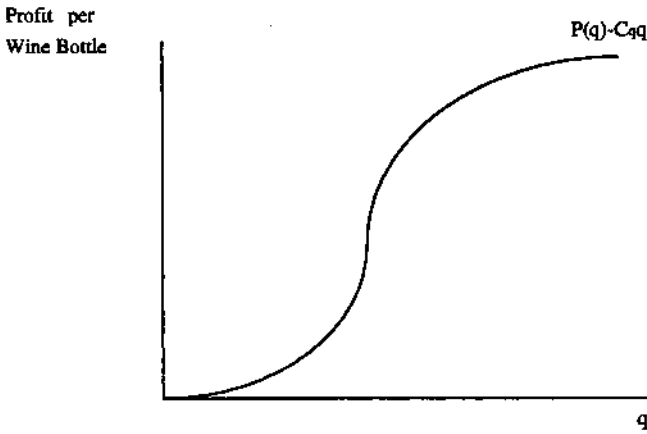


FIG. 1

a  $\theta$  such that both  $\lambda$  and  $\partial\lambda/\partial\theta$  are positive, the transversality condition on  $\lambda(\theta)$  will not be satisfied.<sup>10</sup>

The proof also implies that  $\lambda$  can be equal to zero only at the extremes, and therefore the choice of quality is distorted for all values of  $\theta$  except  $\underline{\theta}$  and  $\bar{\theta}$ . For all values of  $\theta$  in the interval  $(\underline{\theta}, \bar{\theta})$ , consumers buy wine of lower quality than the wine they would buy without price discrimination. Such a result resembles the proposition discussed in the literature on optimal taxation, where the marginal tax rate must equal zero at the lowest and highest income brackets and must be positive elsewhere (Seade 1977).

Figure 1 illustrates a markup schedule consistent with our findings. We assume that  $\underline{\theta}$  is zero. The inequality in (18) then implies that for individuals with low  $\theta$  the price of quality must be close to zero so the curve starts at the origin.<sup>11</sup> Everywhere else the markup is positive. Furthermore, it is increasing in  $q$  for all values of  $q$  different from the extremes. The zero-profit condition dictates that the revenues extracted from the consumption of wine will be used to subsidize the meal. Another interesting implication of the results is that for wines

<sup>10</sup> The increasing markup with quality is an implication that helps distinguish this model from at least one other explanation that comes to mind for the high price of wine at restaurants. The basic idea of this alternative explanation is to realize that the marginal cost of wine at restaurants may be higher than at supermarkets and liquor stores. Restaurants, after all, have to serve the wine and clean up afterward. But such costs would seem to be roughly independent of quality, and therefore the markup should be constant.

<sup>11</sup> Equation (22) and the subsequent discussion imply that the equality  $\theta_2 h(q) = P(q)$  (where  $q \neq 0$ ) can hold at most for a single  $\theta$ . For all  $\theta$  greater than  $\theta_2$ , the value of  $h(q)$  must exceed  $P(q)$ .

of very high quality the percentage markup  $[P(q) - C_q q]/C_q q$  must decrease with  $q$  because the marginal markup is less than the average.

One possible deviation from the case illustrated in figure 1 arises when the optimal markup schedule has corners, that is, when at certain values of  $q$  the function  $P(q)$  is nondifferentiable. This possibility exists because if the distribution  $f(\theta)$  has a capricious form, the solution to (21) and (23) may require that for some  $\theta$  the value of  $q$  has to decrease with  $\theta$ . This is inconsistent with the second-order conditions for the consumer's utility maximization. In such a case the constraint  $dq/d\theta \geq 0$  must be incorporated. When the constraint is binding, the price schedule has corners, and it is optimal to bunch customers of different tastes onto the same product. We do not analyze this case in detail here, but the interested reader is referred to Mussa and Rosen (1978) for a discussion of this issue in the context of a monopoly.

#### IV. Conclusion

We have shown how group purchasing can sufficiently constrain individuals to allow even competitive firms to price-discriminate. In the specific case of two-part pricing, this paper's models imply that the second part of the price—the price of popcorn or of wine—will exceed marginal cost, and the entrance fee will be lower. Not only is the markup positive, but if firms can vary it (as in the pricing of wine above), the markup will rise with quality.

Though the case is not analyzed here, it is clear that group purchasing can provide additional motivation for price discrimination on the part of monopolistic firms. Such firms could exploit differences in consumers both within and across groups. Presumably the relative importance of the two types of discrimination would depend on whether differences are greater within or across groups.

One final point about group purchasing. Some alternative decision rules for group purchases will not allow price discrimination by competitive firms. Imagine, for example, a family in which one altruistic member decides not only what firm to patronize but also the consumption of each family member, so as to maximize some family utility function. Such groups would behave as individuals, providing no possibility for firms to exploit within-group differences. While such an extreme decision rule may not be typical of groups—perhaps not even of groups consisting of families—the example does point out that certain group structures can reduce the scope for within-group price discrimination.

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