Coming to Terms with One’s Choices:
Post-Decisional Cognitive Dissonance in Rational Choice

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Abstract

This paper modifies the standard preference framework to incorporate post-decisional cognitive dissonance. It focuses on the “sour grapes” aspect of post-decisional cognitive dissonance, where preferences and even perceptions are altered due to the process of dissonance reduction. First, we propose axioms which introduce cognitive dissonance into the standard rational choice framework. From this, we derive a model that illustrates the basic principles of dissonance. Then, the model is “reality checked” against previous studies which have looked at dissonance to see if it produces the observed results. Next, the model is translated into utility terms and applied to a budget constrained choice situation. The model is then applied to explain the following observed phenomena: the “foot-in-the-door” method of inducing compliance; habit formation and the evolution of cooperation oriented norms; the high returns that can be obtained by switching jobs during one’s career; and the long term detrimental effects to introducing monetary incentives for “intrinsically motivating” activities like social service. Finally, we generalize the model to account for some additional phenomena that cannot be explained by the basic model.
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# Contents

1 Introduction 1

2 Intuition 5

3 Definitions, Axioms and the Model 7

3.1 Definitions 7

3.2 Axioms and Simplifying Assumptions 7

3.3 A Model of Dissonance Reduction 13

3.4 An Example 16

3.5 Interpreting the Model in Terms of Utilities 19

4 Applications 23

4.1 Induced Compliance: Foot in the Door 23

4.2 Habit Formation 25

4.3 Wage Contracts 27

4.4 Over-Incentivization of Social Work 31

5 Generalizing the Model of Dissonance Reduction 37

5.1 Additional Axiom 37

5.2 A New Example 38

5.3 Applications 40

5.3.1 Overestimation 40

5.3.2 Job Safety 40

6 Conclusion 42
List of Figures

2.1 Indifference Curve Alterations ........................................... 6
3.1 Resistance to Change as a Function of Preference Strength Alteration ........ 10
3.2 Mental Cost of Altering the Preference Strength between 2 Cognitions .......... 11
3.3 Variation of Mental Costs with Preference Alteration for a Revealed Partially Consistent Cognition .................................................. 15
3.4 Indifference Curve Alterations ........................................... 22
3.5 The Different Equilibria due to the Altered Indifference Curves .................. 23
4.1 Indifference Curve Alterations ........................................... 24
4.2 Indifference Curves ...................................................... 26
4.3 The Wage Contract for One Period ...................................... 28
4.4 Indifference Curve Alterations ........................................... 30
4.5 The Utility Function and Reservation Utility .................................. 33
4.6 The Altered Utility Function and Reservation Utility .......................... 35
4.7 Indifference Curve Alterations ........................................... 36
5.1 Indifference Curves ...................................................... 40
1 Introduction

In a study in 1956 [Brehm, 1956], several women were shown 8 different household appliances and were asked to rate them in order of preference. As a reward for participating in the study, they were told that they could choose between 2 of the ones they had rated as being roughly equally attractive. The chosen appliance was wrapped and given to the participant. Some time later, before the appliance had been unwrapped, each participant was asked again to rate the appliances. The participants rated the appliance they had chosen as being more attractive than before and the one they had rejected as being less attractive than before, though no new information had been gained about either one.

In another study [Knox and Inkster, 1968], researchers asked people at a race track how confident they were that they would win their bet. They asked this to people who were just about to place their bets and also to those who had just placed their bets, so that the only difference between the 2 random groups was that the latter had placed their bets (no new information had been gained). The researchers found that in general, the people who had already placed their bets thought their horses were more likely to win while the ones who had not yet placed their bets had lower perceived probability of success.

Standard utility theory is unable to explain the results of the 2 studies cited above. One of the axioms on which the standard rational man paradigm is founded is that there is a fixed preferential ordering of choices. Paul Samuelson [Samuelson, 1947] made a significant contribution to choice theory by introducing the revealed preference framework in which the underlying preference structure is “revealed” by observing the choices made by individuals in various situations. Through the work of Arrow, Houtakker, Sen and others, this line of thought was developed into a powerful theory of rational choice. Though this model provides an elegant framework with which we can look at choices, recent evidence from social psychology suggests that it might be lacking in applicability to certain observed phenomena.

The theory of cognitive dissonance [Festinger, 1957] is a powerful theory in social psychology whose incorporation into the revealed preference framework could significantly improve the applicability and explanatory power of the latter and possibly explain confounding studies like the ones cited earlier. According to Aronson [Aronson, 2004], “Cognitive dissonance is a
state of tension that occurs whenever an individual simultaneously holds two cognitions (ideas, attitudes, beliefs, opinions) that are psychologically inconsistent”. Since being in this state of tension is uncomfortable to the individual, he or she will try to reduce this feeling of discomfort. A very straightforward way of doing this would be to reverse the decision that caused the dissonance. But if the decision had been a rational one, then changing that to choose another (suboptimal) option would only cause more dissonance. Thus, revoking a decision is not a good way to reduce post-decisional dissonance when the choice is rational. Also, in a wide variety of decision scenarios, the choice is irrevocable. Another way of reducing the dissonance is by justifying to oneself the choice that led to the inconsistent cognitions. During this process of self-justification, in order to arrive at a certain “cognitive consistency” within oneself, one’s attitudes and beliefs can undergo changes which will help to lower or remove the dissonance experienced by the individual. Thus, preference structures, which are generally considered to be fixed in the standard rationality model, become malleable as the mind tries to line up stronger and stronger justifications for one’s choice. For example, if there is an inconsistency between 2 cognitions that one is holding simultaneously, the mind might alter one of those cognitions or one’s preferences over it to make it consonant with the other cognition, or it might lower the importance of one of the cognitions and through this, lower the tension experienced due to the inconsistency. Alternatively, one could bring into consideration more cognitions that are consistent with either or both of the inconsistent cognitions and hence bridge the gap between them.

Though there have been many previous papers which have looked at the effects of post-decisional cognitive dissonance in economic activity, they have considered dissonance as a fixed effect which changes the utility of an action, or preferences, in some arbitrary manner. Though this type of modeling does give results which are observed in actual life or in experiments, it does not give a complete understanding of what is happening. For instance, Akerlof and Dickens [Akerlof and Dickens, 1982] proposed a model of how people who do hazardous jobs could come to view their jobs as less dangerous due to cognitive dissonance. They then go on to examine how such an alteration would effect the adoption of safety equipment by these workers. They say that “If the cost imposed by future wrong decisions is not too great, workers in the hazardous industry will, because of cognitive dissonance, come to believe that the job
is really safe”. But upon inspection of this argument, the following objection could arise: If the workers decided to work in the hazardous industry after weighing the costs and benefits and finding a net benefit, they should not experience any dissonance as they have acted in line with their cognition that they are smart, rational, benefit maximizing people. Thus a question arises - how can cognitive dissonance occur in a rational person?

Another approach that has been taken is that which focuses on another aspect of dissonance - that of selective intake of information after a decision [Eckwert and Drees, 2005, Lévy-Garboua, 2004]. An instance where this effect is seen is after a car purchase [Ehrlich et al., 1957]. Suppose that one bought a certain model A, choosing it over model B. After the purchase, the consumer will selectively read more advertisements about her car and avoid positive information about model B. Also, she will pay more attention to the times when she sees cars of model B lying on the shoulder of the highway, awaiting repair. These actions will bolster her choice and degrade the rejected option, reducing her dissonance. While that is a way of reducing dissonance, i.e., by selectively taking in consonant cognitions, that is not the only method.

This paper will attempt to modify the standard preference framework in order to incorporate post-decisional cognitive dissonance. It will focus on the “sour grapes” aspect of post-decisional cognitive dissonance, where preferences and even perceptions are altered due to the process of dissonance reduction. First, we propose a few alterations to the standard axioms of rational choice and introduce a few axioms that account for cognitive dissonance. From this, we derive a model that illustrates the basic principles of dissonance. Then, the model will be “reality checked” against previous studies which have looked at dissonance to see if it produces the observed results. Next, the model will be translated into utility terms and applied to a budget constrained choice situation to produce results that are different from those given by standard neoclassical theory. Translation of our model from preferences to utilities makes it easily applicable to standard scenarios in economics. The model will then be applied to explain the following observed phenomena: the “foot-in-the-door” method of inducing compliance; habit formation and the evolution of cooperation oriented norms; the high returns that can be obtained by switching jobs during one’s career; and the long term detrimental effects to introducing monetary incentives for “intrinsically motivating” activities like social service. It should be noted that all these instances are applications of one basic
model. Finally, we will generalize the model by relaxing some of the initial assumptions (and
hence, all the previous results would still hold). This generalized model is able to account for
some phenomena that cannot be explained by the basic model.

Thus, by making a few modifications to the existing choice theory framework, this paper
aims to propose a model which can incorporate post-decisional cognitive dissonance, is robust
to the existing experimental findings in the field, and predicts results which, while being counter
to those predicted by standard neoclassical theory, have some empirical backing.
2 Intuition

In this section, we will attempt to give the reader an intuitive feel for what is to come. We will look at the gist of dissonance theory as will be applied in our model. It might be helpful to think of preferences that are stated below in terms of standard utility functions.

As described earlier, dissonance is a state of tension that occurs whenever an individual simultaneously holds two cognitions (ideas, attitudes, beliefs, opinions) that are psychologically inconsistent. Consider the choice between 2 bundles, \( \{a_1, b_1\} \) and \( \{a_2, b_2\} \) as shown in fig 2.1. Assume that more of each component \( a \) or \( b \) is preferred to less. Let \( a_1 > a_2 \) but \( b_1 < b_2 \). The initial indifference curves are labeled C. Assume that the first bundle is chosen as it gives the higher net utility. But this choice has resulted in the person receiving \( b_1 \) while she preferred \( b_2 \). This “partial” inconsistency will cause the person to experience dissonance. This dissonance will be reduced to an extent by the fact that \( a_1 \) was chosen over \( a_2 \) and that was in line with her preferences. But if this reduction is not strong enough, there will still be some unmitigated dissonance within her.

Given this state of tension that imposes a mental cost on her, her preferences over \( a \) and \( b \) will undergo changes in such a way that they decrease her feeling of dissonance. The importance of \( b \) will be downplayed and the slope of the indifference curves decreases with respect to \( b \). On the other hand, the importance of \( a \) will be emphasized and the slope of the indifference curves increase with respect to \( a \). This results in the pivoting of indifference curves, indicated by \( C' \), in the figure. They become flat with respect to the “rejected” \( b \), indicating a weaker preference ordering over it than before. On the other hand, they become steeper with respect to the “chosen” \( a \), indicating a stronger preference ordering over it than before. Thus, the Marginal Rate of Substitution changes as the marginal utility of \( a \) increases and that of \( b \) decreases. This alteration is important as it is long lasting and thus, will be carried over into other decision situations where either \( a \) or \( b \) is involved.

Thus, the core idea is that once a choice is made, preferences are altered in such a way as to justify that choice. It will be this same idea and figures similar to the one above that will be used throughout this paper in all the different cases, except at the end, where this idea is generalized.
Figure 2.1: Indifference Curve Alterations
3 Definitions, Axioms and the Model

3.1 Definitions

Cognition: A cognition is the mental representation of anything that is perceived by someone. It can be thought of as a quantum of information. Thus, objects, actions and beliefs, all have a mental representation when processed by the brain and these representations constitute cognitions. Since every choice is a comparison between groupings of such quanta of information, cognitions are the basic units of all choices.

Cognition Sets: A set whose elements are all the possible values of one particular type of cognition. For example, if the cognition under consideration is an apple, then, the apple cognition set is the set of all possible numbers of apples. If the cognition is the perceived probability of winning a bet, then, the cognition set would be the closed interval \([0, 1]\).

Option: Options are the entities between which a choice is made. Note that an option could even be a single element of a cognition set. For instance, in the choice between one and 2 apples, the options are 2 elements of the apple cognition set. In the choice between 2 levels of spiciness, the 2 options are 2 elements of the cognition set that contains all possible levels of spiciness that one can experience. (Thus, choice can occur between sensations like taste, and not just concrete objects or goods). The implicit assumption here is that the mind is able to hold each cognitive element independently and compare it with other similar elements.

Mental Cost: This is a feeling of discomfort or tension that could be induced due to various reasons. In this paper, cognitive dissonance and alteration of preferences are 2 factors that induce mental costs.

3.2 Axioms and Simplifying Assumptions

1. The Ordering Axiom: There exists a complete and transitive preference ordering (\(\succeq\)) among all options that an individual could face. If \(x_1 \succ x_2\), then \(x_1\) is chosen over \(x_2\). \(^1\)

2. Preference Strength and Cost of Change Axiom: The preference relation that

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\(^1\)Note that this excludes options which can never occur. For instance, this will exclude the option of going to both a movie and a play at 6 p.m. on a certain day as both these events cannot occur simultaneously, and hence this is not a possible option.
exists between any 2 options has a strength associated to it. If an option \( o_1 \) is preferred to \( o_2 \), then the preference strength for \( o_1 \) over \( o_2 \), given by \( s(o_1, o_2) \) will be positive, and the stronger the preference for \( o_1 \), the greater will be the preference strength. Similarly, if \( o_2 \) is preferred to \( o_1 \), then \( s(o_1, o_2) \) will be negative, and the stronger the preference for \( o_2 \), the more negative will be \( s(o_1, o_2) \). (Though not necessary, it might be useful to think of preference strengths between 2 options as the difference in the utilities of the 2 options). The preference relation also has associated with it a positive resistance to change, \( r(s(o_1, o_2)) \), for changing the strength from that value. (This can be compared to the friction at any point for moving an object from that point).

**Corollary 1: Total Mental Cost of Preference Strength Alteration:** The total mental cost of changing the preference strength between 2 options from an initial level \( s_{\text{initial}} \) to a final level \( s_{\text{final}} \) along a certain path is the path integral of the resistance to change of each of the values of \( s \) along the path of change. (This in turn can be compared to the total work done in moving the object from the initial point to the final point, overcoming the friction at each point along the way).

We define \( C \) to be the sum of the mental costs of preference strength alterations (for altering the preference strengths between many cognitions).

3. **Component Cognition Axiom:** Each option is a set comprising of elements from cognition sets such that if any one option has one element from a certain cognition set, then all options have an element from that cognition set. (Note that 0 is an element of all cognition sets).

**Corollary 2: Preference Ordering Among Clusters of Cognitions:** For every choice that is made between options, with each option being \( o_i = \{a_i, b_i, \ldots\} \), \( \exists \) at least one smallest subgrouping of elements of each option, \( \{m_i, n_i, \ldots\} \), such that \( \exists \) a complete and transitive preference ordering (with strengths and resistance to change attached to the preference relations) among all corresponding subgroups from each option, with this preference ordering being independent of the other elements in the options. (I.e., \( \exists \) a complete and transitive preference ordering among \( \{m_1, n_1, \ldots\}, \{m_2, n_2, \ldots\}, \ldots \)). In the case where each option is a single element from a certain cognition set, say, the
number of apples, then, the smallest subgrouping is each option in entirety.

**Simplifying Assumption #1:** We will assume that for all the choice situations and the options that we will consider in this paper, the smallest subgroupings of elements from each option that have independent preference orderings (as defined in Corollary 2) are the individual elements of each option. Thus, we are assuming that for options \( o_i = \{a_i, b_i, \ldots \}; i = \{1, 2, \ldots \} \), the smallest subgroupings of elements of that have independent preference orderings are \( \{a_i\}, \{b_i\}, \ldots \).²

**Simplifying Modeling Assumption # 2:** Consider the case where the preference strength between 2 elements of a cognition set is being altered from its original value \( s_{\text{initial}}(x_1, x_2) \), through some intermediate values, to a final value \( s_{\text{final}}(x_1, x_2) \). We assume that \( r(s_{\text{final}}(x_1, x_2)) = |s_{\text{final}}(x_1, x_2) - s_{\text{initial}}(x_1, x_2)| \) for all cognitions \( x \).

Thus, at any intermediate value of the preference strength \( s(x_1, x_2) \), we assume that the resistance to changing the preference strength from that value is proportional to the magnitude of preference strength alteration that has occurred from the initial value of the preference strength. This can be seen in fig 3.1. The more a preference strength has been altered from its original value, the harder it gets to change it further. This is to reflect the fact that the individual’s original preference strengths were the result of some prior information and its alteration would move the individual away from reality, which is disliked by the individual.

The total mental cost of altering the preference strength between \( x_1 \) and \( x_2 \) from \( s_{\text{initial}}(x_1, x_2) \) to \( s_{\text{final}}(x_1, x_2) \) is \( 0.5(s_{\text{final}}(x_1, x_2) - s_{\text{initial}}(x_1, x_2))^2 \).³ This is seen in fig 3.2.

The implicit assumption is that the mental cost of changing the preference strength between 2 elements of a cognition set is the same for any 2 elements of the cognition set, and that it is the same over all cognition sets, an assumption that can be easily relaxed if needed.

²We concede that this is a strong assumption and that there can exist cases where the options cannot be decomposed into their component cognitions where each of the components is part of an independent preference ordering. But in those cases, instead of considering each individual cognition, by looking at the smallest subgroupings for which there do exist independent preference orderings and by considering these clusters of cognitions to be the components of the options, the model can still be used.

³Since these preference strength alterations involve a positive mental cost and since the individual minimizes these costs as will follow from Axiom 5, one would expect to see preference strength alterations only along the shortest possible path, i.e., linear. Hence, we use the normal straight line integral to derive this expression.
Thus, $C$, which was defined earlier as the sum of the mental costs of altering the preference strengths (i.e., for multiple cognitions) would be:

$$C = \sum_{\text{All cognitions x with preference alterations}} 0.5(s_{\text{final}}(x_i, x_j) - s_{\text{initial}}(x_i, x_j))^2.$$ 

**Definition: Revealed Partial Inconsistency:** A choice is Revealed to be Partially Inconsistent with respect to a certain cognition iff the choice resulted in at least one rejected option that had an element from that certain cognition set and that element was strictly preferred to the corresponding element (i.e., from that same cognition set) in the chosen option. 

We use the preference ordering as defined by Assumption 1 in keeping with the propositions made by Festinger in his original work on Cognitive Dissonance. He specifically states that the cognitions are “considered by themselves”. Further research would be required to verify that this is indeed the preference relation that is actually used, and not a conditional one, which would instead depend on the other cognitions in the options.

As mentioned in footnote 2, if an element of an option is not part of an independent preference ordering, then we would consider the smallest subgrouping of elements from that option that includes that element and for which there does exist an independent preference ordering, and the Revealed Partial Inconsistency would be with respect to that subgroup of cognitions.
and \( s(x_1, x_i) < 0 \) for any \( i \neq 1 \), then the choice is Revealed Partially Inconsistent with respect to \( x \).

**Definition: Revealed Partial Consistency:** A choice is Revealed to be Partially Consistent with respect to a certain cognition iff the choice resulted in a chosen option that had an element from that certain cognition set and that element was strictly preferred to the corresponding element(s) (i.e., from that same cognition set) in all the rejected options. I.e., if \( o_1 \) is the chosen option, \( x_1 \in o_1 \), \( x_i \in o_i \) such that \( i \neq 1 \), and \( s(x_1, x_i) > 0 \forall i \neq 1 \), then the choice is Revealed Partially Consistent with respect to \( x \).

4. **Dissonance Axiom:** Whenever a choice is made between options such that it is Revealed to be Partially Inconsistent with respect to at least one cognition, there exists as a result of this choice a mental cost called cognitive dissonance that increases with the number of Revealed Partially Inconsistent cognitions and increases with the strength of the preference relations among those cognitions. If the choice is Revealed Partially

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6. Thus, we can see that for dissonance to exist in a rational choice, each of the options in the choice should
Inconsistent with respect to a cognition $x$, then there exists a mental cost called cognitive dissonance $d_x(s(x_1, x_2)) > 0$ such that $\frac{\partial d_x(s(x_1, x_2))}{\partial s(x_1, x_2)} > 0$. The total amount of dissonance experienced due to all the Revealed Partially Inconsistent cognitions is given by:

$$D = \sum_{(All \ inconsistent \ cognitions \ x)} \sum_{(All \ i \neq 1 \ s.t. \ s(x_1, x_i) < 0)} d_x(s(x_1, x_i)).$$

**Dissonance Mitigation:** The mental cost (cognitive dissonance) that is imposed by Revealed Partially Inconsistent choices is reduced by a mitigation factor (to a minimum of 0) and this mitigation increases with the number of Revealed Partially Consistent cognitions and the strength of the preference relations among those cognitions. If the choice is Revealed Partially Consistent with respect to a cognition $x$, then the amount of mitigation of cognitive dissonance is given by the function $m_x(s(x_1, x_2)) > 0$ such that $\frac{\partial m_x(s(x_1, x_2))}{\partial s(x_1, x_2)} > 0$. The total amount of mitigation experienced due to all the Revealed Partially Consistent cognitions is given by:

$$M = \sum_{(All \ consistent \ cognitions \ x)} \sum_{(All \ i \neq 1 \ s.t. \ s(x_1, x_i) > 0)} m_x(s(x_1, x_i)).$$

The remaining mental cost will be called unmitigated cognitive dissonance. We let the unmitigated dissonance $= D - M$.  

**Simplifying Modeling Assumption # 3:** We assume that if $o_1$ is the chosen option and $\exists x_1 \in o_1$, $x_2 \in o_2$ and $x_2 \succ x_1$ (and so, $s(x_1, x_2) < 0$), then $d_x(s(x_1, x_2)) = -\delta s(x_1, x_2)$ is the function that determines the amount of cognitive dissonance experienced by the individual due to the Revealed Cognitive Inconsistency, where $\delta > 0$. Thus, if $\exists x_1, y_1 \in o_1$, $x_2, y_2 \in o_2$, $x_2 \succ x_1$ and $y_2 \succ y_1$, then the total amount of dissonance created is

$$D = d_x(s(x_1, x_2)) + d_y(s(y_1, y_2)) = -\delta s(x_1, x_2) - \delta s(y_1, y_2).$$

We also assume that if $o_1$ is the chosen option and if $\exists p_1 \in o_1$, $p_2 \in o_2$ and $p_1 \succ p_2$ (and so, $s(p_1, p_2) > 0$), then $m_p(s(p_1, p_2)) = \gamma s(p_1, p_2)$ is the function that determines have at least 2 subgroupings of elements that have a complete and transitive preference ordering among the corresponding subgroupings in each of the options (as defined in Corollary 2) that is independent of the other cognitions. This is because if there was only one such subgrouping, that subgrouping would the option itself and since the choice was rational, this would mean that the choice did not result in any Revealed Partially Inconsistent cognitions and thus, there would be no dissonance. In our models and examples, we keep the number of component cognitions in each option down to this minimum of 2 for simplicity.

7 Other functional forms have also been proposed, including a “dissonance ratio” with the unmitigated dissonance $= \frac{D}{M + \delta}$.  

12
the amount of dissonance mitigation experienced by the individual due to the Revealed Cognitive Consistency, where $\gamma > 0$. Thus, if $\exists p_1, q_1 \in o_1, p_2, q_2 \in o_2, p_1 \succ p_2$ and $q_1 \succ q_2$, then the total amount of mitigation of dissonance is

$$M = m_p(s(p_1, p_2)) + m_q(s(q_1, q_2)) = \gamma s(p_1, p_2) + \gamma s(q_1, q_2).$$

5. **Mental Cost Minimization axiom**: Individuals try to minimize the total mental costs they experience at any point of time. The total mental cost experienced by an individual at any point of time is given by $TMC = D - M + C$.  

The table 3.1 summarizes the notation for the concepts and the modeling assumptions introduced so far.

### 3.3 A Model of Dissonance Reduction

This model of dissonance reduction follows from the axiomatic framework that we have set up. The gist of the process is captured by the following quotation of Festinger: “The alternative which had been chosen would seem much more attractive, and the alternative which had been rejected would seem less attractive than it had been”. This is in fact an experimentally observed phenomenon [Brehm, 1956, Knox and Inkster, 1968]. This alteration is long lasting and the new preference structure will be the one that the individual uses in future choice situations involving any or all of the cognitions involved in the current choice.

Suppose that an individual makes a rational choice that results in some cognitions, $x, y, z, \ldots$, being Revealed Partially Inconsistent. By the Dissonance axiom, this causes the decision maker to experience the mental cost of cognitive dissonance $D$. But also suppose that the choice results in the remaining cognitions $a, b, c, \ldots$ being Revealed Partially Consistent. By the Dissonance Mitigation subaxiom, this reduces the dissonance experienced by the individual by $M$ (up to a minimum of 0). If the remaining mental cost, called unmitigated dissonance,

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8This axiom automatically subsumes the “Principle of Cognitive Consistency” which states that people try to reduce the inconsistency between the different cognitions that they hold. This can be seen by noting that the inconsistencies would cause dissonance, which is a mental cost. And by the Mental Cost Minimization axiom, this cost will be reduced.

9Pg 34, A Theory of Cognitive Dissonance
<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$o_i$</td>
<td>Option i</td>
</tr>
<tr>
<td>$x_i$</td>
<td>The element of the cognition set $x$ that is in option $i$</td>
</tr>
<tr>
<td>$s(x_1, x_2)$</td>
<td>The strength of the preference for $x_1$ over $x_2$</td>
</tr>
<tr>
<td>$r(s(x_1, x_2))$</td>
<td>The resistance to changing the preference strength, $s(x_1, x_2)$, from that value</td>
</tr>
</tbody>
</table>
| $C$          | The mental cost imposed in changing the preference strengths from their initial to final levels for all cognitions  
|              | $= \sum_{\text{All cognitions } x \text{ with preference alterations}} 0.5(s_{\text{final}}(x_i, x_j) - s_{\text{initial}}(x_i, x_j))^2$ |
| $d_x(s(x_1, x_2))$ | The amount of dissonance created when the choice resulted in a “partial cognitive inconsistency” in the cognition $x$ |
| $D$          | The total amount of dissonance created by the choice. Thus, it involves all the individual $d_x$’s of each “inconsistent” cognition  
|              | $= \sum_{\text{All inconsistent cognitions } x} \sum_{\text{All } i \neq 1 \text{ s.t. } s(x_1, x_i) < 0} -\delta s(x_1, x_i)$ |
| $m_x(s(x_1, x_2))$ | The amount of mitigation of dissonance created when the choice resulted in “partial cognitive consistency” in the cognition $x$ |
| $M$          | The total amount of mitigation of dissonance created by the choice. Thus, it involves all the individual $m_x$’s of each “inconsistent” cognition  
|              | $= \sum_{\text{All consistent cognitions } x} \sum_{\text{All } i \neq 1 \text{ s.t. } s(x_1, x_i) > 0} \gamma s(x_1, x_i)$ |
| $TMC$        | The total mental cost experienced by the individual at a point of time $= D - M + C$ |

Table 3.1: Notation Summary
is greater than 0, then, by the Mental Cost Minimization axiom, the individual will try to remove this mental cost.

From our axioms, we see that one of the ways in which this can be achieved is by altering the preference strengths between the cognitions (both Revealed Partially Inconsistent as well as Consistent). This is because the dissonance $D$ and mitigation $M$ are determined by the preference strengths between the cognitions involved. But by Preference Strength and Cost of Change axiom, this alteration will impose new mental costs on the individual. Again, by the Mental Cost Minimization axiom, the individual will try to minimize the total mental cost experienced by her at any point of time. This process will continue until a minimum total mental cost is achieved such that any alteration of preference strengths from that point would only increase total mental costs. This is seen graphically in Fig. 3.3. for one Revealed Partially Consistent Cognition. (From our simplifying assumption, note that $D - M$ and $C$ will be convex.

![Figure 3.3: Variation of Mental Costs with Preference Alteration for a Revealed Partially Consistent Cognition](image)

Figure 3.3: Variation of Mental Costs with Preference Alteration for a Revealed Partially Consistent Cognition
The final levels of preference strengths that will result from this process can be obtained by a Lagrangian optimization. The function to be minimized would be the total mental cost at any point of time, which would be a summation of the remaining unmitigated dissonance at that time and the mental cost imposed by the preference strength alterations up to that point. The constraint would be that $D - M \geq 0$. Thus, the rational choice made by the individual causes predictable preference alterations. One will come to justify one’s choice in such a way that she will have a stronger preference for the option and cognitions that were chosen in a consistent way and a reduced strength for the cognitions that were chosen in an “inconsistent” way.

This alteration can be looked at from the viewpoint of preference strength alterations in the following manner. If there were no costs to changing the preference strengths, i.e., if $C = 0$, then we would expect the preference strength of the “inconsistent” cognition to become less negative as this would lower $D$ (as $\frac{\partial d}{\partial s(x_1, x_2)} > 0$) and thus, unmitigated dissonance $D - M$. Similarly, for the “consistent” cognition, the preference strength would increase as this would raise $M$ (as $\frac{\partial m}{\partial s(x_1, x_2)} > 0$) and thus lower unmitigated dissonance. Now, if costs were imposed for changing preference strengths, to minimize total cost, we would expect as little preference strength change as possible for getting any given amount of reduction in unmitigated dissonance. Since change of preference strength in any direction (i.e., increase or decrease) has a positive cost, we would expect to see the strengths to move in the same direction as they did when there were no costs, but in a way that minimizes the changes, and hence the costs. The value of $s(x_1, x_2)$ will increase in the positive direction for cognitions that were “Revealed Partially Consistent” while it will become less negative for the “Revealed Partially Inconsistent” cognitions. Movement in any direction other than this would only increase costs.

Note that the model depends only on the axioms, and not on the simplifying assumptions.

3.4 An Example

A simple demonstration of the model will be given by means of the following example. Consider a choice between a bundle consisting of 3 apples and 2 bananas and another bundle consisting of 2 apples and 3 bananas. We will assume that 3 apples are preferred to 2 and 3 bananas are
preferred to 2. We will also assume that \( \{3a, 2b\} \succ \{2a, 3b\} \) where \( a \) stands for apples and \( b \) for bananas. \( s(a_1, a_2) \) is the strength of the preference between the 2 amounts of apples \( a_1 \) and \( a_2 \). Let \( s(3a, 2a) = 3 \) in this case. \( s(b_1, b_2) \) is the strength of the preference between 2 amounts of bananas \( b_1 \) and \( b_2 \). Let \( s(2b, 3b) = -2 \). Finally, given a choice between these 2 bundles, the rational person will choose \( \{3a, 2b\} \), and the choice will be “overall” consistent. The strength of the preference for option 1 (i.e., \( \{3a, 2b\} \)) over option 2 (i.e., \( \{2a, 3b\} \)) is \( s(o_1, o_2) \). Let \( s(o_1, o_2) = s(a_1, a_2) + s(b_1, b_2) = 1 \). \(^{10}\)

Though this choice is rational, the choice was Revealed Partially Inconsistent with respect to bananas as \( s(b_1, b_2) = -2 < 0 \). By the Dissonance axiom, the individual experiences cognitive dissonance that is captured by the expression \( D = -\delta s(2b, 3b) \). \( D \) exists only as long as \( s(2b, 3b) \leq 0 \). Since the choice was Revealed Partially Consistent with respect to apples, the dissonance is mitigated (by mitigation subaxiom). This mitigation is captured by \( M = \gamma s(3a, 2a) = 3 \). So, the unmitigated dissonance would be \( D - M \). For ease of calculation, we will let \( \delta = 4 \) and \( \gamma = 1 \). \(^{11}\) So, \( D = 8, M = 3, D - M = 5 \).

According to the Mental Cost Minimization axiom, the individual will try to minimize this unmitigated dissonance. This will be done by an alteration of the strength of the preference ordering between individual cognitions. But such a change imposes a mental cost by the Preference Strength and Cost of Change axiom. By the Mental Cost Minimization axiom, she acts in a way that minimizes the TMC experienced at any point of time. This process brings unmitigated dissonance as low as possible (\( \geq 0 \)). The optimal preference strengths can be calculated using a Lagrangian optimization, minimizing \( TMC \) subject to the constraint that \( D - M \geq 0 \).

We will now derive the expressions for \( D - M \), \( C \) and \( TMC \). For notational simplicity, we let \( s_{\text{initial}}(a_1, a_2) = s_{1a}, \ s_{\text{final}}(a_1, a_2) = s_{2a}, \ s_{\text{initial}}(b_1, b_2) = s_{1b} \) and \( s_{\text{final}}(b_1, b_2) = s_{2b} \). At

\(^{10}\)We will assume for analytical simplicity that there are no interaction effects between the apples and the bananas.

\(^{11}\)It would be interesting to investigate how the proposed preference alterations would be different for different values of \( \delta \) and \( \gamma \).
any one point of time:

\[ D - M = -4s_{2b} - s_{2a} \]

\[ C = 0.5(s_{2a} - s_{1a})^2 + 0.5(s_{2b} - s_{1b})^2 = 0.5(s_{2a} - 3)^2 + 0.5(s_{2b} + 2)^2 \]

\[ TMC = -4s_{2b} - s_{2a} + 0.5(s_{2a} - 3)^2 + 0.5(s_{2b} + 2)^2 \]

The constraint is that \( D - M \geq 0 \), i.e., \(-4s_{2b} - s_{2a} \geq 0\).

To account for the inequality in the constraint, we will introduce a slack variable \( p \) such that the constraint becomes \(-4s_{2b} - s_{2a} \geq p^2\). Now, we do a Lagrangian optimization to obtain our optimum \( s_{2a} \) and \( s_{2b} \).

\[ L = -4s_{2b} - s_{2a} + 0.5(s_{2a} - 3)^2 + 0.5(s_{2b} + 2)^2 + \lambda(-4s_{2b} - s_{2a} - p^2) \]

Calculations (please refer to Appendix A for details) show that the optimum solution is:

\[ s_{2a} = 3.294, \quad s_{2b} = -0.824, \quad s_{2o} = 2.470 \]

This result is exactly as predicted. The strength of the “consistent” cognition, i.e., \( s_a \), increased from 3 to 3.294 while that of the “inconsistent” cognition, i.e., \( s_b \), became less negative from -2 to -0.824. Also, the strength of the preference for the chosen option over the rejected option, i.e. \( s(o_1, o_2) \), also increased from 1 to 2.470.

Finally, we will look at how the mental cost experienced by the individual has changed. The initial mental cost imposed by the unmitigated dissonance on the individual was \( D - M = 8 - 3 = 5 \). After undergoing the preference strength alteration, the total mental cost incurred was

\[ -4s_{2b} - s_{2a} + 0.5(s_{2a} - 3)^2 + 0.5(s_{2b} + 2)^2 = 0.735. \]

This resulted in the removal of the dissonance (as at the equilibrium, \( p = 0 \)).

With this model, we are now able to explain the first example that was cited in the introduction - the one where participants changed their preference rating over appliances after making a choice. After the initial choice where the participants chose an appliance that was almost equally preferable to another, they would have experienced dissonance for rejecting the other one. To reduce this dissonance, they would have altered their preferences in such a way that the chosen appliance was rated as more preferable than before and the rejected one as
less preferable. This alteration would lower the dissonance experienced.

3.5 Interpreting the Model in Terms of Utilities

The existence of a complete and transitive preference ordering among all elements of a cognition set would mean that we could use a utility function that assigns each element (i.e., each level of the cognition) a number such that if one level of cognition is preferred to another, it will have a higher utility than the other. We could let the strength of the preference be proportional to the difference in utility between the 2 cognition levels. The cost of changing the preference strengths would be the cost of changing the utility function.

With these slight modifications to the previous framework, we will now use the model proposed earlier in an example of rational decision making where the modification gives results which are different from those given by a standard rationality approach. We will use this as a numerical workout to demonstrate how exactly the indifference curves would change given certain assumptions for utility functions. So, we will use a ham-fisted approach here, assigning specific functional forms and numerical values for computational clarity.

Note: The different results are proposed to be over and above any information effects (revealing of heretofore unknown information about the good) created due to the choice. Thus, we will assume that such effects do not exist. Such effects may be included into the example as an extra factor.

2 Goods, 2 Periods of Choice

Let x and y be 2 goods (which are cognitions too by definition) with a joint utility function given by $U(x, y) = x^{0.5} + y^{0.5}$. Also, let the individual utility function for each good be $U(x) = x^{0.5}$, $U(y) = y^{0.5}$. As before we will assume that there are no interaction effects (as the joint utility is just the sum of the 2 individual utilities). Let the income be $I$.

Period 1: Free Choice: Suppose that the individual is given the option of having either 4 units of x or 4 units of y. Since she is indifferent between the 2 options, she randomizes her choice. Assume she chooses x. Putting the options into our framework:

$$o_1 = \{4x, 0y\}, \quad o_2 = \{0x, 4y\}$$
\( \alpha_1 \) is the chosen option. So,
\[
 s(4x, 0x) = U(4x) - U(0x) = 4^{0.5} = 2, \quad s(0y, 4y) = U(0y) - U(4y) = -(4^{0.5}) = -2
\]

At the end of period 1, she will experience some dissonance due to the fact that she rejected \( 4y \) and chose \( 0y \) which was against her initial preference ordering for \( y \). Using the same functions as in the earlier model with \( \delta = 4, \gamma = 1, \)
\[
 D = -4 \times s(0y, 4y) = -4(U(0y) - U(4y)) = 4 \times 2 = 8
\]
\[
 M = 1 \times s(4x, 0x) = U(4x) - U(0x) = 2
\]
\[\text{Unmitigated Dissonance} = D - M = -4 \times s(0y, 4y) - s(4x, 0x) = 6\]

To reduce and maybe even remove this dissonance, she will change her preference strengths. Using the same functional form as before for the total mental cost of preference strength alterations, \( C \), we note that the mental cost of changing \( s(4x, 0x) \) from \( s_{\text{initial}}(4x, 0x) \) to \( s_{\text{final}}(4x, 0x) \) is
\[
0.5(s_{\text{final}}(4x, 0x) - s_{\text{initial}}(4x, 0x))^2.
\]
The corresponding mental cost for \( y \) is
\[
0.5(s_{\text{final}}(0y, 4y) - s_{\text{initial}}(0y, 4y))^2.
\]

We will apply the same procedure as before to minimize the total mental cost, as defined by the sum of the unmitigated dissonance at any point, and the mental cost incurred in altering the preference strengths to that point. This will be done given the usual constraint, i.e., \( D - M \geq 0 \).

So, the function to minimize would be:
\[
\text{TM} = -4 \times s_2(0y, 4y) - s_2(4x, 0x) + 0.5(s_2(4x, 0x) - s_1(4x, 0x))^2 + 0.5(s_2(0y, 4y) - s_1(0y, 4y))^2
\]
where \( s_1 \) is the initial and \( s_2 \) is the final preference strength. Since we know the initial preference strengths, we substitute them into the expression. So, \( s_1(4x, 0x) = 2 \) and \( s_1(0y, 4y) = -2 \). We let \( s_{2x} = s_2(4x, 0x) \) and \( s_{2y} = s_2(0y, 4y) \) for simplicity.
\[
L = -4s_{2y} - s_{2x} + 0.5(s_{2x} - 2)^2 + 0.5(s_{2y} + 2)^2 + \lambda(-4s_{2y} - s_{2x} - p^2)
\]
The Lagrangian Optimization would give us the following values for the preference strengths:
(Please refer Appendix C for details)
\[
s_{2x} = 2.353, \quad s_{2y} = -0.588
\]

Thus, we are no longer indifferent between the 2 options. Since we defined preference strength
between 2 cognition levels as the difference in their utilities, the altered preference strength
would imply altered utility functions. The alteration in preference strength between the 2
values of x (from 2 to 2.353) is captured by an alteration in the individual utility function for
x. (We assume that the function changes for all values of x). For simplicity, we will consider
the linear scaling alteration of utility functions. (This transformation meets the criteria for
VNM utilities).
Letting \( U'(x) = \alpha x^{0.5} \) signify the new utility function for x, we have

\[
s_{2x} = s_{final}(4x, 0x) = U'(4) - U'(0) = \alpha 4^{0.5} - \alpha 0^{0.5}
\]
\[
\Rightarrow 2.353 = 2\alpha \Rightarrow \alpha = 1.177
\]

Similarly, letting \( U'(y) = \beta y^{0.5} \) signify the new utility function for y, we have

\[
s_{2y} = s_{final}(0y, 4y) = U'(0) - U'(4) = \beta 0^{0.5} - \beta 4^{0.5}
\]
\[
\Rightarrow -0.588 = -2\beta \Rightarrow \beta = 0.294
\]

And the new joint utility function will be

\[
U'(x, y) = 1.177x^{0.5} + 0.294y^{0.5}
\]

The indifference curves for such an alteration are shown in Figure 3.5. Here, C represents the
indifference curves before the alterations of \( s \)'s and C' represents the altered ones. We see
that the indifference curves pivot in such a way that they become steep with respect to the
consonant cognition \( x \) while flat with respect to the dissonant cognition \( y \). This alteration
is long lasting and will be carried over into other decision situations where either of these
cognitions are involved.

**Period 2: Budget Constraint:** In the second period (which we will assume was
not foreseen by the individual when making the initial choice), the individual faces a budget
constraint. The total income available to be spent on purchasing \( x \) and \( y \) is $10. Also, the
prices of \( x \) and \( y \) are both assumed to be $1. With the new utility function that we obtained
earlier, this would lead to the following Lagrangian:

\[
L = 1.177x^{0.5} + 0.294y^{0.5} + \lambda(10 - 1x - 1y)
\]
A straightforward constrained optimization would give us the optimal amounts of $x$ and $y$ as follows (please refer to Appendix C for details):

$$x = 9.413, \quad y = 0.587$$

We will now compare these numbers to the standard case where no dissonance reduction would have taken place, and thus, we would have had the original utility function. Since the original utility function was symmetric for $x$ and $y$ and since the prices of both $x$ and $y$ are the same, the optimum amounts of $x$ and $y$ would also have been the same. In fact, the optimum would have occurred when the $10 was split equally between them, meaning,

$$x = 5, \quad y = 5$$

We can see how our results are very different from this symmetrical result. The different equilibria in the 2 cases are seen in Figure 3.6.
4 Applications

4.1 Induced Compliance: Foot in the Door

Jonathan Freedman and Scott Fraser conducted a study [Freedman and Fraser, 1966] in which they looked a technique of inducing compliance. In the control condition, they went up to homeowners and asked them if they would agree to put up a large and obtrusive sign, that asked people to drive safely, on their front lawn. As expected, most people refused. But in the experimental condition, they went up to homeowners and asked them if they would agree to place a small sticker on their windows which asked people to drive safely. Most people agreed to this request. The researchers then went back to these people later and asked them if they would be willing to put up the large sign on their lawn. A much larger proportion of the people agreed to that request in this condition. This technique, called the foot-in-the-door technique, is the method of getting someone to do something which they would have rejected if they had been asked directly, but would agree to once they have agreed to some related but
smaller request earlier.

This study and technique can be explained using our model. The initial decision of agreeing to place a sticker on one’s window would produce some dissonance, and it would be removed by altering one’s preferences in such a way that one comes to have a stronger preference for supporting the “drive safely” cause, and would care less if that intruded into one’s personal space - as shown in Figure 3.4. With this altered preference structure, the individual is more likely to agree to the other request, which involves a high commitment to the cause but also a very large invasion of space.

![Indifference Curve Alterations](image)

Figure 4.1: Indifference Curve Alterations

One interesting implication of this is that gradual changes toward some final objective might be more acceptable to the public than a sudden one-shot change. This can be seen intuitively as follows - suppose that a policy change desired by the government is very different from status-quo. Also, suppose that it is so different that a one-shot introduction would involve rejection by the public. But instead, if an intermediate policy is introduced that is tending towards the final policy but is not as drastically different from status-quo, then the
public would be more willing to accept it. But once that choice is made, by the dissonance reduction mechanism, the preferences of the public change in such a way that they become more supportive of policies that are different from status-quo but in the direction of the intermediate policy. Once this has happened, if the final policy is introduced, we would see much higher acceptance rates. The same principle could be applied to the introduction in the market of a new product that is very different from existing products.

4.2 Habit Formation

From our utility theory example in Chapter 3.5 where the individual’s choice in period 1 altered her choice in period 2, we can see that the choice in the first period in some ways led to a “habit” formation for the individual. For instance, this framework could be used to explain the often observed phenomenon of an individual “acquiring” a taste for an expensive item that she initially disliked. If she decided to go ahead and pay the large amount of money for the good she disliked (say Cuban cigars, whose taste she disliked) just to meet “social status requirements”, she would experience dissonance for having paid the money for something she did not like intrinsically. But our model would predict that after the choice has been made, her preferences for the item will alter in such a way that she will come to actually like the item, hence removing the dissonance. Then, the next time she is faced with a choice of whether or not to purchase the expensive item, she is more likely to do so. She has “acquired” a taste for the item.

This model can also be used explain the evolution of cooperative norms in societies. In this explanation, the proposed effect is in addition to any information effects. Suppose that an individual is faced with a choice between 2 options, one of which has a high monetary payoff but involves cheating on the other player while the second option has a lower monetary payoff but is cooperative and does not involve cheating. Also, suppose that the individual values being considered an honest player. Now, if the cooperative option gave a higher utility overall to the individual (combining both the monetary and “moral” utilities), and thus chose that option, after the choice, she would experience dissonance for having rejected the option with the higher monetary payoff. This is mitigated by her choice of the option with the higher moral
payoff. To remove the unmitigated dissonance, the individual would alter her preferences in such a way that she will come to place more importance on the “moral” payoff and reduce that of the “monetary” payoff. This would cause her indifference curves to alter as shown in fig 4.2.

![Figure 4.2: Indifference Curves](image)

Repetitions of such choices and the ensuing dissonance reduction could alter the preferences for the “moral” and “monetary” payoffs to a great extent, with the “moral” payoff being given a great deal of importance (i.e., its utility being increased greatly). Such a preference structure would in turn lead the individual to choose options with higher moral payoff while foregoing options with large monetary payoffs. An example of such behavior is that of an individual leaving a tip at a restaurant in a foreign town just before flying out. Here, she can gain monetarily by choosing not to pay the tip. But the moral payoff of leaving the tip (or rather, the negative payoff or regret of not leaving a tip) is so great that even though no one (except the waitress of course) would know whether she left a tip or not, she will choose to do so. Such a preference structure is also useful for cooperation in deals where cheating on
those deals could have given the cheater a greater monetary payoff. This is because cheating
would involve a serious negative moral payoff which could outweigh the monetary gains. Thus,
the above mentioned preference alteration could be a mechanism by which cooperative norms
evolve.

4.3 Wage Contracts

One of the puzzles in labor economics is that individuals can gain a large wage increase by
switching jobs. Though the standard explanation for this is that there are large costs to
switching jobs, empirical estimates for these switching costs reveal very large numbers - per-
haps too high for this explanation to be the only one. For instance, according to a recent
paper [Yamaguchi, 2007], “...a typical ten year experienced male worker needs 22% permanent
wage premium to be fully compensated for a career change”. According to the author, “[An]
open question is why the direct costs of job changes are high”. Also, Altonji and Williams
[Altonji and Williams, 2005] say that their “main conclusion is that the returns to seniority
are modest”.

We will attempt to account for at least a part of this wage differential by introducing
cognitive dissonance into an instance of wage contract theory. In this scenario, we will assume
that neither the employer nor the worker can foresee either the dissonance which will occur after
the first period or the dissonance reducing preference changes. This is also in line with most
studies conducted which tend to show that the effects of dissonance are generally unanticipated
by the individual.

Consider the following 2 period contract game. At the beginning of each period, the
employer offers a contract to a worker, with wage = $w$. (We assume that the employer can
observe the level of effort put in by the worker with complete accuracy). The work involves
an effort $e = 2$. The worker can then choose whether or not to accept the offer. The wage will
be paid after effort has been observed by the employer. We assume that $e = 2$ produces $10
worth of output. Assume that the disutility of effort = 2. The reservation wage for the worker
is $1. We will use very simple utility functions for analytical simplicity. For the employer,
$U(\pi) = \pi$ where $\pi$ is the profit earned. For the worker, $U(w, e) = w - e$. Once again, we will
assume that there are no interaction effects.

\[ w - 2 \geq 1 \Rightarrow w \geq 3. \]

To maximize the payoff for the employer, \( w \) should be low.

\[ \Rightarrow \text{Optimal } w = 3; \quad \pi_E = 7 \text{ (Actually, optimal } w \text{ will be just above } 3 \text{ and } \pi_E \text{ will be just below } 7). \]

We will obtain the same equilibrium in period 2 too, as nothing would have changed from period 1.

Now we introduce cognitive dissonance into this model. Since nothing changes in period 1, the results are still the same as what they were without cognitive dissonance. But at the end of period 1, the worker will experience dissonance for having worked hard and put in the effort that caused disutility. The 2 options that the worker faced are:

\[ o_1 = \{w, e\}, \quad o_2 = \{1, 0\} \]

Let \( s(w, 1) = U(w) - U(1) = 3 - 1 = 2, \ s(e, 0) = U(e) - U(0) = -2 \)

\[ D = -\delta * s(e, 0), \ M = \gamma s(w, 1) \]
Also, we assume that the mental cost of changing preference strengths has the same functional form as before. We let $0 < \gamma < \delta$ so that $D - M > 0$ in order for there to be dissonance reduction. Notice how this formulation is similar to that in the Utility Example in Ch. 3.6 (end of Period 1). So, we will obtain similar results as in that case after optimization of preference strengths. We will look at the case where $\delta > 2$. In this case, the Lagrangian optimization would cause all dissonance to be removed and we would have the following values for the preference strengths:

$$s_{\text{final}}(w, 1) = \frac{2\delta(\gamma + \delta)}{\delta^2 + \gamma^2}, \quad s_{\text{final}}(e, 0) = \frac{-2\gamma(\gamma + \delta)}{\delta^2 + \gamma^2} \Rightarrow \frac{s_{\text{final}}(w, 1)}{s_{\text{final}}(e, 0)} = \frac{-\delta}{\gamma}$$

Now, if we let $\delta = 4, \gamma = 1$ as before,

$$s_{\text{final}}(w, 1) = 2.353, \quad s_{\text{final}}(e, 0) = -0.588$$

Furthermore, for simplicity, we will assume that the employer does not experience much dissonance.

The alteration in preference strengths would imply an alteration in the utility functions, by definition. As before, we will use the linear scaling alteration of utility functions. Letting $U'(w_i) = \alpha w_i$ signify the new utility function for $w_i$, we have:

$$s(w_h, 1) = s_{\text{final}}(w_h, 1) = U'(3) - U'(1) = \alpha * 3 - \alpha * 1 = 2\alpha$$

$\Rightarrow 2.353 = 2\alpha \Rightarrow \alpha = 1.177$

Similarly, letting $U'(e_i) = \beta e_i$ signify the new utility function for $e_i$, we have:

$$s(e_h, 0) = s_{\text{final}}(e_h, 0) = U'(2) - U'(0) = \beta * 2 - \beta * 0$$

$\Rightarrow -0.588 = 2\beta \Rightarrow \beta = -0.294$

And the new joint utility function will be

$$U'(w_i, e_i) = 1.177 w_i - 0.294 e_i$$

The alteration is captured by Fig. 4.4. This result is in line with the observation that long time workers come to attach a value to working hard (especially in their particular job), discounting
the effort involved. This tendency to discount the disutility of effort is captured by the new utility function.  

Thus, \( U'(w, e) = 1.177w - 0.294e \) will be the utility function that will be used by both the employer and the worker in period 2. (We are assuming that the employer is aware of the worker’s disutility of effort as well as the worker’s utility from wages in both periods). Assume the employer can offer another contract now, with a different \( w \) but with the same \( e = 2 \) as before.

So, in period 2 the participation constraint is

\[
U'(w, e) \geq U'(1, 0) \Rightarrow 1.177w - 0.294e \geq 1.177 \times 1 - 0 \Rightarrow w \geq 1.502
\]

\( \Rightarrow Optimal \ w = 1.502 \). Profit for the employer is increased to $8.498.

Thus, the introduction of cognitive dissonance lowers the optimum wage required in

\[1\] Another possibility is that the preference alteration will only happen for the preference strength with regards to the disutility of effort as that would seem more malleable than the preference strength between different amounts of money. If that were the case, the entire dissonance reduction would be transferred onto the effort term and hence, it would experience a much larger alteration in preference strength. But the following results would remain qualitatively similar.
period 2 and increases the employer’s profit. Though this is a counter-intuitive result, one has to remember the assumptions we have made. We have assumed that the productivity of the person does not increase with time and that her reservation wage is low and unchanging. But if those assumptions are relaxed, the prediction from our model could be modified to say that after the first period, the employer can offer a wage that is lower than what would be required if the productivity increase and other factors are taken into account. Thus, the real wage might not increase as much as it should. ²

Though this is still seems to be a counter-intuitive result, it could be seen as an explanation for the wage differential puzzle stated earlier. If the effect that we have postulated is considered in addition to switching costs, then the estimates of the switching costs that are required to account for the wage differentials might be at a more plausible level. Also, our result could explain the finding of Altonji and Williams [Altonji and Williams, 2005] stated earlier, that the returns to seniority are modest. Thus, though our result need not be the only explanation for the puzzling differential described above, it could definitely be a factor in it.

4.4 Over-Incentivization of Social Work

Now, we will apply our model to the case of providing monetary incentives to people to induce them to do more social work. Standard economic theory would predict that providing more incentives would only increase the supply of people willing to do to job, as the payoff exceeds the reservation utility of more and more people. But many studies have found the inverse effect [Frey and Oberholzer-Gee, 1997, Mellström and Johannesson, 2005, Titmus, 1970]. In many instances involving social work, civic duty or charitable work, introduction of a remuneration scheme significantly decreased the number of people who agreed to do the job. There have been a lot of papers which have dealt with this issue [Benabou and Tirole, 2000, Deci, 1975, Frey and Oberholzer-Gee, 1997, Frey and Jegen, 2001], and a model grounded in the standard rationality paradigm will suffice to explain it.

But there is another puzzle in this scenario. What would happen once the monetary incentives stop?

²An actual nominal wage cut will not work in reality as this will introduce many other cognitions, such as “I am being extorted”. Instead, we are saying that the real wage increase would be lower than what it would be by the standard model.
incentives were removed? Standard rationality would predict that the situation would revert to the way it was before the incentives were introduced. But some empirical studies [Deci, 1975, Deci and Flaste, 1996] have shown that in fact, once the incentives are removed, even fewer people would volunteer. We will now propose a very simple model in which normal rationality theory would suffice to explain the first effect (introduction of monetary incentives causing fall in labor supply). But then, we will apply our dissonance reduction model to this framework and produce a prediction - that paying people do to the task will result in fewer people doing the task when the payment is removed, compared to the number of people who would have done the task before the payment was introduced. This prediction is thus able to explain the second puzzle.

Consider a task which has significant intrinsic motivation attached to it - like social service, civic duties or charitable work. We will now propose a simple model for this task. Assume that the utility obtained from the task is given by the following function:

$$U = \alpha (\text{perceived amount of social service}) + \beta (\text{payment received}) - \text{effort}$$

Now, assume that the more payment the person receives for the task, the less she will consider it to be social service.

$$\text{perceived amount of social service} = \frac{k}{\text{payment received} + 1}$$

(We introduced the +1 in the denominator to remove the asymptote at the point of no payment).

$$\Rightarrow U = \frac{\alpha k}{\text{pay} + 1} + \beta \text{pay} - \text{effort}$$

To make the model simple, let $\alpha = 1, \beta = 1, \text{effort} = 1, k = 3$. Also, let $y$ denote the amount of money received.

$$\Rightarrow U = \frac{3}{y + 1} + y - 1$$

Let reservation utility = 1.5. Also, assume that the utility function is separable additively.

Thus, we have a situation where $U > 1.5$ for $0 \leq y < 0.5$ and $y > 1$ (Figure 4.5). This model is able to explain the first “abnormal” scenario described above, where the introduction of payment reduced the number of people willing to do a task. If an individual had a utility function similar to the one we described above, then she would have done the task when $y = 0$ as at that point, $U = 2$. But if a payment between $0.5 < y < 1$ is introduced, then $U < 1.5$ and she will no longer do the task. She will have to be paid significantly higher ($y > 1$) to
make her do the same task that she would have done for no pay. But according to this model, once the payment is removed, she should still do the task as her utility at no pay is above reservation utility.

Now, we will introduce cognitive dissonance into this picture. Assume that a policy has been introduced that pays people to do social service. Let $y = 2.1$. At this pay, the utility that our person (with the utility function described earlier) derives from accepting the money and doing the task will be higher than what she would derive from rejecting the money and doing the task. Here, we are assuming that the person can either take the $2.1 or nothing. She cannot decide to take a smaller amount of money.

Let $x$ denote the perceived amount of social service. So, when she rejects the money, the perceived level of social service is high; $x = 3$. But if she accepts the money, the perceived level of social service is low; $x = 0.968$. Now consider her 2 options:

Option 1 = $o_1 = \{x_1 = 0.968, y_1 = 2.1\}$  
Option 2 = $o_2 = \{x_2 = 3, y_2 = 0\}$

She will choose option 1 as it gives her more utility. Using the same functional forms for

---

These are the best and second best options. For instance, doing nothing would give her only her reservation utility of 1.5 which is lower than the utility either option.
preference strengths and dissonance (with $\delta = 4, \gamma = 1$), we have

\[
s(x_1, x_2) = U(x_1) - U(1) = -2.032
\]
\[
s(y_1, y_2) = U(2.1) - U(0) = 2.1
\]
\[
D = -\delta * s(x_1, x_2) = -4 * s(x_1, x_2) = 8.128
\]
\[
M = \gamma s(y_1, y_2) = s(y_1, y_2) = 2.1
\]
\[
D - M = 6.026
\]

Also, we let the mental cost of changing preferences have the same functional form as before. The Lagrangian Optimization would give us the following values for the preference strengths:

(Please refer Appendix C for details)

\[
s_{final}(y_1, y_2) = 2.353, \quad s_{final}(x_1, x_2) = -0.588
\]

The alteration in preference strengths would imply an alteration in the utility functions, by definition. As before, we will use the linear scaling (Figure 4.6). Letting $U'(x_i) = \alpha x_i$ signify the new utility function for $x_i$, we have

\[
s(x_1, x_2) = s_{final}(x_1, x_2) = U'(0.968) - U'(3) = \alpha * 0.968 - \alpha * 3 = -2.032\alpha
\]
\[
\Rightarrow -0.588 = -2.032\alpha \Rightarrow \alpha = 0.289
\]

Similarly, letting $U'(y_i) = \beta y_i$ signify the new utility function for $y_i$, we have

\[
s(y_1, y_2) = s_{final}(y_1, y_2) = U'(2.1) - U'(0) = \beta * 2.1 - \beta * 0
\]
\[
\Rightarrow 2.353 = 2.1\beta \Rightarrow \beta = 1.120
\]

And the new joint utility function will be

\[
U'(x_i, y_i) = 0.289x_i + 1.120y_i - 1
\]

Putting it solely in terms of $y$, we get:

\[
U' = \frac{0.867}{y + 1} + 1.12y - 1
\]
From Figure 4.7, we note that in this case too, the indifference curves pivot in the same manner as before. But in this case, since one of the cognitions \((x)\) is determined by the level of the other cognition, not all points on the indifference curves are attainable. In fact, the only attainable points are those where the curve \(x = \frac{3}{y+1}\), shown in the figure by the bold, curved line, crosses the indifference curves. This is because the values of one cognition are constrained by the value of the other cognition according to that very relationship.

Now, if the payment scheme is removed, we find that \(U' = -0.132\). Thus, the person would no longer do the task if the payment is removed. In a sense, the policy to induce more social work has backfired as it had reduced the intrinsic motivation of the person to do the task. This result is in line with certain empirical studies [Deci, 1975, Deci and Flaste, 1996] and theoretical predictions including the literature on “Motivation Crowding Theory” [Frey and Jegen, 2001]. Benabou and Tirole (2000) say that “extrinsic rewards may have hidden costs as stressed by psychologists in that they undermine intrinsic motivation. As a result, they may be only weak reinforcers in the short run, and become negative reinforcers once withdrawn”. Our model is able to give an explanation for why “intrinsic motivation” could be killed by over-incentivization. This is a result which could have
significant policy implications for implementation of monetary incentives, especially for tasks which have a significant “intrinsic motivation” component.
5 Generalizing the Model of Dissonance Reduction

Though our model is able to explain the results of some of the studies which involved preference alterations, there is another class of studies that it cannot explain. A good example of such a study would be the second one mentioned in the introduction, involving the perceived probability of winning a bet at the race-track. Here, rather than the preference strength between the cognitions, it was the cognition itself which was altered. (The cognition is the perceived probability of winning the bet). This leads us to another method of dissonance reduction, that of cognition alteration. We can generalize the earlier model to include this feature by adding another axiom and fleshing out the definition of a cognition a little.

In Chapter 3, we defined a cognition as a quantum of information. By making the plausible assertion that people are aware of their preferences, these preferences themselves become classified under cognitions. Then, our earlier model becomes an instance where one type of cognitions, namely, preferences, are altered. This can be considered a specific case of a more general model where cognitions in general are malleable. So, our earlier model could be thought of as the case where the mental costs of altering cognitions other than preferences were prohibitively high. This seems very plausible because so far, all the cognitions that we have considered have been very clearly defined things or concepts, whose values could be ascertained in an objective fashion. But we will now relax this assumption of prohibitively high mental costs for cognition alterations. Note that all our earlier results will hold for the general model too. But first, we add a new axiom.

5.1 Additional Axiom

6. Cognition Cost of Change Axiom: Each element of a cognition set has associated with it a positive resistance to changing it from that value. This, in essence, means that coming to believe that a cognition is something else than what one perceived it as being initially involves a mental cost.

Corollary 3: Total Mental Cost of Cognition Alteration: The total mental cost of changing an element of a cognition set from the original element $x_1$ to another element $x_2$ in the same cognition set along a certain path is the path integral of the resistance to
change at each of the values of $x$ along the path of change.

In this general model, the mental costs associated with dissonance remain the same.

5.2 A New Example

We will now introduce a new choice situation which will be the example on which we will apply our general model. In order to demonstrate the changes that occur from our earlier model, we will only look at the additional effects of this model. Thus, we will assume that the mental costs of altering preferences are prohibitively high in this example. Thus, when we use the term “cognitions” from now on in this example, we are referring to the alterable cognitions, i.e., the cognitions minus preferences.

**Example 2:** Consider the choice between 2 options:

1. Pay $2 for a 0.5 probability of winning $10.
   
   i.e., $o_1 = \{a_1 = \text{probability } 0.5\text{ of winning }$10, $b_1 = -$2\} \ (\text{E(winnings)}=$3)$

2. Pay nothing and win nothing.
   
   i.e., $o_2 = \{a_2 = \text{probability } 0\text{ of winning }$10, $b_2 = $0\} \ (\text{E(winnings)}=$0)$

⇒ Option 1 $\succ$ Option 2 and so, $o_1$ is chosen.

Let $s(a_1, a_2) = 20(a_1 - a_2)$; $s(b_1, b_2) = 2(b_1 - b_2)$; $s(o_1, o_2) = s(a_1, a_2) + s(b_1, b_2)$.

As before, assume that the interaction effects do not exist. Also, similar to preference strength alterations, assume that when the value of a cognition is being altered from its initial value, through some intermediate values, to a final value, the farther the changed value is from the initial, the higher will be the resistance to changing it from that value. Specifically, we let $r_a = 10|a_{1\text{ final}} - a_{1\text{ initial}}|$, where $a_{1\text{ initial}}, a_{1\text{ final}} \neq 0$. If $a_{1\text{ initial}}$ or $a_{1\text{ final}} = 0$, the cost of changing it from that value is assumed to be $\infty$. So, probability of 0 is a discontinuity in the cognition set, because it is very hard to convince oneself that there is even a tiny chance of winning the bet if one has not placed it in the first place. But once it is placed, the probability of winning as perceived by the bettor becomes malleable. Also, for cognition b, the resistance to change of the cognition can be assumed to be $\infty$ too, as it is much harder to think of
the price paid for the lottery as malleable, compared to the probability of winning. Thus, 
\[ r(b_i) = \infty. \]

We use the same functional forms as before and let \( \delta = 4, \gamma = 1. \)

Unmitigated dissonance \( = D - M = -\delta * s(b_1, b_2) - \gamma(a_1, a_2) = -4 * s(b_1, b_2) - s(a_1, a_2) \)
\[ = -8(b_1 - b_2) - 20(a_1 - a_2) = 16 - 20a_{final} \]

Mental cost of altering cognition \( a_1 \) from \( a_{initial} \) to \( a_{final} = C \)
\[ = 10(a_{final} - a_{initial})^2 = 10(a_{final} - 0.5)^2 \]

Constraints: \( D - M \geq 0 \Rightarrow 16 - 20a_{final} \geq 0 \) and \( 0 < a_{final} \leq 1 \)
Function to Minimize: Total Mental Cost \( = D - M + C \)
\[ = 10(a_{final} - 0.5)^2 + 16 - 20a_{final} = 10a_{final}^2 - 30a_{final} + 18.5 \]

The minimum for this function, given the constraints, is at \( a_{final} = 0.8. \) (Please refer to Appendix B for details). From Fig 5.1, we notice that the indifference curve scenario in this case is different from before. Here, the curves have not pivoted (as we have assumed that preference alteration is prohibitively costly). Instead, the perceived option (the option that was chosen) has shifted to a different indifference curve.

This is again in line with our expectations, and the results of the Knox & Inkster study [Knox and Inkster, 1968] about horse betting that we discussed in the introduction. The expected probability of winning increases after the bet was placed due to dissonance reducing mechanisms. Also, 
\[ s(a_{final}, a_{final}) = 8, \ s(b_{final}, b_{final}) = -4 \] and \( s(o_{final}, o_{final}) = 4 \) compared to 1 initially. Thus, one comes to prefer the choice even more strongly.

It should also be mentioned that this is the simplest case of this general model - wherein only 1 cognition has been altered. If we allow for the alteration of more cognitions, we would expect the perceived options to shift further to different indifference curves. Furthermore, if preferences had been allowed to alter, then, in addition to this shifting of the options, one would also obtain the by-now-familiar pivoting of the indifference curves that we had seen in the initial model.

\footnote{The expression for C is similar to that done earlier for the total cost of altering a preference strength from one level to another. Note that the cost of changing \( a_2 \) would be \( = \infty \) as it is 0. Similarly, the costs of changing \( b_1 \) and \( b_2 \) would also be \( = \infty \) due to their respective resistances. Thus, in our optimization, only \( a_1 \) will vary.}
5.3 Applications

5.3.1 Overestimation

Using this model, we are able to explain the horse betting study mentioned in the introduction. This could also be a possible explanation for the commonly observed phenomenon of people having inflated expectations of success once they have invested significantly in a choice [Friesen and Weller, 2006]. Thus, it could also be used to explain speculative bubbles.

5.3.2 Job Safety

As mentioned earlier in the introduction, Akerlof and Dickens [Akerlof and Dickens, 1982] talk about how a laborer who chose to work in a hazardous occupation would, at the end of the first period, come to believe that the job is pretty safe. We can explain this effect using our second model. Here, the actual perception of how safe the job is has been altered. Thus, we have to use the second model. Once the worker chooses the dangerous job, she experiences dissonance for having put herself at risk while she would have preferred not to. This dissonance is mitigated.
by the fact that the pay is higher for the dangerous job. The remaining unmitigated dissonance will be reduced or removed by cognition alteration as in the general model (and maybe even by preference alteration, using the initial model). She will come to believe that the job is not as dangerous as she had thought, and this would help lower her dissonance. Such effects have been observed in people who work in dangerous places [Ben-Horin, 1979].
6 Conclusion

We have shown that the introduction of cognitive dissonance into the rational decision making process is able to explain some counter-intuitive yet commonly observed phenomena. We have also made a few non-standard predictions about the result of certain strategic interactions and the impact of certain policies.

But our approach is a very basic one at most. We have used only very basic functional forms for the dissonance and mitigation. The utility functions implicitly assumed that there were no interaction effects between the different cognitions. While converting the model into utility terms, we assumed the existence of utility functions, even after preference alterations. This would imply that the preferences have to be transitive even after change. Also, we have assumed that alterations happen to the entire utility functions. This need not be the case and we could have changes in the functions at localized points. But in general, this assumption of ours seems plausible, though further investigation is necessary to verify it.

Furthermore, we have only focused on one type of dissonance - the post-decisional. Consideration of other forms of dissonance could also lead to some very interesting results. For instance, with this same basic structure, we could consider the type of dissonance that is induced when an individual comes to realize something bad about her action, but after the action has been committed. It could also be adapted to consider other forms of dissonance reduction, like selective intake of new information. This will be a nice analytical continuation of some existing work, like that of Lévy-Garboua [Lévy-Garboua, 2004]. We have also left out certain factors that determine dissonance, like the degree of irrevocability of the decision.

Even though we have not addressed the above issues, we have, with a very basic model, still generated some results that seem plausible. We hope that this simple model can work as a platform for further work in this direction.
Appendices

Appendix A:
Model 1 : Example Optimization Calculations

\[
\begin{align*}
\frac{dL}{dx} &= -1 + s_{2m} - 2 - \lambda = 0 \Rightarrow \lambda = s_{2m} - 3 \\
\frac{dL}{dy} &= -4 + s_{2y} + 2 - 4\lambda = 0 \Rightarrow \lambda = \frac{s_{2y} - 2}{4} \\
\Rightarrow s_{2d} &= 4s_{2m} - 14 \\
\frac{dL}{dp} &= 0 \Rightarrow p^2 = -4s_{2d} - s_{2m} = -4(4s_{2m} - 14) - s_{2m} = -17s_{2m} + 56 \\
\frac{dL}{dp} &= -2p\lambda = 0 \Rightarrow p = 0 \text{ or } \lambda = 0
\end{align*}
\]

If \( p = 0 \) (i.e., constraint is 0, i.e., all dissonance is removed):

\[
\begin{align*}
p^2 &= 0 = -17s_{2m} + 56 \\
\Rightarrow s_{2m} &= 3.294 \\
\Rightarrow s_{2d} &= -0.824 \\
\Rightarrow s_{2o} &= s_{2m} + s_{2d} = 2.470
\end{align*}
\]

If \( \lambda = 0, \lambda = 0 = \frac{s_{2d} - 2}{4} \Rightarrow s_{2d} = 2. \) But this violates \( s_d \leq 0 \) and so, the case of \( \lambda = 0 \) is not applicable.

Appendix B:
Model 2 : Example Optimization Calculations

Function to minimize: \( 10a_1^2f_{final} - 30a_1f_{final} + 18.5 \). This function has a minimum at \( a_1f_{final} = 1.5 \). But since \( 0 < a_1f_{final} \leq 1, a_1f_{final} \leq 1 \). Function is decreasing in the region \( (0,1] \). But at \( a_1f_{final} = 1, D - M = 16 - 20 \neq 0, D - M = 0 \Rightarrow a_1f_{final} = 0.8 \). Thus, given the constrains, the function is minimized at \( a_1f_{final} = 0.8 \).

Appendix C:

2 Goods, 2 Periods of Choice : Optimization Calculations for Dissonance Reduction

\[
\begin{align*}
\frac{dL}{dx} &= -1 + s_{2m} - 2 - \lambda = 0 \Rightarrow \lambda = s_{2m} - 3 \\
\frac{dL}{dy} &= -4 + s_{2y} + 2 - 4\lambda = 0 \Rightarrow \lambda = \frac{s_{2y} - 2}{4} \\
\Rightarrow s_{2y} &= 4s_{2x} - 10 \\
\frac{dL}{dp} &= 0 \Rightarrow p^2 = -4s_{2y} - s_{2x} = -4(4s_{2x} - 10) - s_{2x} = -17s_{2x} + 40 \\
\frac{dL}{dp} &= -2p\lambda = 0 \Rightarrow p = 0 \text{ or } \lambda = 0
\end{align*}
\]

If \( p = 0 \) (i.e., constraint is 0, i.e., all dissonance is removed):

\[
\begin{align*}
p^2 &= 0 = -17s_{2x} + 40 \\
\Rightarrow s_{2x} &= 2.353 \\
\Rightarrow s_{2y} &= -0.588
\end{align*}
\]

If \( \lambda = 0, \lambda = 0 = \frac{s_{2y} - 2}{2} \Rightarrow s_{2y} = 2. \) But this violates \( s_y \leq 0 \) and so, the case of \( \lambda = 0 \) is not applicable.

Lagrange Optimization Calculations in Period 2:

\[
\begin{align*}
L &= 1.177x^{0.5} + 0.294y^{0.5} + \lambda(10 - 1x - 1y) \\
\frac{dL}{dx} &= 0.5885x^{-0.5} - \lambda = 0 \Rightarrow \lambda = 0.5885x^{-0.5} \\
\frac{dL}{dy} &= 0.147y^{-0.5} - \lambda = 0 \Rightarrow \lambda = 0.147y^{-0.5} \\
\Rightarrow 0.5885x^{-0.5} &= 0.147y^{-0.5} \Rightarrow x = \frac{(0.5885)^2}{0.147} = 16.0272y \\
\frac{dL}{dx} &= 0 \Rightarrow 10 - x - y = 0 \Rightarrow 10 - 16.0272y - y = 0 \Rightarrow y = 0.587296 \Rightarrow x = 9.4127
\end{align*}
\]
Bibliography


