Modeling Creditor Interactions in Complex Bankruptcies

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Chapter 1

Kmart as an Illustrative Case

Chapter 11 reorganization fulfills an important function in the economy: facilitating the most efficient disposal of the assets, tangible and intangible, of an insolvent or troubled company. As a result, bankruptcy also provides the best returns for the firm’s creditors and investors. Any policy or regulation which affects the returns to investors deserves careful consideration as it will affect new investment. One such legal regulation is the treatment of companies composed of multiply incorporated entities. Such cases arise often as firms, for various reasons, form subsidiaries to conduct aspects of their business. The treatment of such companies in bankruptcy is not rigidly prescribed by law. At the filing, the separate entities continue as such; however, it is within the bankruptcy court’s power to ”substantively consolidate” the subsidiaries. This nullification of corporate boundaries can have significant effects on the ultimate recovery made by various classes of creditors. This alternative procedure treats all creditors of the firm and its subsidiaries as equals. By using a model of the bargaining amongst the creditors, I analyze the effects of these essentially opposite treatments of the corporate entity on compromises reached by the creditors.

Kmart Corporation’s 2002 bankruptcy is a good example of how the court’s treatment of subsidiaries affected the bargaining. This example, which sparked my interest
in the issue, introduces the type of cases I intend to analyze. The motivations and opportunities of the creditors in this case are, I believe, representative of those of most similarly situated creditors. Moreover, the strategic progression of the case provides a primer on how the alternative legal standards affect the treatment of subsidiaries, and correspondingly affect the outcome reached.

Kmart filed for bankruptcy protection and reorganization under Chapter 11 on January 22, 2002. A bankruptcy filing essentially freezes the finances of the bankrupt company in order to assess the claims of the creditors and resolve them. At the time of Kmart’s filing there were two primary classes of creditors: (i) prepetition lenders, mostly banks, whose claims arose from previous loans to Kmart and its subsidiaries; and (ii) trade creditors: vendors, service providers and landlords whose claims arose at the time of filing from unfinished business with Kmart, but generally not with the subsidiaries. The assets and equity of Kmart and a number of Kmart of [] subsidiaries (Kmart of Indiana, etc.) were to be divided amongst these competing classes of creditors.

The prepetition lenders were owed more than $1 billion. These prepetition lenders had received guarantees on these loans to Kmart from the Kmart of [] subsidiaries. These subsidiaries did not owe any debt to third parties other than the prepetition lenders because of Kmart’s business structure. Kmart contracted with trade vendors for the goods, which were then transferred to Kmart subsidiaries. The disclosure statement explaining the ultimate agreement succinctly lays out the prepetition lender’s claim:

In short, the prepetition lenders hold substantially all of the prepetition claims against the Kmart of [] subsidiaries. The prepetition lenders therefore have asserted that they believe they are entitled to substantially all of the value in the Kmart of [] subsidiaries, and that [their] claims thus must be paid in full, before any value attributable to the assets of Kmart
of [] subsidiaries may be upstreamed to Kmart on account of its ownership interests in these subsidiaries.(Butler,2003,p.68)

They sought a rigorous enforcement of the existing corporate structure. This would result in the default treatment of claims in the bankruptcy of a complex company: structural subordination.

To put the assertion in general terms: prepetition lenders have debt claims against the subsidiaries as well as the primary firm while trade creditors have debt claims against the primary firm and only have access to the subsidiary assets because the primary firm owns the subsidiary firms. Because debt claims supersede ownership claims, prepetition lenders would be entitled to the assets of the subsidiaries before trade claims are addressed. Kmart’s trade creditors, hoping for a greater recovery countered the prepetition lender’s claim; requesting the boundaries of Kmart’s several corporate entities be disregarded, and their assets pooled. Their assertion corresponds to the second treatment of complex firms in bankruptcy: substantive consolidation. The trade creditors argue that because they dealt with Kmart and its subsidiaries as a single economic unit, the assets of all Kmart firms should be consolidated into one estate from which to resolve all claims.(Butler,2003,p. 71) The effect of this consolidation would be to negate the structural subordination as it is a legal act which substantively removes the ”structure” to which the term refers.

Ultimately, in the Kmart case an agreement was reached in which the prepetition lenders agreed to less than what they might have received if the pure structural subordination had been upheld, while the trade creditors received somewhat more. It is clear that while expediency certainly played a role, uncertainty about the legal decision with regard to corporate structure also played a part in the compromise. The authority to substantively consolidate exists as part of the court’s power to approve a plan of reorganization under Section 1123(a)(5)(C) of the Bankruptcy code. However, there exists no prescribed standard within the Bankruptcy code for
when to consolidate. (Butler, 2003, p. 70) Various precedents have identified a number of justifications for consolidation which are explored below. However, these guidelines are varied in their structure and application, and therefore may have offered little insight to the involved parties. While the trade creditors did treat Kmart as a single unit, the prepetition lenders argue that they saw fit to contract with the Kmart of subsidiary firms. Without further speculation as to the parties’ expectation, it is at least clear that they chose a certain compromise over an uncertain juridical outcome.
Chapter 2

Legal Justification: Substantive Consolidation and Corporate Disregard

Mary Elisabeth Kors undertook an expansive review of the scholarship and case law surrounding substantive consolidation which I have found invaluable. Substantive consolidation falls in a class of treatments with equitable subordination (which reduces claims of related entities to the level of equity) and piercing the corporate veil (which allows the recovery of assets from related entities). These remedies constitute what is called “corporate disregard law” which seeks to address wrongful use of the corporate structure to harm creditors. The remedies, each in a different way, selectively “disregard” the partition of incorporation and allow for the recovery of assets from owners who would otherwise be protected by limited liability. (Kors, 1998, p.3)

Throughout the 19th century with the expansion of corporations, new laws progressively limited liability on the part of shareholders. With the advent of limited liability, courts began finding cases which called for disregarding this shareholder protection. In analyzing a number of such cases in 1912, Professor I.M. Wormser drew the fol-
lowing generalization:

When the conception or corporate entity is employed to defraud creditors, to evade a statute, to achieve or perpetuate monopoly, or to protect knavery or crime, the courts will draw aside the web of entity, will regard the corporate company as an association of live, up-and-doing, men and women shareholders, and will do justice between real persons.

(Wormser, 1912, p. 517)

In the cases examined by Wormser, the effective ”merger” of two bankrupt debtors: a precursor to substantive consolidation, was among the remedies applied by the court. From these ideas the first clear cases in which courts applied substantive consolidation arrived. In the 1941 Sampsell v. Imperial Paper & Color Corp., decision the Supreme Court prescribed substantive consolidation as the remedy to the fraudulent transfer of assets to an associated entity prior to bankruptcy.(1941) While the case is often pointed to as the basis for substantive consolidation, it poses more difficulties than explanations: if the wrong was the fraudulent transfer, why not order piercing of the corporate veil to recover the asset? A year later the Fourth Circuit consolidated a subsidiary and parent in Stone v. Eacho. The court noted the subsidiary was a ”mere instrumentality” of the parent and pointed to a set of criteria for determining instrumentality laid out in the earlier Tenth Circuit decision Fish v. East.(Stone v. Eacho (In re Tip Top Tailors, Inc.), 1942)

Courts continued to apply substantive consolidation, using a variety of tests, sometimes without reference to precedent. However, in her study of substantive consolidation, Kors notes four classes of standards: Alter Ego/Factors Analysis, Balancing Test, Difficulties of Disentanglement and the Augie/Restivo Test.(Kors, 1998, pp. 7-10) These standards will provide a survey of the scope of justifications for substantive consolidation without becoming mired in the multitude of cases and the specific tests applied.
**Alter Ego/Factors Analysis:**

These analyses focus on the traditional justification of alter ego. The tests seek to identify if the companies at issue were "alter egos", "one entity" or if the subsidiary is a "mere instrumentality" of the parent. In *Fish v. East*, this issue of instrumentality was recognized as a question of degree and provided factors for determining the interrelation of the entities. The practice of developing such factors became popular and a number of these checklists were proposed. The factors set forth in *In re Vecco Construction Industries, Inc.* have been used in over 20 substantive consolidation cases and are representative:

1. The degree of difficulty in segregating and ascertaining individual assets and liability;

2. The presence or absence of consolidated financial statements;

3. The profitability of consolidation at a single business location;

4. The commingling of assets and business functions;

5. The unity of interests and ownership between various corporate entities;

6. The existence of parent and intercorporate guarantees on loans; and

7. The transfer of assets without formal observance of corporate formalities.

*(in re Vecco Construction Industries Inc., 1980)*

Although the Vecco factors and like tests introduce pure bankruptcy concerns (difficulties of disentanglement primarily arise with bankruptcy), they are largely concerned with the demonstration of "alter ego" status to justify disregard for the corporate structure. Other courts have used this "alter ego" question as a portion of a more complicated balancing test of the type explored below.
Balancing Test:

A balancing test of one sort or another is the most popular method for justifying substantive consolidation. These tests shift criteria toward an ends based analysis, weighing the harm from consolidation against the harm from maintaining the corporate entity, or alternatively, the benefits of consolidation against the costs of consolidation. Kors, in her review of the consolidation decisions decided by balancing tests, offers a summary of the points to be balanced:

For substantive consolidation:

- The fact that one entity is the alter ego of another (i.e., excessive unity among the entities (typically as evidenced by a factor test));

- The expectations of creditors (i.e., did creditors of the entity seeking consolidation rely on the unity of the entities);

- The difficulties of disentanglement (i.e., the assets and liabilities of the subject entities are so intermingled that disentangling them would be impossible or prohibitively expensive);

- Other administrative benefits resulting from consolidation (i.e., the increased likelihood of successful reorganization, the reduction of administrative expense); and

- The misappropriation of one entity’s assets by another (or analogous incurrence of liabilities).

Against substantive consolidation:

- The prejudice and harm to creditors of the wealthier entity, which is virtually always measured in reliance (i.e., did these creditors rely to their detriment on the independent status of the ”wealthier” debtor). (Kors, 1998, p.8)
Courts have offered varying interpretations, differing on the extent to which benefits of consolidation must outweigh the harms therein and how transfers between creditors arising out of the consolidation are to be considered. These tests have for the most part integrated many aspects of the alter ego tests in assessing the existence of a substantial identity between entities, but have added the pragmatic concern of benefits and harms to creditors as well as the administrative gains from consolidation.

**Difficulties of Disentanglement:**

While "difficulties of disentanglement" appear both in tests of the Vecco type and in balancing tests, they have also been used as a stand alone justification for consolidation. In deciding *Chemical Bank v. Kheel*, the court focused largely on difficulties of disentanglement as a sufficient condition for consolidation. (1966) Courts have applied widely varying measures to these difficulties. While many courts are relatively lenient, others require disentanglement to be so costly as to threaten any recovery by the involved creditors before justifying consolidation.

**Augie/Restivo Test:**

The Augie/Restivo Test developed in *Union Savings Bank v. Augie/Restivo Banking Co.,Ltd.* suggests that substantive consolidation tests are merely a variant of two critical factors: (i) creditors expectations; and (ii) whether the assets and liabilities of the entities are hopelessly intermingled.

The test focuses on the expectations of creditors at the time of their contracting with the entities, consolidating when the entities were dealt with as a single unit and refusing consolidation if the creditors extended credit "on the basis of the financial status of a separate entity." (Union Savings Bank v. Augie/Restivo Banking Co.,Ltd., 1988) The decision specifically refers to interest rates set by creditors based on these contractual expectations and the potential damage substantive consolidation may do
to the efficiency of capital markets. This standard allows consolidation for entangle-
ment sparingly, demanding the consolidation be to the benefit of all creditors.
Chapter 3

The Creditor Bargaining Model

Douglas Baird is the most prominent author of the previous academic work examining the effects of legal rules on bargaining in bankruptcy. He models the effects of legal rules as affecting the bargaining environment, creating or denying exit options and perhaps affecting the impatience of each party. He uses this model to analyze the effect of the absolute priority rule and the new value exception on the interaction between a senior creditor and manager/owner. (Baird, 1991)

Initially my hope was to use Baird’s model to analyze the effects of structural subordination and substantive consolidation on the outcomes of the class of complex bankruptcies in which they occur. However, for a number of reasons Baird’s model fails to capture the bargaining dimension across which the legal rules I am interested in operate. Primary among the problems is that these rules affect the relative positions of various classes of creditors in the final outcome. Therefore the corresponding bargaining is likely to occur amongst these creditors rather than between one creditor and the manager. What’s more, Baird’s model is premised on a manager/owner while many of the complex firms involved in bankruptcies are publicly held. While Baird’s model could not serve to analyze my problem, his insight that bargaining models could predict the effects of legal rules on bargaining in bankruptcy seems valid.
Although my model captures specifically the relevant strategies, it is essentially the same competitive interaction: two parties seeking to maximize the return on their claims in the face of costs to not reaching agreement. I have constructed a model to analyze how the prospects of the alternative treatments of subsidiaries affect the outcomes of bargaining amongst creditors, specifically prepetition lenders and trade creditors.

### 3.1 The Simple Iteration

The initial model captures the interaction in its most basic iteration for simplicity. This model forms the basis for the more comprehensive model I discuss in the following section. The essential layout of the game theoretical model is as follows:

**The Firm:**

In the simplest case I consider a single firm with a single subsidiary. The relevant parameter in each case is the value of the assets; the value of the primary company is $F_P$ while the value of the subsidiary firm I call $F_S$. For ease of calculation and for analysis it may become helpful to consider the relative size of the firm and its subsidiary, I therefore define:

$$F_S = \alpha F_P$$  \hspace{1cm} (3.1)

Throughout the future iterations of the model I maintain the assumption of only one subsidiary. While a firm may in fact have numerous subsidiaries, as long as the debt contracts are the same for each, the relevant value remains the value of all subsidiaries, and in more complex cases $F_S$ may simply be thought of as this sum of subsidiaries.
The Players:

In the simple model, the primary players are the creditors amongst whom the firm and its subsidiaries will be divided. The prepetition lenders are those that have security agreements on their claims with the subsidiaries. The claims of the prepetition lenders has the value $C_{PL}$. The trade creditors are owed money due to ongoing commercial relationships at the time of bankruptcy. This class enjoys only an ownership claim on the subsidiaries through their debt claims against the primary company. Their claims have the value $C_{TC}$. Because the firm is insolvent we assume, for the time being:

$$C_{PL} + C_{TC} > F_P + F_S = (1 + \alpha) * F_P$$  \hspace{1cm} (3.2)

Also, for this example I assume that the claim of the prepetition lenders can not be satisfied by the subsidiaries alone:

$$C_{PL} > F_S = \alpha * F_P$$  \hspace{1cm} (3.3)

In addition to the two primary players, there is a judge who is not technically a player because he acts in a probabilistic way, deciding in the absence of agreement either to consolidate or not and divides the company accordingly. In the simple model he merely decides to uphold structural subordination with probability $p$ and consolidate with probability $(1 - p)$.

Bargaining and Payoffs:

In the simplest model I consider a single offer with acceptance or rejection and judgment. First, the prepetition lender offers a division of the total value of the firm
and its subsidiary to the trade creditor. The offer is of the form:

\[(\Pi_{TC}, \Pi_{PL}) = (x, (1 + \alpha) \cdot F_P - x)\]  \hspace{1cm} (3.4)

where \(x\) is said to be the offer. The trade creditor may then accept the offer or bring a motion to consolidate the company. If the motion is brought, the judge upholds the structure with probability \(p\) and consolidates with probability \((1 - p)\).

In the case of consolidation, the payoffs are easily calculated. The entire value of the firm and subsidiary is divided merely by weight of claims so all creditors receive the same percentage recovery; the payoffs to trade creditors and prepetition lenders are respectively:

\[(\Pi_{TC}, \Pi_{PL}) = \left(\frac{(1 + \alpha) \cdot F_P \cdot C_{TC}C_{TC} + C_{PL}}{C_{TC} + C_{PL}}, \frac{(1 + \alpha) \cdot F_P \cdot C_{PL}C_{TC} + C_{PL}}{C_{TC} + C_{PL}}\right)\]  \hspace{1cm} (3.5)

The case of structural subordination provides somewhat more complicated payoffs. In this case the prepetition lenders receive the full value of the subsidiary up to the value of their claim. Assuming their claims are not satisfied by the subsidiary firms, the parent firm is then divided based on the weights of the remaining prepetition lender’s claim and the trade creditor’s claim, or:

\[(\Pi_{TC}, \Pi_{PL}) = \left(\frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P}, \frac{F_P \cdot (C_{PL} - \alpha \cdot F_P) + \alpha \cdot F_P}{C_{TC} + C_{PL} - \alpha \cdot F_P} + \alpha \cdot F_P\right)\]  \hspace{1cm} (3.6)

In the case of an accepted bargain the payoffs are of course defined by the offer. However, the court outcomes may serve to define the bargaining space in which we may expect to find an outcome. A prepetition lender will never offer more than the amount the trade creditor could get in consolidation, and the trade creditor will accept no worse than his payoff under structural subordination. The offer \(x\) will than
be bounded as follows:

\[
\frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P} < x < \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}}
\]  

(3.7)

Our goal is to seek a subgame perfect Nash equilibrium\(^1\) in which an offer is made based on \(p\) and that offer is accepted. We may calculate the expected profit of the trade creditor in court:

\[
E(\Pi_{TC})_{\text{Court}} = p \cdot \frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P} + (1 - p) \cdot \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}}
\]  

(3.8)

\(x = E(\Pi_{TC})_{\text{Court}}\) is the only offer to which neither creditor strictly prefers the expected court outcome. Assume some pairs of creditors are in fact playing:

\[
(S_{PL}, S_{TC}) = (x = E(\Pi_{TC})_{\text{Court}}, \text{Accept for } x \geq E(\Pi_{TC})_{\text{Court}})
\]  

(3.9)

These players are indifferent between this agreed upon outcome and the expected court outcome. No equilibrium exists in which the chosen strategies strictly dominate all alternatives because I have yet to introduce additional costs associated with litigation. The motivation to negotiate rather than go to court is the first complication I add to the model.

### 3.2 Adding Costs

In the costless litigation case above risk aversion on the part of the trade creditors would create an equilibrium. However, in the real world, it is more than merely this risk aversion which drives creditors to strike compromises and avoid court. Litigation imposes a number of pecuniary costs on the parties. Additionally there are likely to be opportunity costs incurred if assets are not optimally employed or even devalue while

\(^1\)See Appendix 1
the firm languishes in bankruptcy proceedings. Any cost triggered by the refusal of an offer and a move to court will encourage the parties to agree. This cost of litigation would include the non-pecuniary loss from uncertainty stemming from risk aversion. For this reason, it is unnecessary to add a diminishing utility function with respect to revenue, to capture the effects of risk aversion.

**Legal Costs**

The most easily modeled costs would be those of the litigation itself. For simplicity, I will use a uniform cost $C_L$ applied to the recovery of both prepetition lenders and trade creditors. How do these modifications to the structure of our model affect the recoveries, and perhaps more importantly do they drive the parties to a sub-game perfect Nash equilibrium? In determining the effects of the trial costs on the offer we must first calculate the new recovery faced by trade creditors under substantive consolidation:

$$
\Pi_{TC} = \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}} - C_L
$$

Figure 3.1: The Simple Model
and under structural subordination:

$$\Pi_{TC} = \frac{F_P \ast C_{TC}}{C_{TC} + C_{PL} - \alpha \ast F_P} - C_L$$ \hfill (3.11)

giving:

$$E(\Pi_{TC})_{\text{Court}} = p \ast \frac{F_P \ast C_{TC}}{C_{TC} + C_{PL} - \alpha \ast F_P} - C_L$$

$$+ (1 - p) \ast \frac{(1 + \alpha) \ast F_P \ast C_{TC}}{C_{TC} + C_{PL}} - C_L$$

$$= p \ast \frac{F_P \ast C_{TC}}{C_{TC} + C_{PL} - \alpha \ast F_P} + (1 - p) \ast \frac{(1 + \alpha) \ast F_P \ast C_{TC}}{C_{TC} + C_{PL}} - C_L$$ \hfill (3.12)

A comparison of equation 3.12 to equation 3.8 shows that the expected recovery of the trade creditor is merely $E(\Pi_{TC})$ from results of the previous model less the legal cost. The prepetition lender then stands to enhance his recovery by $C_L$ by making an offer equal to the trade creditor’s expected recovery in court:

$$x = p \ast \frac{F_P \ast C_{TC}}{C_{TC} + C_{PL} - \alpha \ast F_P} + (1 - p) \ast \frac{(1 + \alpha) \ast F_P \ast C_{TC}}{C_{TC} + C_{PL}} - C_L$$ \hfill (3.13)

For the trade creditor accepting offers of value $x$ is a dominant strategy; by definition, there exists no set of circumstances under which the creditor could rationally expect a recovery greater than $x$ in court. Finally, to check that such an $x$ would produce a sub-game perfect Nash equilibrium, we must show that the prepetition lender prefers to make such an offer as opposed to one which would lead to rejection and litigation. He does so if:

$$(1 + \alpha) \ast F_P - x > E(\Pi_{PL})_{\text{Court}}$$ \hfill (3.14)

$$p \ast \frac{F_P \ast C_{PL}}{C_{TC} + C_{PL} - \alpha \ast F_P} + \alpha \ast F_P + (1 - p) \ast \frac{(1 + \alpha) \ast F_P \ast C_{PL}}{C_{TC} + C_{PL}} + C_L$$

$$> p \ast \frac{F_P \ast C_{PL}}{C_{TC} + C_{PL} - \alpha \ast F_P} + \alpha \ast F_P + (1 - p) \ast \frac{(1 + \alpha) \ast F_P \ast C_{PL}}{C_{TC} + C_{PL}} - C_L$$
Thus, we have a subgame perfect Nash equilibrium in which the parties choose
strategies:

\[(S_{PL}, S_{TC}) = (x = p \frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P} + (1 - p) \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}} - C_L,\]

\[\text{Accept for } x \geq E(\Pi_{TC})_{\text{court}}\] (3.16)

In this equilibrium, the default status of respect for the corporate entity plays a pivotal
role, as it puts the burden on the trade creditors to challenge the status. Because
litigation is the only exit option open to the trade creditors, they can not rationally
hope to avoid incurring the majority of the litigation cost. It is apparent through
parallel construction that should consolidation be the norm, the trade creditors could
capture the larger share of the firm.

**Opportunity Costs**

Trade creditors face a second specific cost of entering litigation. Because the
company in bankruptcy was a customer of the trade creditor, a litigation which
postpones the company’s return to solvency itself leads to a cost. To analyze this
situation, let \(C_T\) be the cost from lost trade resulting from the extension of bankruptcy
by litigation. Intuition would suggest that here too, the prepetition lender may
"capture" these costs as the trade creditor must incur them to challenge his offer. To
confirm this suspicion note:

\[(\Pi_{TC})_{SC} = \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}} - C_T\] (3.17)

\[(\Pi_{TC})_{SS} = \frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P} - C_T\] (3.18)
giving:

\[ E(\Pi_{TC})_{Court} = p \cdot \frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P} + (1 - p) \cdot \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}} - C_T. \]  (3.19)

Therefore the prepetition lender will offer:

\[ x = p \cdot \frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P} + (1 - p) \cdot \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}} - C_T \]  (3.20)

Finally note that the prepetition lender will be willing to make such an offer:

\[ (1 + \alpha) \cdot F_P - x > E(\Pi_{PL})_{Court} \]  (3.21)

\[ p \cdot \left( \frac{F_P \cdot C_{PL}}{C_{TC} + C_{PL} - \alpha \cdot F_P} + \alpha \cdot F_P \right) + (1 - p) \cdot \frac{(1 + \alpha) \cdot F_P \cdot C_{PL}}{C_{TC} + C_{PL}} + C_T > 0 \]  (3.22)

Once again, a subgame perfect Nash equilibrium develops. The players strategies in equilibrium are:

\[ (S_{PL}, S_{TC}) = \left( p \cdot \frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P} + (1 - p) \cdot \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}} - C_T, \right. \]

\[ Accept \ for \ x \geq E(\Pi_{TC})_{Court} \]  (3.23)

These costs can be integrated in a third model. However, redefining terms slightly will simplify the arithmetic. Notice that as litigation costs increase while holding the remainder of the model constant, the creditors’ expectations from the original case (without costs) are unchanged. These original expectations:

\[ E(\Pi_{TC})^0 = p \cdot \frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P} + (1 - p) \cdot \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}} \]
\[ E(\Pi_{PL})^0 = p \left( \frac{F_P \cdot C_{PL}}{C_{TC} + C_{PL} + \alpha \cdot F_P} \right) + (1 - p) \left( \frac{(1 + \alpha) \cdot F_P \cdot C_{PL}}{C_{TC} + C_{PL}} \right) \]

Substitution for these terms into the strategy sets defined in equations 3.16 and 3.23 equations provide a clearer way of describing the equilibria:

\[(S_{PL}, S_{TC}) = (x = E(\Pi_{TC})^0 - C_L, acceptx \geq E(\Pi_{TC})^0 - C_L) \quad (3.24)\]

\[(S_{PL}, S_{TC}) = (x = E(\Pi_{TC})^0 - C_T, acceptx \geq E(\Pi_{TC})^0 - C_T) \quad (3.25)\]

These costs, and indeed all costs which accrue at the point of litigation, may be represented as such.\(^2\) The equilibrium strategies when both creditors face costs of litigation, and where the trade creditor suffers an additional opportunity cost of litigation given by:

\[(S_{PL}, S_{TC}) = (x = E(\Pi_{TC})^0 - C_L - C_T, acceptx \geq E(\Pi_{TC})^0 - C_L - C_T) \quad (3.26)\]

Before complicating the model further, what results has this simple model provided? In this model, outcomes are correlated exclusively with the probabilistic expectation in court. Because creditors have only a probabilistic expectation for the court outcome, all bankruptcies reach the same settlement as a function of the entities’ sizes, the creditors’ claims, and litigation costs. Additionally, it is notable that all cases are settled rather than litigated. This certainly fails to mirror actual outcomes and suggests a respect in which the model is failing to represent reality. Finally, these settlements may be described: in crafting his offer, the prepetition lender may “capture” costs associated with litigation as the trade creditor may not refuse without actually incurring the same costs.

\(^2\)Any cost triggered by litigation is, in this model, a cost to refusing the offer. Hence, the prepetition lenders may “capture” any such cost in their offer, knowing that to refuse the offer trade creditors must incur the costs.
3.3 A More Complex Iteration

The simple model provided an important insight into how trial costs and the burden of bringing litigation affect the bargained for shares among creditors. However, the simple model hinges on the expectations for the court outcome. Both parties have only a probabilistic belief about the court outcome. To add complexity, the model may account more completely for the decision between structural subordination and substantive consolidation. To do so, I relax the assumption that all firms face an equally likely probability of being consolidated. Consider, instead, two types of firms: a multiple entity type which courts will treat by upholding structural subordination and a single entity type which courts will treat with consolidation. This idea is entirely plausible if the court has a specific test it uses to determine if consolidation is justified.

The introduction of firm "types" demands the introduction of beliefs on the part of the players. In this model prepetition lenders will know the type of firm they are bargaining over. This assumption will rest of course on the type of standard the court applies however because the prepetition lender is the only creditor to have contracted with the subsidiary entities, they likely have a greater understanding of the degree of separation of the parent firm and its subsidiary. The trade creditor however may not be privy to the exact terms of original agreements between firm, subsidiary, and prepetition lender. The trade creditor will, however, develop a belief about the type faced from the offer presented by the prepetition lender. These additions will make for interesting equilibria in which the settlement offers, selected from a position of knowledge, will act as a signal to the trade creditors.

3 Any of the tests from Chapter 2, universally applied, would serve. Note this model is different from the current state in that with the existence of multiple accepted and preceded tests, their can not be certainty of type.

4 This assumption seems particularly strong in the case of a test such as Augie/Restivo which considers creditors expectations that may be specifically documented in contract agreements or other evidence.
Figure 3.2: The Complex Model
Specifying the Complex Model

In a model in which there may be distinct types, nature makes the first move in deciding the type. Nature first chooses a particular bankrupt group of companies as distinct multiple entities with probability $m$ or a single entity with probability $1 - m$. Knowing the type of company, the prepetition lender then makes an offer for an out of court settlement much like in the previous model. The trade creditor may now face either certain subordination or certain consolidation in court. To make a rational decision about whether or not to accept the offer, the trade creditor must have an idea of the probability he will face one or the other based on the offer he receives. To simplify the specification, prepetition lenders will choose between two offers: a low offer ($x_{\text{Low}}$), based on the division under structural subordination; and a high offer ($x_{\text{High}}$), based on the division under substantive consolidation.\(^5\) The trade creditor believes the company to be a multiple entity with probability $p$ in the face of $x_{\text{High}}$ and $q$ in the face of $x_{\text{Low}}$. Likewise, the trade creditor believes the company to be a single entity with probability $1 - p$ in the face of $x_{\text{High}}$ and $1 - q$ in the face of $x_{\text{Low}}$. These beliefs need not be related, and will depend instead on the strategies chosen by prepetition lender. The trade creditor may accept or litigate. In litigation, the treatment corresponding to the company’s type will be applied along with a court cost. The payoffs to litigation are:

\begin{align*}
(\Pi_{TC}, \Pi_{PL})_{SC} &= \left( \frac{(1 + \alpha) \cdot F_P \cdot C_{TC}}{C_{TC} + C_{PL}} - C_L, \frac{(1 + \alpha) \cdot F_P \cdot C_{PL}}{C_{TC} + C_{PL}} - C_L \right) \quad (3.27) \\
(\Pi_{TC}, \Pi_{PL})_{SS} &= \left( \frac{F_P \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_P} - C_L, \frac{F_P \cdot C_{PL}}{C_{TC} + C_{PL} - \alpha \cdot F_P} + \alpha \cdot F_P - C_L \right) \quad (3.28)
\end{align*}

\(^5\)It’s possible to directly extend the continuous offer space from the previous model. However by limiting the prepetition lender to discrete plausible offer strategies, the beliefs of trade creditors are more easily specified and the interesting equilibria are more easily elucidated.
Pre-petition lenders might offer only slightly more than these in court expectations such as in equations 3.24 and 3.25, however it is illustrative for a few reasons\(^6\) to consider offers which directly correspond to the divisions under structural subordination and substantive consolidation as follows:

\[
x_{High} = \frac{(1 + \alpha) \cdot F_p \cdot C_{TC}}{C_{TC} + C_{PL}} \\
x_{Low} = \frac{F_p \cdot C_{TC}}{C_{TC} + C_{PL} - \alpha \cdot F_p}
\]

(3.29) (3.30)

To more easily compare payoffs, the payoffs in litigation (equations 3.27 and 3.28 may be rewritten in terms of the offers in equations 3.29 and 3.30).

\[
(\Pi_{TC}, \Pi_{PL})_{SC} = (x_{High} - C_L, (1 + \alpha) \cdot F_p - x_{High} - C_L) \\
(\Pi_{TC}, \Pi_{PL})_{SS} = (x_{Low} - C_L, (1 + \alpha) \cdot F_p - x_{Low} - C_L)
\]

(3.31) (3.32)

### 3.4 Seeking Plausible Equilibria

In this model, the addition of uncertainty and the accompanying beliefs requires application of a more stringent equilibrium condition: perfect Bayesian equilibrium.\(^7\) For simplicity, first seek any pure-strategy equilibria\(^8\). The strategies of the prepetition lender as a function of type will define the space of possible equilibria:

1. Offer low if multiple entity, high if single entity\((p = 0, q = 1)\).
2. Offer high if multiple entity, low if single entity\((p = 1, q = 0)\).
3. Offer low if multiple entity, low if single entity\((p = ?, q = m)\).

---

\(^6\)First, offers which closely correspond to the expected court payoff would be costless to challenge through litigation. A trade creditor offered \(x_{Low}\) offer may check that the company is, in fact, a multiple entity type by litigating and if they fail to "catch" a single entity type, they have lost very little.

\(^7\)See Appendix 2

\(^8\)In which players in a given situation players choose a single strategy rather than a "mixed" strategy in which players randomize among strategies.
4. Offer high if multiple entity, high if single entity \((p = m, q = ?)\).

However the second and fourth may be eliminated by inspection. Players facing the multiple entity type are guaranteed a high portion (less the litigation costs) if their offer is rejected. Why then would a prepetition lender facing a multiple entity ever offer a high share? The first (a separating equilibrium) and the third (a pooling equilibrium) demand investigation, as they are the remaining plausible pure strategy equilibria. Barring an equilibrium in which both creditors choose a pure strategy, it may be that an equilibrium exists in which players randomize among strategies.

**Pooling Equilibrium on Low Offer**

First, to test the third potential equilibrium, consider under which conditions a pooling equilibrium on low offers may be sustained. How would bargaining and beliefs evolve in such an equilibrium? Because prepetition lenders pool on Low:

\[ q = m \]  \hspace{2cm} (3.33)

With these beliefs, will the trade creditor accept the low offers? Consider his expected profit in both cases:

\[ E(\Pi_{TC}(Accept)) = x_{Low} \]  \hspace{2cm} (3.34)

\[ E(\Pi_{TC}(Litigate)) = m(x_{Low} - C_L) + (1 - m)(x_{High}) \]  \hspace{2cm} (3.35)

For prepetition lenders to pool on low offers the offers must be accepted else some prepetition lenders of the single entity type offer \(x_{High}\) to induce \(Accept\). The trade
creditor chooses to accept if:

$$E(\Pi_{TC}(Accept)) > E(\Pi_{TC}(Litigate))$$  \hfill (3.36)

$$x_{Low} > m(x_{Low} - C_L) + (1 - m)(x_{High} - C_L)$$

$$(1 - m)x_{Low} > (1 - m)x_{High} - C_L$$

$$C_L > (1 - m)(x_{High} - x_{Low})$$  \hfill (3.37)

A pooling equilibrium of this type will develop if the litigation costs outweigh the benefit from litigating against a single entity weighted by the probability of encountering a single entity (equation 3.37). Under such conditions the pure Bayesian equilibrium:

$$(S_{PL}; S_{TC}) = (x_{Low}; accept x_{Low}, refuse x_{High}; p = ?, q = m)\big| C_L > (1 - m)(x_{High} - x_{Low})$$  \hfill (3.38)

Is such an equilibrium plausible? It would demand either that the costs of litigation be very large relative to the gains from litigation or that the chance of successfully litigating be small. Under conditions such as prohibitive court costs or extremely rare success it is easy to imagine trade creditors universally accepting low offers when the alternative in court is relatively undesirable. However, it seems that under normal conditions, such an equilibrium would be unlikely.

**Separating Equilibrium**

While it is possible for seemingly unlikely parameters to produce a pooling equilibrium in which the prepetition lender offers $x_{Low}$ and the trade creditor never litigates, this equilibrium does not exist under many parameters. Now consider the conditions which the model might produce a separating equilibrium in which prepetition lenders offer $x_{Low}$ when a multiple entity and $x_{High}$ when a single entity. The trade creditor
beliefs:

\[ q = 1; p = 0 \]  

(3.39)

Notice \( 1 - q \): the chance that an offer of \( x_{Low} \) is received when \( x_{High} \) is deserved, is nonexistent. Because these "opportunities" of litigation don’t exist, none will litigate. Given this behavior on the part of trade creditors in response to separating, is separating chosen by prepetition lenders? Multiple entity firms will continue separating and continue receiving their maximum payout. However, in the face of unconditional \textit{Accept}, \( x_{High} \) does not maximize the prepetition lender’s receipts. Therefore, prepetition lenders of the single entity type will offer \( x_{Low} \), at least occasionally, as though they were multiple entities deserving of the larger share. Because the beliefs and corresponding response created by the separating behavior themselves encourage behavior other than separating, a separating equilibrium in this bargaining environment does not exist.

\textbf{Mixing Strategies in Equilibrium}

The manner in which the potential separating equilibrium of the previous section ultimately failed points to the most plausible equilibrium outcome of the complex model. Having exhausted the possible pure strategy equilibria of this model; equilibria in which players randomize between some strategies should be tested. However, as discussed earlier, a multiple entity will never offer \( x_{High} \). Likewise, trade creditors won’t litigate against offers \( x_{High} \). These conditions suggest an equilibrium in which multiple entity types offer \( x_{Low} \) and single entities offer \( x_{High} \) except with probability \( \theta \) when they offer \( x_{Low} \) to impersonate a multiple entity type. Likewise, trade creditors always accept \( x_{High} \) and litigates against \( x_{Low} \) with probability \( \phi \) to discourage single entities from offering \( x_{Low} \). In equilibrium, both competitors choose \( \theta \) and \( \phi \) to make...
the opposing creditor indifferent between his choices. For the trade creditor:

\[
\text{Choose } \phi \text{ s.t. } E(\Pi_{PL}(x_{High})) = E(\Pi_{PL}(x_{Low})) \quad (3.40)
\]

\[
\phi((1 + \alpha)F_p - x_{High} - C_L) + (1 - \phi)((1 + \alpha)F_p - x_{Low}) = (1 + \alpha)F_p - x_{High}
\]

\[
-\phi(x_{High} + C_L) - (1 - \phi)x_{Low} = -x_{High}
\]

\[
(1 - \phi)(x_{High} - x_{Low}) = \phi * C_L \quad (3.41)
\]

Verbally, choose \( \phi \) so that the gain to single entity types from offering a low offer and not being litigated against is exactly outweighed by the losses from being litigated against. Likewise for the prepetition lender:

\[
\text{Choose } \theta \text{ s.t. } E(\Pi_{TC}(Accept)) = E(\Pi_{TC}(Litigate)) \quad (3.42)
\]

\[
x_{Low} = \frac{\theta(1 - m)}{\theta(1-m) + m} (x_{High} - C_L) + \frac{m}{\theta(1-m) + m} (x_{Low} - C_L)
\]

\[
C_L = \frac{\theta(1-m)}{\theta(1-m) + m} (x_{High} - x_{Low}) \quad (3.43)
\]

This condition requires that the gain from litigation weighted by the likelihood of success in litigation must exactly counter balance the certain costs of litigation. With both players randomizing according to equations 3.41 and 3.43, the pure Bayesian equilibrium:

\[
(S_{PL}; S_{TC}) = (x_{Low}, x_{Low}) \text{ w/ probability } \theta \text{ s.t } C_L = \frac{\theta(1-m)}{\theta(1-m) + m} (x_{High} - x_{Low});
\]

refuse \( x_{Low} \) w/ probability \( \phi \) s.t (1 - \( \phi \))(x_{High} - x_{Low}) = \( \phi * C_L \), accept \( x_{High} \);

\[
p = 0, \quad q = \frac{m}{\theta(1-m) + m}) \quad (3.44)
\]
This mixed strategy equilibrium is held together by the balance struck by each creditor to create indifference in the other. Were the trade creditor to litigate slightly more often, single entity prepetition lenders would strictly prefer $x_{High}$, but the trade creditors would be incurring uncompensated litigation fees. Likewise, if prepetition lenders impersonated multiple entities by offering $x_{Low}$ more often, the would increase $\phi$ and increase their own legal fees.
Chapter 4

Analysis

The jurisprudence of structural subordination has yet to be completed. While the issue has been addressed and re-addressed, precedents handed down, disregarded and replaced, the standards for applying the treatment are manifold and for any direction given by a particular standard, there is no legislated choice of a particular standard. This uncertainty on the part of creditors affects both their recovery at bankruptcy and likely more importantly, their investment decisions at the outset. The two models developed in this study attempt to shed light on the interactions of creditors in the absence of a legislated standard. What implications may we draw from these models, and what principles of optimal legal structure might these implications suggest?

4.1 The Simple Model

The simple model represents conditions of least certainty on the part of creditors. Consider the legal standards best approximated by the probabilistic judge in the first model. A capricious judicial system which assigns structural subordination or substantive consolidation without standard would be seen by creditors as probabilistic. However, other regulatory systems might also produce this probabilistic judgment. For example, a system in which a number of objective standards exist, but the appli-
cation of the standards is random would be viewed by creditors as equivalent to the capricious judge.

Without drawing a direct analog to our judicial system, what are the implications of this uncertain judgment revealed by the simple model? First, in the face of uncertainty and court costs, creditors reach equilibria in which offers based on the expected court outcomes are accepted. Under this regime, court costs are avoided as are opportunity costs of prolonged negotiation. However, there is a danger under regimes of uncertainty that investment may be affected by the distorted returns. The equilibrium payouts in the simple model are neither of the theoretically justified payouts (those prescribed by structural subordination and substantive consolidation). Moreover, payouts are unbalanced in favor of the party whose preferred corporate treatment is the defacto treatment. These parties are able to capture legal costs because the law forces their competitor to litigate.

Notice, there is a third system for which the case of uncertainty is an analog. If in the complex model neither party had information about type, their only information would be $m$ the distribution of types. This $m$, would in effect be $p$ from the first model.1

4.2 The Complex Model

In the complex model, information about company types is added to model the addition of specificity into a standard. Standards involving contractual terms and creditor expectations are likely good examples, as particular "types" of contracts or expectations would define the firm type. However, tests of this type (Augie/Restivo) are likely to rely heavily on the records of the prepetition lenders as it is their behavior which potentially differentiates among the entities. To the extent that the contracts,

1The major difference is that the first model employs a continuous offer space, if the second model did so, the case of no information about types would be equivalent to the dimple model.
agreements, and correspondence are material, the prepetition lender receives insight into the type of firm he faces that the trade creditor does not. The complex model represents this by leaving trade creditors ignorant of type.

The introduction of information changes the resulting equilibria in two notable ways. First, the likely equilibria (mixed strategy pure Bayesian equilibrium) includes creditors litigating and incurring legal costs. These fees are a loss to both creditors and would not ultimately be desirable. However, returns seem likely to move toward expectations in the complex model as information about type is revealed. While single entity creditors will occasionally receive high shares through impersonation, and deserving multiple entity creditors will occasionally be litigated against; payoffs will at least become correlated with type. To the extent creditor expectations are correlated with type, outcomes will become more representative of expectations. Here litigation imposes another problem: distortion of the return for firms incurring legal costs.
Chapter 5

Conclusion

What characteristics in equilibria might we hope to create through our legal system? The optimal system would, as much as is possible, prevent creditors from litigating. Additionally, the returns in such a system would match expectations to minimize any negative effect on investment. What characteristics would foster such equilibria in the complex model?

Consider a case where both creditors have full information about type. Such a system would represent a legal system in which a single standard has been decided upon and the applicable information is available to all. In such a system, prepetition lenders would have no hope of receiving a high share from a single entity type, so a true separating equilibria would form. Legal costs would be eliminated and returns would be directly in line with expectations. Here then a legal code with a single clear standard would be an optimal standard by our definition.

Might another system produce optimal equilibria? The legal problem addressed by these remedies is created when prepetition lenders either loan to subsidiaries or, more often, write security agreements which include subsidiary assets for loans to the parent. If the problem itself grows out of contractual specificity on the part of prepetition lenders, expecting the same of all parties may solve the problem. If con-
tracts were taken at face value, structural subordination or substantive consolidation would be assigned formulaically. Not only will creditor expectations be represented in bankruptcy but in the presence of heterogeneous capital structures firms will have more specificity in contracting.

Substantive consolidation, and corporate disregard more broadly, developed as a remedy for lack of contractual specificity associated with the novelty of the corporation and limited liability. A century later, expecting contractual specificity would not be burdensome. Moreover, presumably the adoption of such a unified policy on subsidiary entities in bankruptcy would benefit the creditors themselves in the form of saved litigation fees and more efficient investment.
Appendix A

Subgame Perfect Nash Equilibria

The first model is a member of a class of games called dynamic games of perfect information. (Gibbons, 1992, p. 71) This means that the game is played sequentially and at each step players have knowledge of the previous moves and all payoff functions. In determining which equilibria are plausible in dynamic games of perfect information, the important nuance is credibility. Consider dynamic game of perfect information in figure A.1. Two Nash equilibria exist in the game:

\[(s_E, s_I) = (\text{Enter, Accommodate}) \& (\text{Don't Enter, Fight})\]

![Figure A.1: A Simple Dynamic Game of Perfect Information](image-url)

Figure A.1: A Simple Dynamic Game of Perfect Information
In both outcomes neither player would alter their strategy given their opponent’s strategy. However, it seems like the second, \((Don't\ Enter, \ Fight)\), seems suspect. Were the entrant to enter; is the incumbent’s threat to fight credible? In the case of entry, the incumbent prefers to accommodate, so his threat to fight is not credible. The condition for Nash equilibrium has failed to eliminate an implausible equilibrium; the dynamic nature of the game demands an extra test to separate implausible equilibria. The problem of non-credible threats involves an understanding of strategies off of the path of play. In the second equilibria, the entrant’s decision not to enter is based on a strategy off of the path of play which will not be exercised. To address beliefs off the path of play we may require not only that the equilibrium be Nash for the entire game, but that for all subgames, the player has chosen their best response. A subgame begins at a single node other than the first. The example has one subgame shown in figure A.2.

Imposing the new condition, the second equilibrium fails. In the subgame it is clear that \(\text{Fight}\) is not a best response for the incumbent, therefore it may not be part of an equilibrium leaving:

\[(s_E, s_I) = (\text{Enter}, \text{Accommodate})\]
This equilibrium is called a subgame perfect Nash equilibrium and is the equilibrium applicable to dynamic games of perfect information.
Appendix B

Perfect Bayesian Equilibria

The more complex model is a member of a class of games called dynamic games of incomplete information. (Gibbons, 1992, p. 173) This means that the game is played sequentially and while players have knowledge of all payoff functions; they may not know all of the previous moves and correspondingly, from which node they themselves are moving. In determining which equilibria are plausible in dynamic games of imperfect information, beliefs become pivotal. Consider the game of incomplete information in figure B.1 Two Nash equalibria exist in the game:

\[(s_1, s_2) = (Up, Left) \& (Back, Right)\]

In both outcomes neither player would alter their strategy given their opponent’s strategy. Because a proper subgame may begin only at a single node other than the first, the example has no subgames, therefore both Nash Equilibria are trivially subgame perfect. However the second, \((Back, Right)\), is driven by Player 1’s belief that Player 2 will play \(Right\). In reality Player 2 would never play \(Right\) because he would never expect to be at the node following \(Down\). Perfect Bayesian equilibrium strengthens the concept of subgame perfect Nash equilibrium to distinguish unreasonable equilibria of this type by adding beliefs to the equilibria. Perfect Bayesian
Figure B.1: A Simple Dynamic Game of Incomplete Information

equilibria demands that at each node, players’ strategies maximize their payoffs given their beliefs about which node they are deciding from and their opponents strategy profiles. Consider adding beliefs to the simple dynamic game of imperfect information. Assuming Player 2 is allowed to move, he believes Player 1 has chosen Up with probability $p$ and Down with probability $1 - p$. From these probabilities we may calculate:

$$E(\Pi(\text{Right})) = p \times 0 + (1 - p) \times 1 = 1 - P$$  \hspace{5cm} (B.1)$$

$$E(\Pi(\text{Left})) = p \times 1 + (1 - p) \times 2 = 2 - P$$  \hspace{5cm} (B.2)$$

$$E(\Pi(\text{Right})) < E(\Pi(\text{Left})) \mid p > 0$$  \hspace{5cm} (B.3)$$

Perfect Bayesian equilibrium demands that strategies are optimal given beliefs, therefore Player 2 will not play Right in equilibrium. Given that Player 2 never plays Right, this leaves a single perfect Bayesian equilibrium:
\[(s_1, s_2) = (\text{Up, Left}; p = 1)\]
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