

# **Electoral Competition and Duverger's Law**

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## 1 – Introduction

The aim of this thesis is to explain the robustness of the two-party equilibrium prevalent in the United States. In other words, why do third parties rarely enter national electoral competition, and why do they almost never win? There are two points which make this question particularly interesting. First, the two-party equilibrium has strong implications for policy formation: voter welfare is not necessarily maximized when options are so limited. While having fewer political parties likely leads to economies of scale, a larger spectrum of parties would result in more policy diversity, satisfying more voters. Understanding the cause of the two-party result is informative for thinking about maximizing voter welfare. Second, there is a discrepancy between the basic game-theoretical model of electoral competition and reality. Unlike the prediction of that model, parties differentiate their policies and do not simply appease the median voter.

Sociologist Maurice Duverger was the first to note that a first-past-the-post voting system consistently yields two-party outcomes. His most robust claim, titled “Duverger’s Law”, is that a *plurality voting* (first-past-the-post) system leads to domination of elections by two political parties. Duverger also hypothesized that *run-off majority voting* and *proportional representation* lead to a multi-party result (Duverger, 1963), however this idea is not as widely accepted.

Duverger explained his law through a *mechanical effect* and a *psychological factor* (Riker, 1982, p. 761). The mechanical effect is an empirical observation that third parties in elections end up more underrepresented than anticipated. In other words, a third party receives a lower percentage of seats in Congress than the percentage of votes it garnered. Rae (1971) and Sprague (1980) provide evidence that this mechanical effect concept is correct. The psychological factor explains that voters may abandon preferred parties to vote for second-preferred candidates, which may have a higher chance of winning the election. Voters do not want to waste their votes. This and related concepts have been rigorously explored by social choice theory, and formalized into the idea of *strategic voting* by Cox (1997).

Multiple models have attempted to move beyond Duverger's explanation. The two party equilibrium has been justified using district composition, ideological candidates, presidentialism, differing party and candidate incentives, and ethnic heterogeneity among others. Chhibber and Kollman (1998) even propose that party number is influenced over time by changes in the concentration of political and economic power. This paper builds upon relevant game theory literature to formulate a model that mirrors national party competition over multiple districts. It provides a rationale for the two-party equilibrium in a large country with a complex voting mechanism like the United States, and reaffirms Duverger's conclusion.

The organization is as follows: Chapter 2 provides a literature review that summarizes basic models of electoral competition and entry. The most important revisions of these models are discussed next, providing a conceptual grounding for the approach taken in this paper. In Chapter 3, I develop my own model of electoral competition on a national scale, with parties that compete on a continuum of districts. This is meant to mirror Senate elections in the United States. After discussing definitions and assumptions, I explore various specifications of two-party competition using a restricted and unrestricted party budget, as well as two different formulations of a party's utility function. One utility function is based simply on payoffs from winning elections, while the other includes a utility from savings. Next, I examine the incentives of potential new entrants, focusing on expected utility of this third party and the effects of entry on the incumbents. Lastly, I create a model of entry which provides a rationale for Duverger's Law.

## 2 – Literature Review

### 2.0 The Downs Model

The fundamental model of electoral competition was developed by Downs (1957), adopting Hotelling's spatial product competition model to explain policy formation. In a Downsian world, political parties and candidates are synonymous, and are only interested in winning the election. They seek to maximize the probability of victory and are not encumbered by other motivations. There exists a continuum of voters with symmetric *single-peaked*<sup>1</sup> preferences regarding a specific policy  $p$ , where  $p \in [0, 1]$ . The range  $[0, 1]$  is a *uni-dimensional policy space*, on which voters are distributed, with median voter preferring policy  $p^m$ . Voters vote for policies closest to their "bliss point". For example, a voter with ideal  $p = .4$  will prefer  $p = .2$  over  $p = .8$ . It is assumed that parties have perfect information regarding the distribution of voters.

Parties 1 and 2 decide to announce a policy  $p^i$  that maximizes their payoff  $\Pi^i$ . The payoff function is just the probability of winning the election given  $p^1$  announced by party 1 and  $p^2$  announced by party 2:

$$\text{Payoff of party 1:} \quad \Pi^1(p^1, p^2) = P(p^1, p^2)$$

$$\text{Payoff of party 2:} \quad \Pi^2(p^1, p^2) = 1 - P(p^1, p^2)$$

$P(p^1, p^2)$  is probability that party 1 wins

The players competing in Downs will play any political position necessary to win the election. Each will choose  $p^i$  such that  $p^i$  is a *Condorcet winner*, defined as a policy that "can beat all others ... in a series of pairwise majority votes" (Heap et al, 1992, p.301). In other words a Condorcet winner policy  $p^*$  can defeat or tie any other policy  $p \neq p^*$ . The Downs political equilibrium is thus policy pair  $(p^*, p^*)$ , where  $p^* = p^m$ . Both parties converge towards the preferred policy of the median voter,

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<sup>1</sup> A function is single peaked if it possesses a unique local maximum. This concept is explained in Roemer (2001, p.20).

and the election is a tie. By definition, a candidate wins a tie with probability 1/2. No profitable deviation is available for either player, since a move away from  $p^m$  will mean certain loss for the diverging party.

The Downs result is elegant but not realistic. It has since been amended for ideological motivations, multiple policy issues, uncertainty regarding voter behavior, as well as various other issues. This paper's model will feature a pure Downs competition in one of the stages of the national electoral competition game. It is important to remember that Downs candidates converge to the median to win the election, and split the payoff with probability 1/2 at equilibrium  $(p^m, p^m)$ .

## 2.1 The Wittman Model (Ideological Candidates)

One of the primary criticisms of the Downs model is its limited representation of the true motivations of political parties. In reality, politicians are usually partisan and likely care about what policy is implemented. Wittman's (1977) model uses a payoff function that incorporates both probability of winning and utility gained from a specific policy. The payoff functions are below:

$$\text{Payoff of party 1: } \quad \Pi^1(p^1, p^2) = P(p^1, p^2) U^1(p^1) + (1 - P(p^1, p^2)) U^1(p^2)$$

$$\text{Payoff of party 2: } \quad \Pi^2(p^1, p^2) = P(p^1, p^2) U^2(p^1) + (1 - P(p^1, p^2)) U^2(p^2)$$

$P(p^1, p^2)$  is probability that party 1 wins and  $p^1$  is played

$U^i(p^i)$  – utility of party  $i$  from policy  $j$

Utility functions  $U^1$  and  $U^2$  represent policy preferences of the party leaders, which can also be conceptualized as ideological citizen-candidates. These leaders have some ideological bliss point  $p^{i_{\text{bliss}}}$  where  $U^i$  is maximized. Persson and Tabellini (2000, p. 98) separate the payoff function into *centrifugal* and *centripetal* forces. The centrifugal force pulls the party towards the ideological bliss point; the centripetal force pushes the party towards the median  $p^m$  in order to increase the probability of winning. Despite this tension, the Wittman model has the same convergent Nash equilibrium

( $p^* = p^m$ ,  $p^* = p^m$ ) as before. This is because the centripetal force dominates as a result of the winner-takes-all formulation: if party 1 does not play the median and the other party does, party 1 will receive a payoff of  $U^1(p^2 = p^m)$ . If both parties play the median, party 1 receives the same payoff:  $1/2(U^1(p^m) + 1/2(U^1(p^m) = U^1(p^m)$ . Therefore, the parties will both edge closer to  $p^m$  until they converge; afterwards there is no incentive to diverge.

While Wittman's original formulation does not lead to divergence, it is possible to amend slightly the rationale behind the model to achieve differentiated policy. Instead of winner-takes-all, consider a situation when a party can achieve some payoff even if the other party is closer to the median. This type of payoff function would reflect Congressional elections. The centripetal force no longer dictates discontinuous payoffs, and a meaningful tension can be explored between the desire to win more votes and to stay true to ideology. The model in this paper explores a similar dynamic, defining the centrifugal force as the desire to save money, rather than Wittman's ideological preferences.

## 2.2 Uncertainty

In reality, political parties do not have perfect information regarding the distribution of voters. Campaigning and media efforts need to be used by politicians to understand voter preferences and accurately position themselves. Roemer (2001) proposes models of uncertainty that can be applied to Downs and Wittman: the state-space approach and the error-distribution model. These models formulate uncertainty on a "macro" level, rather than tracking it in individual preferences. The state space model essentially portrays a number of states of nature, occurring under some probability distribution. These states of nature differ in voter behavior, resulting from some exogenous shock such as bad weather, or political debates gone awry. Given these circumstances, in some of states of nature policy  $p^1$  defeats policy  $p^2$ , and in others  $p^2$  defeats  $p^1$ . The probability of party 1's victory is the probability of a state of nature occurring in which  $p^1$  defeats  $p^2$ . The second approach to uncertainty, the error-distribution model, focuses instead on the candidates' knowledge

of voters' preferences. The political actors are imperfectly aware of the voter distribution, predicting the fraction of voters that prefer to  $p^1$  to  $p^2$  with some margin of error.

Uncertainty is useful because it is more realistic and can induce divergent equilibria. Roemer (2001) shows that Downs remains unaffected by uncertainty and the two parties continue to converge. Wittman's partisan politicians model, on the other hand, achieves differentiated equilibria under either uncertainty approach. While the model in this paper does not directly introduce uncertainty into party decision making, it nonetheless relies on something like the state space approach. It will be shown that a third party faces uncertainty regarding how much money it can fundraise, which in turn significantly affects its incentives.

### **2.3 Multidimensionality and Probabilistic Voting**

So far, parties have been portrayed as competing over a single issue. Yet, legislatures do not only consider issues of "left" and "right", but those on multiple ideological levels: tax policy, foreign relations, social welfare, among many others. It is possible to have different individual rankings for multiple policy proposals of political candidates, making the voting decision more complicated. Multidimensional policy competition attempts to model this situation.<sup>2</sup> Unfortunately, it is very difficult to find an equilibrium given multiple dimensions. The condition required for some policy to be a Condorcet winner is prohibitively strong – it must be a "median in all directions", requiring extreme symmetry in the distribution of bliss points (Persson and Tabellini, 2000, p.29). Heap et al (1992, p.223) explain: "the distribution must be such that one voter occupies the median of both dimensions, and remaining voters are strung out in such a way that they are all diametrically opposed to one another around this median point and exactly paired with one another". The probability of this scenario is nonexistent. The lack of an equilibrium leads to *cycling*, which means that a candidate may always propose a policy that defeats the original, and there is no best response policy available for any of the players. Both the Downs and Wittman model lack a stable equilibrium in this

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<sup>2</sup> Multidimensional policy competition is covered in Heap et al (1992, p.223), Persson and Tabellini (2000, p.29) and Roemer (2001, p.111).

formulation (Roemer, 2001). *Probabilistic voting* models, pioneered by Hinich (1977) and Coughlin and Nitzan (1981) are used to find equilibria in multiple dimensions. By introducing uncertainty into voter behavior, probabilistic voting makes the probability of a voter choosing a particular candidate into a continuous function, fixing the discontinuities inherent in multidimensional competition.<sup>3</sup>

## 2.4 Citizen Candidate model and Entry

The citizen-candidate model, developed by Osborne and Slivinski (1996) and Besley and Coate (1997), is the starting point for analyzing the incentives candidates face when entering an election. Each candidate has a particular bliss-point, is motivated by both probability of winning and policy implemented. There are multiple feasible Nash equilibria, including single candidate and multi-candidate equilibria (Persson and Tabellini, 2000, 104). Essentially, a voter enters the electoral competition if the gain in utility from implementing her preferred policy over the incumbent policy is greater than the cost of entry. One relevant pitfall of this model is the absence of established players prior to the entry calculus.

Other entry models have been developed by Palfrey (1984), Feddersen, Sened and Wright (1990) and Shvetsova (1995). In general, most models show that a third party is unlikely to compete in the election given positive entry cost, as predicted by Duverger. Palfrey (1984) finds a non-centrist equilibrium where the incumbents win with probability 1/2. The intuition is as follows: if parties 1 and 2 play  $p^1 < p^m$  and  $p^2 < p^m$ , then party 3 can steal the election by playing  $p^3 = p^m$ . Thus  $p^1$  and  $p^2$  are on opposite sides of the median. If the third party would only enter if there is positive probability of victory, it does not enter because cost is too high and the incumbents face the regular Downs game. If the third party enters even if it bound to lose,  $p^1$  and  $p^2$  do not converge in order to protect their flank – a third party could come from the side, capturing votes and causing the opposite incumbent to win. Callander (1999, p.19) extends Palfrey's model to include a continuum of districts across which parties choose a single policy, also finding a unique non-centrist two-party equilibrium.

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<sup>3</sup> The discussion of probabilistic voting by Persson and Tabellini (2000, p. 33) is particularly apt.

Feddersen et al (1990) model another approach to the entry decision. Citizens vote strategically, and the cost of entry is  $\alpha$ . The payoff function is simply the probability of winning times the utility gained from playing a preferred policy  $[P^i(p^i)U^i(p^i)]$ . Given simplifying assumptions,  $P^i(p^i) = 1/n$  in equilibrium,  $n$  being the number of entrants. The entry decision  $P^i(p^i)U^i(p^i) > \alpha$  can be represented as  $U^i(p^i)/\alpha > n$ ; in other words the benefit-cost ratio for the given election must exceed the number of candidates. The model in my paper uses multiple concepts discussed in this section, such as entry cost, the utility of victory and the desire of parties to protect their flanks.

## 2.5 Separation of candidates and parties

Recent work on electoral competition looks closer at the incentives of the different actors in the political game. Specifically, models can be made more realistic by separating the concept of “political party” from that of “candidate”. Eyster and Kittsteiner (2004) propose a novel approach to finding a non-centrist equilibrium. They separate political actors into parties that set a policy across multiple districts in the first round of the game, and local candidates that deviate from their party’s policy in the second round. The districts have different distributions of voters, and therefore different local median policies. The candidates are Downs-motivated to win the local election, and move towards that local median at a cost. This cost represents “advertising a new policy ... unpalatable payments or promises to special interests necessary to finance publicizing a change from party policy” (Eyster and Kittsteiner, 2004, p.4). The payoff function for the parties balances the probability of winning the national election against the campaign costs incurred by the candidates. Maximizing this function leads parties to diverge in policy and decrease direct competition. The model offers a historically feasible result using an elegant qualification of Downs and is the starting point for the specification used in this paper.

### 3 – Model and Results

#### 3.0 Definitions

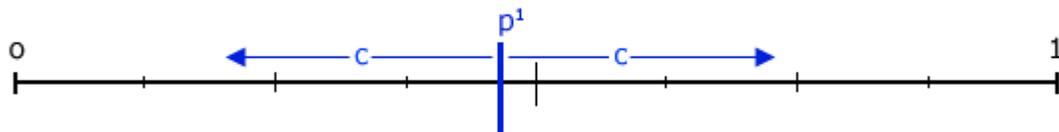
Two parties, party 1 and party 2, compete in a national election with a continuum of districts uniformly distributed on the uni-dimensional national policy space  $[0, 1]$ . The preferred median policy  $p^m_{\text{local}}$  of each district is the location of that district on the national policy space. The parties announce national policies  $p^1$  and  $p^2$ . In each of the districts, local candidates compete in a Downsian election. This implies that local candidates must converge to the  $p^m_{\text{local}}$  to maximize the chance of victory. The candidates start out at the national policy positions of their parties ( $p^1, p^2$ ). The candidates' move to a local median is costly for the party. If a candidate can credibly represent the national party in a district, the national party finances this move. Some districts are so far away from a party's position that its candidate cannot credibly represent the party in that district, and the party does not field a candidate. In other words, the candidate would move so far away from the party on the policy space, that she could no longer be associated with the party platform. The range of districts in which a party finances its candidates is called the *credibility range*, spanning a distance of  $c$  from  $p^i$  in both directions.<sup>4</sup> To summarize:

**Party 1 national platform =  $p^1$ ; Party 2 national platform =  $p^2$**

**National median policy =  $p^m_{\text{national}} = 1/2$**

**Parties compete on  $[0, 1]$**

**Credibility range =  $[p^1 - c, p^1 + c]$  spanning a distance of  $2c$  on  $[0, 1]$**



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<sup>4</sup> For example, incumbent parties in the United States have a large credibility range, such as the one used throughout this model. In countries such as India, it is likely that the credibility range is very small.

The payoff structure mimics US Senate races.<sup>5</sup> Some states send both a Democrat and Republican to Washington: a payoff of 1 senate seat for each party. Those states would be included in the credibility range of both parties, and the parties would finance their candidates move to the local median. In other states, the local median voter is ideologically close to the Democratic national platform and prohibitively far from the Republican national platform. There, the Democrats will tend to win 2 seats in the Senate. These states are not included in the Republican credibility range, and yield 0 seats. Thus a party can achieve payoffs of 2, 1 or 0.

A district election is *contestable* if the district is located within  $[p^1 - c, p^1 + c]$ , the credibility range  $2c$ . An election can be *challenged* only if it is contestable, meaning that a party finances its candidate's move towards the local median.<sup>6</sup> If an election is not contestable, party 1 will win districts closer to  $p^1$  than  $p^2$  and earn a 2 seats, but lose districts closer to  $p^2$ , earning 0 seats. If an election is contestable and challenged by both parties, each party wins one seat, receiving a payoff of 1. If the election is contestable, but challenged by only a single party, the challenging party will win both seats, receiving a payoff of 2, and its opponent will receive 0. A contestable election not challenged by either party yields payoffs equivalent to that of an election that is not contestable. Another way to understand the payoffs is the following: parties achieve the highest payoff of 2 with probability 1 in an unchallenged election, and with probability 1/2 in a challenged one. Probability 1/2 comes from the convergence result in Downs.

Each party has a utility function  $U^i$  derived from the payoffs possible in the national election, which depend on the position of both parties. Function  $\Pi^i(p^i, p^j)$  aggregates all the payoffs from local districts, and is defined precisely in Appendix 6.2. It reflects something like the average number of Senate seats won by the party. Essentially,  $\Pi^i(p^i, p^j)$  multiplies the payoff won in a particular district by the district portion that provides that payoff. The district portions always sum to unity.

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<sup>5</sup> This is notably different from Downs, where winner takes all. Instead, it is possible for a party to capture less than 1/2 of the entire policy space, and still achieve a satisfactory utility.

<sup>6</sup> A summary of these payoffs is shown in Appendix 6.0.

$U^i$  = Utility of party  $i$

$U^i = U [\Pi^i(p^i, p^j)] = \Pi^i(p^i, p^j)$

$\Pi^i(p^i, p^j) = (\text{payoff of local district})(\text{district portion})$

**Unchallenged election payoffs: victory = 2, loss = 0**

**Challenged election payoff = 2/(amount of parties challenging district)**

Parties have a limited budget and face concrete costs. For simplicity, the local cost of challenging an election is fixed at 1.<sup>7</sup> This cost represents the party's cost of introducing a local candidate and financing her campaign to the local median through advertisements, rallies and other events. It is only paid by a party when it challenges an election; unchallenged elections are "free". It does not matter how many parties are challenging the election – one, two or three – the cost is constant. National cost is represented by function  $C^i$ , which aggregates costs paid for each district for party  $i$  (Appendix 6.4). Each party has a fixed budget  $B$  for funding the national costs. This is a representation of how many elections a party can challenge. The most elections a party will ever be able to challenge is its entire range of contestable elections, the credibility range  $2c$ . Thus there are two important cases<sup>8</sup>: an *unrestricted* budget  $B = 2c$  and a *restricted* budget  $B < 2c$ . It is assumed that parties challenge all the elections that are contestable by both parties, unless the budget restriction prevents them from doing so.<sup>9</sup> The notation is below:

$C^i(p^i, p^j) = (\text{cost of local district} = 1)(\text{district portion})$

**$B$  = budget of party**

**$C^i(p^i, p^j) \leq B \leq 2c$ ; costs cannot be larger than budget**

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<sup>7</sup> Alternately, it is possible to use variable costs in the model. Instead of paying 1 to finance an election anywhere in the credibility range, a party would pay price  $b$ , where  $b = (1/c) | p^m_{\text{local}} - p^1 |$ . In other words, the party would finance the local candidate's move by paying for the difference between party and local median policy. The payoff for elections challenged by both parties would be  $(1 - b)$ . This formulation is more difficult to track and is not used in this paper. For an example of similar results using variable costs, see Eyster and Kittsteiner (2004).

<sup>8</sup> If  $B > 2c$ , the party is still only able to challenge  $2c$  elections. This is synonymous with  $B = 2c$ .

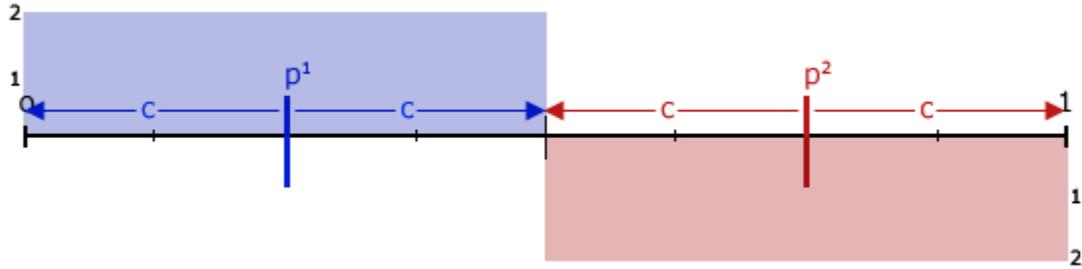
<sup>9</sup> Without this assumption, convergence dominates most of the results.

The sections below explore a number of permutations of the defined model given  $2c = 1/2$ . First, I look for a two-party equilibrium using  $U^i = \Pi^i(p^i, p)$  and an unrestricted budget  $B = 2c$ . This formulation leads to convergence of the two parties at the national median. In the next section the budget is restricted to  $B = c$  in order to simulate fundraising limits. Convergence dominates again, but I find a budgeting strategy that is useful for later sections. Section 3.3 amends function  $U^i$  to include utility from saving the budget, which reflects diminishing marginal utility of money. As a result, the parties are pushed apart by their desire to save and a set of divergent equilibria with a fixed distance between parties becomes possible. Section 3.4 explores the effect of a budget restriction on the new utility function, showing that the distance between parties' policy locations is inversely affected by the size of the budget. A larger budget allows parties to play policies closer together, a smaller budget forces parties to diverge more extremely. Sections 3.5 and 3.6 explore the expected payoffs of a third party entrant into the equilibria previously found. In 3.5 the entrant's budget is equal to that of the incumbents, while in 3.6, the third party's budget is restricted and the maximum utility is consequently smaller. Lastly, I develop a model of entry for the third party which shows why new entrants are extremely rare and Duverger's Law holds.

### **3.1 Two Party Model: Unrestricted Budget, Utility from Payoffs**

Consider the example where  $B = 2c$ ,  $c = 1/4$ . There is no real budget constraint. All contestable elections are potentially challenged. Diagrams on the next page provide an easier method to track payoffs from votes and costs than the definitions of  $\Pi^i(p^i, p)$  and  $C^i$  in the Appendix. The vertical axis represents possible payoffs from a local district on range  $[0, 2]$  and the horizontal axis is the national policy space  $[0, 1]$ . The blue area is  $\Pi^1$ , the aggregate of payoffs for party 1 or the number of Senate seats captured. Similarly the red area is  $\Pi^2$ , the aggregate of payoffs for party 2 or the number of Senate seats captured by party 2. The areas are calculated by multiplying the local district payoff times the district portion with that payoff. For example,  $\Pi^1 = (2)(\text{unchallenged districts of party 1}) + 1(\text{challenged districts by both parties}) + 0(\text{unchallenged districts of party 2})$ . In

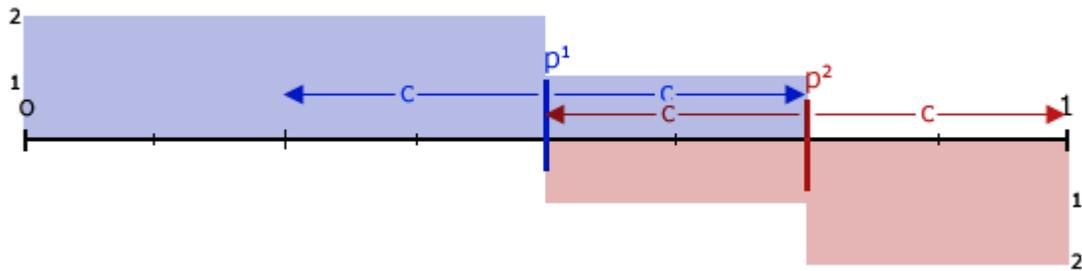
order to find an equilibrium, the parties are initially placed such that their credibility ranges do not overlap. Given the payoffs in that situation, is there a profitable deviation for party 1?



$$\begin{aligned} \Pi^1(2/8, 6/8) &= (2)(4/8) = 8/8 \\ \Pi^2(2/8, 6/8) &= (2)(4/8) = 8/8 \end{aligned}$$

$$\begin{aligned} C^1(2/8, 6/8) &= 0 < B = 1/2 \\ C^2(2/8, 6/8) &= 0 < B = 1/2 \end{aligned}$$

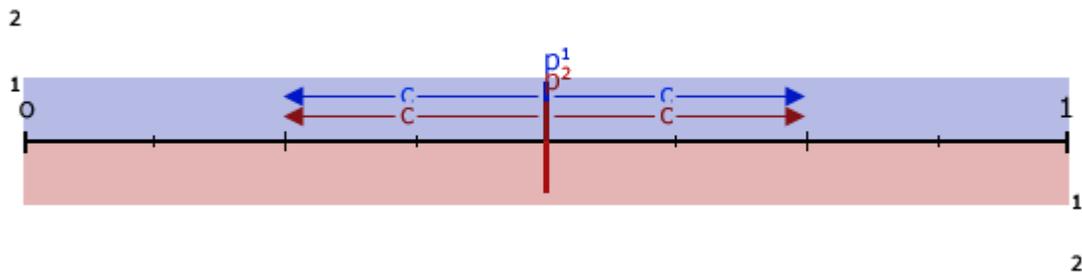
Party 1 can increase its payoffs by moving to the median  $p^m_{\text{national}}$ , as seen in the next diagram.



$$\begin{aligned} \Pi^1(4/8, 6/8) &= (2)(4/8) + (1)(2/8) = 10/8 \\ \Pi^2(4/8, 6/8) &= (2)(2/8) + (1)(2/8) = 6/8 \end{aligned}$$

$$\begin{aligned} C^1(4/8, 6/8) &= 1/4 < B = 1/2 \\ C^2(4/8, 6/8) &= 1/4 < B = 1/2 \end{aligned}$$

Given this move, party 2 will also move towards the median.



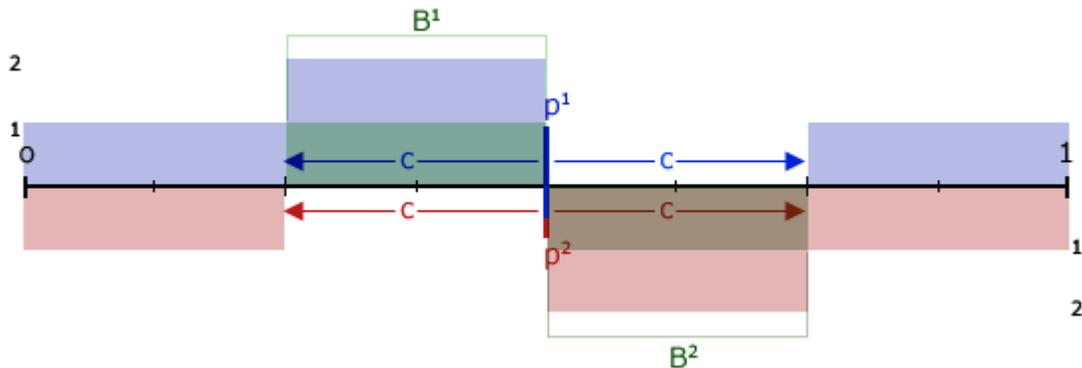
$$\begin{aligned} \Pi^1(4/8, 4/8) &= (1/2)(2)(4/8) + (1)(4/8) = 8/8 \\ \Pi^2(4/8, 4/8) &= (1/2)(2)(4/8) + (1)(4/8) = 8/8 \end{aligned}$$

$$\begin{aligned} C^1(4/8, 4/8) &= 1/2 = B = 1/2 \\ C^2(4/8, 4/8) &= 1/2 = B = 1/2 \end{aligned}$$

Once both parties converge to  $p^m_{\text{national}}$  there are no profitable deviations for either party. The equilibrium is  $(1/2, 1/2)$ . The two parties compete over their entire credibility range for a payoff of 1, and gain a payoff of 2 from all the other districts with probability  $1/2$ . This result is equivalent to Downs. Next I test the possibility that limits on fundraising could lead to divergence.

### 3.2 Two Party Model: Restricted Budget, Utility from Payoffs

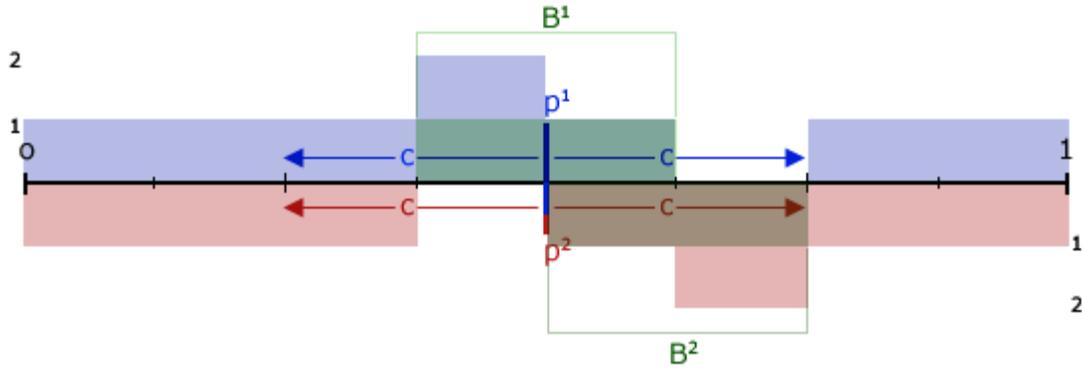
In the above example, parties could afford to challenge all the elections in the credibility range. Here the budget is limited to reflect campaign-finance legislation, or simply smaller budgets resulting from poor fundraising. For example, let  $B = 1c$ ,  $c = 1/4$ . It is assumed that both parties are equally affected by the restriction. Another step is introduced into the electoral competition game. First, the party decides which policy to play, then it decides which contestable elections it will challenge. Does this constraint affect the locations of the national policies? Starting at equilibrium  $(p^1 = 4/8, p^2 = 4/8)$  found in 3.1, the following diagram incorporates the budgets of party 1 and 2. The green areas labeled  $B^1$  and  $B^2$  represent the placement of these budgets, spanning all districts that are challenged by the particular party. Thus in the following diagram, party 1 is challenging elections on  $[2/8, 4/8]$  and party 2 is challenging those on  $[4/8, 6/8]$ .



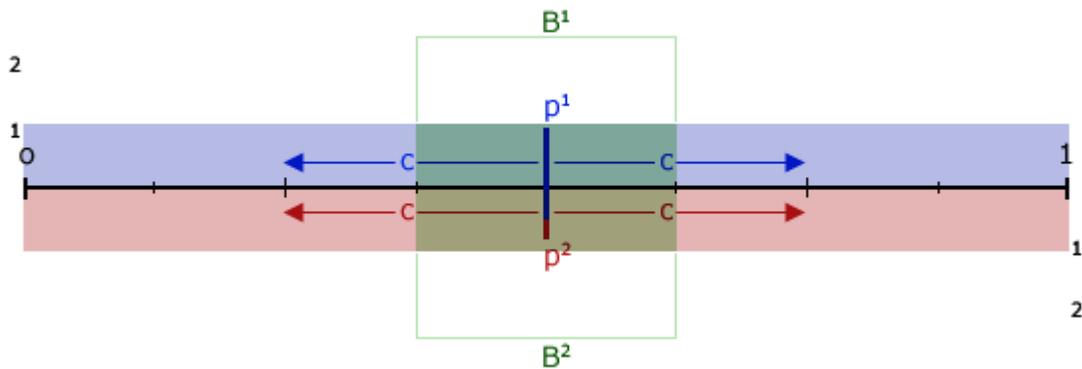
$$\begin{aligned} \Pi^1(4/8, 4/8) &= (2)(2/8) + (1/2)(2)(4/8) + (0)(2/8) = 8/8 \\ \Pi^2(4/8, 4/8) &= (2)(2/8) + (1/2)(2)(4/8) + (0)(2/8) = 8/8 \end{aligned}$$

$$\begin{aligned} C^1(4/8, 4/8) &= 1/4 = B = 1/4 \\ C^2(4/8, 4/8) &= 1/4 = B = 1/4 \end{aligned}$$

To see if there are any profitable deviations in terms of budget allocation, party 1 decides to finance central elections. The party platforms are kept at the median. Then  $B^2$  is adjusted.



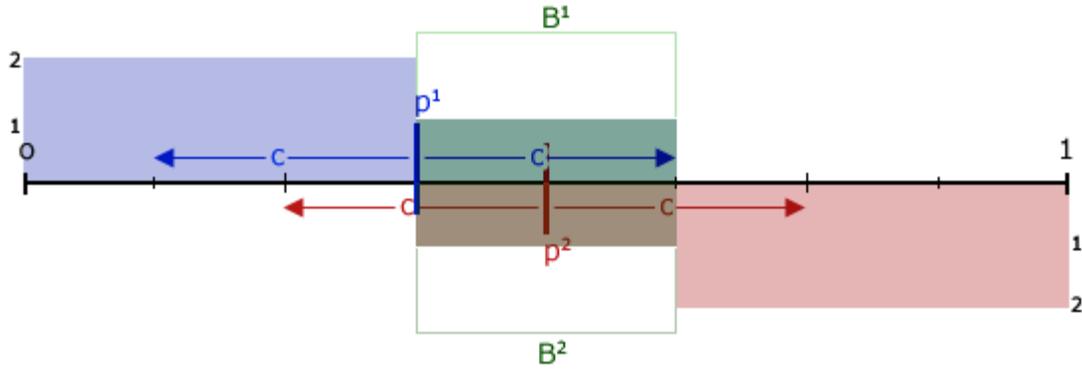
$$\begin{aligned} \Pi^1(4/8, 4/8) &= (2)(1/8) + (1)(3/8 + 1/8 + 2/8) + (0)(1/8) = 8/8 & C^1(4/8, 4/8) &= 1/4 = B = 1/4 \\ \Pi^2(4/8, 4/8) &= (2)(1/8) + (1)(3/8 + 3/8) + (0)(1/8) = 8/8 & C^2(4/8, 4/8) &= 1/4 = B = 1/4 \end{aligned}$$



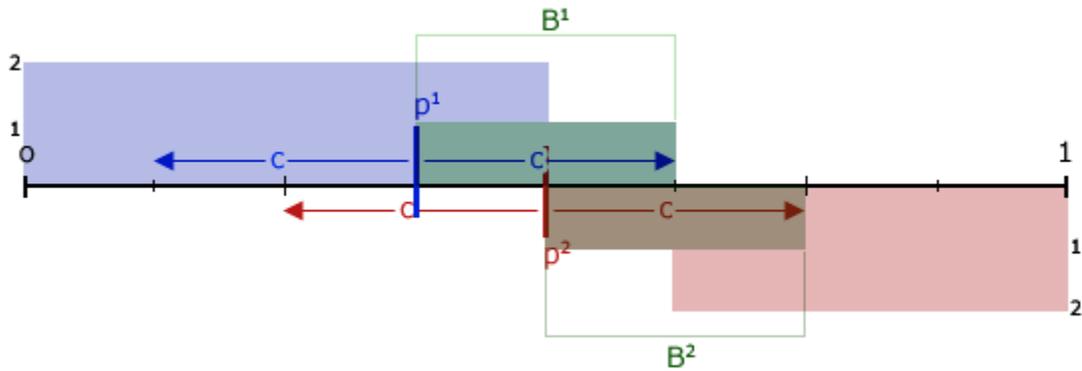
$$\begin{aligned} \Pi^1(4/8, 4/8) &= (2)(0) + (1)(8/8) + (0)(0) = 8/8 & C^1(4/8, 4/8) &= 1/4 = B = 1/4 \\ \Pi^2(4/8, 4/8) &= (2)(0) + (1)(8/8) + (0)(0) = 8/8 & C^2(4/8, 4/8)^{10} &= 1/4 = B = 1/4 \end{aligned}$$

Nothing has changed. The parties have no incentive to move their budgeting as long as they both play  $p^{\text{national}}$ . This result actually turns out to be the unique Downs equilibrium. Unfortunately, there are no incentives leading to divergence. Consider the example on the next page. Party 2 plays  $p^2 = 4/8$  as before; party 1 plays  $p^1 = 3/8$  but keeps its budgeting at the center. Moving  $B^1$  to any other range would decrease the payoffs for party 1. Thus it seems as if the situation is identical to convergence. However, party 2 can move its budget either to the left or to the right. Moving  $B^2$  to the right limit of the credibility range yields a  $\Pi^2$  of  $7/8$ , while placing  $B^2$  at the left limit of the credibility range provides  $9/8$ . Party 2 will deviate and move its budget such that it spans  $[2/8, 4/8]$ . Party 1 will have to respond by converging to the national median policy.

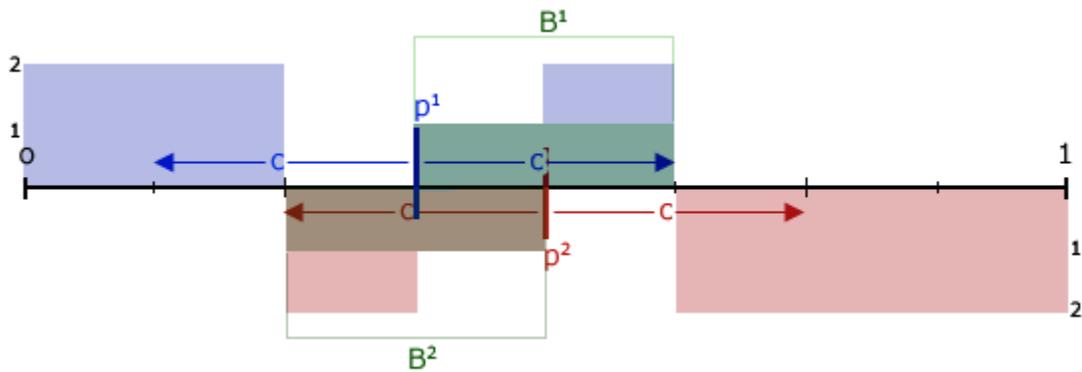
<sup>10</sup> For shorthand,  $C_i(p^i, p)$  will from now on be written as  $C_i$ , and the arithmetic will not be explicitly shown.



$$\Pi^1(3/8, 4/8) = 8/8; C^1 = 1/4 = B = 1/4 \quad \Pi^2(3/8, 4/8) = 8/8; C^2 = 1/4 = B = 1/4$$



$$\Pi^1(3/8, 4/8) = 9/8; C^1 = 1/4 = B = 1/4 \quad \Pi^2(3/8, 4/8) = 7/8; C^2 = 1/4 = B = 1/4$$



$$\Pi^1(3/8, 4/8) = 7/8; C^1 = 1/4 = B = 1/4 \quad \Pi^2(3/8, 4/8) = 9/8; C^2 = 1/4 = B = 1/4$$

If the initial position of the policies is not  $p^1 = p^2 = 4/8$ , both parties will gain the highest payoff by budgeting *aggressively*. Aggressive budgeting means that  $B^i$  is placed on the side of  $p^i$  facing  $p^j$ . This dynamic leads to convergence to the median. Once  $p^1 = p^2 = 4/8$ , the budget can be located anywhere in the credibility range. The spending constraint does not alter the behavior of the parties. Some other force is necessary to push  $p^1$  and  $p^2$  apart – Wittman’s ideological preferences, or

disutility from spending the entire budget (Eyster and Kittsteiner, 2004), which is used in the next section.

### 3.3 Two Party Model: Unrestricted Budget, Utility from Payoffs and Money

Political parties care about more than just winning elections. Consider campaign finance – the assumption that the party is indifferent between spending the entire budget and saving money is unrealistic. Rather, a party is likely to receive utility from unspent money because it can use that money to finance other projects. These projects would not be related to electoral competition. For example, sponsoring events that bring district representatives together and create unity within the party is likely to make governance more effective. Saved money can also be used to lobby and to affect issues that cannot be resolved in the legislative branch. To reflect the party's incentives regarding budget spending, this section amends the utility function to include utility gained by parties from saving money ( $V(S^i)$ ). Given this new utility function, I find a set of divergent two-party equilibria. It will be shown that depending on a party's marginal utility of saved money, a party will desire to save some fixed amount no matter where its opponent is located. In order to save this amount, the party will move a given distance away from its opponent's policy position.

Function  $S^i$  stands for savings, which is  $B$  minus the costs of financing elections.  $V$  is the utility function that indicates how much utility a party receives from  $S^i$ , defined as  $(S^i)^{1/x}$ , where  $1/x$  is an arbitrary rate. A specific formulation of  $V$  is used in this paper to find equilibria:  $x = 2$  and  $V = (S^i)^{1/2}$ . This function reflects diminishing marginal returns from holding money – each additional dollar saved yields less utility. There is now a tension in the party's policy decision: financing elections decreases the total utility of the party, while winning elections increases that utility. The relevant redefinitions are below<sup>11</sup>:

---

<sup>11</sup> It is possible to assign weights to  $\Pi^i(p^i, p^j)$  and  $V(S^i)$  in the utility function:  $U^i = a(\Pi^i) + b(V(S^i))$ . However, it is assumed that  $a = b = 1$  for simplicity. Intuitively, it can be seen that if  $a > b$ , parties will desire to come closer together; if  $a < b$ , parties will wish to move apart and increase savings beyond the result found in this section.

$$U^i = U [\Pi^i(p^i, p^j), D^i] = \Pi^i(p^i, p^j) + V(S^i)$$

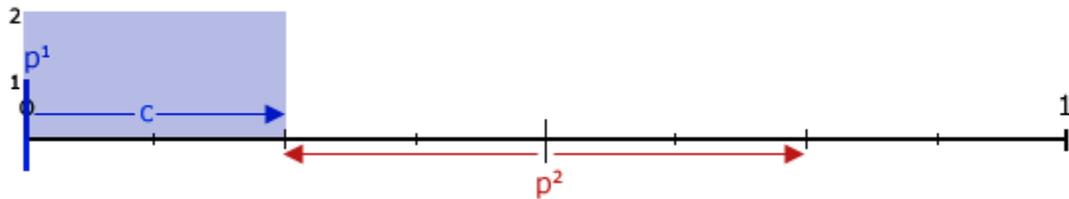
$$S^i = (B - C^i(p^i, p^j))$$

$$V(S^i) = (S^i)^{1/x}; x = 2 \text{ unless stated otherwise}$$

$$\delta U^i / \delta S^i > 0$$

$$C^i(p^i, p^j) \leq B = 2c$$

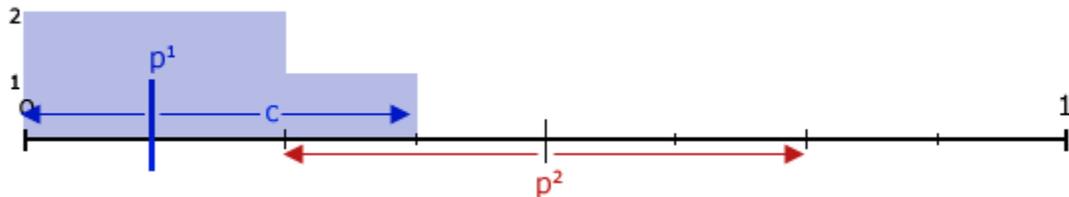
In order to find the equilibrium position of both parties, the policy of party 2 is fixed and the incentives of party 1 are examined. This is done by compiling the payoffs party 1 receives along the policy space into a single graph – a utility frontier. Policy  $p^2$  is fixed at  $4/8$ .



$$\begin{aligned} \Pi^1(0, 4/8) &= (2)(2/8) + (1)(0) + (0)(6/8) = 4/8 \\ U^1(0, 4/8) &= 1.207 \end{aligned}$$

$$S^1 = 4/8$$

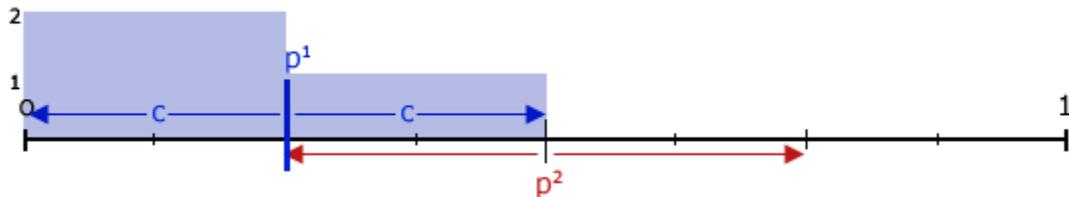
$$V(S^1) = .707$$



$$\begin{aligned} \Pi^1(1/8, 4/8) &= (2)(2/8) + (1)(1/8) + (0)(5/8) = 5/8 \\ U^1(1/8, 4/8) &= 1.237 \end{aligned}$$

$$S^1 = 3/8$$

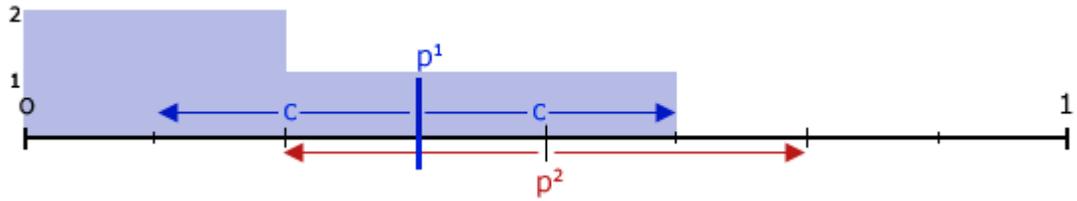
$$V(S^1) = .612$$



$$\begin{aligned} \Pi^1(2/8, 4/8) &= (2)(2/8) + (1)(2/8) + (0)(4/8) = 6/8 \\ U^1(2/8, 4/8) &= 1.250 \end{aligned}$$

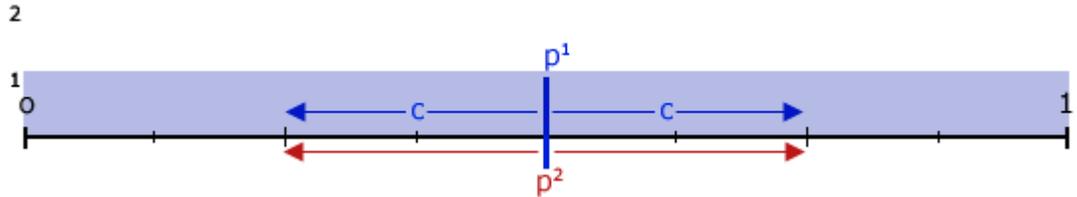
$$S^1 = 2/8$$

$$V(S^1) = .500$$



$$\Pi^1(3/8, 4/8) = (2)(2/8) + (1)(3/8) + (0)(3/8) = 7/8 \quad S^1 = 1/8 \quad V(S^1) = .353$$

$$U^1(3/8, 4/8) = 1.229$$

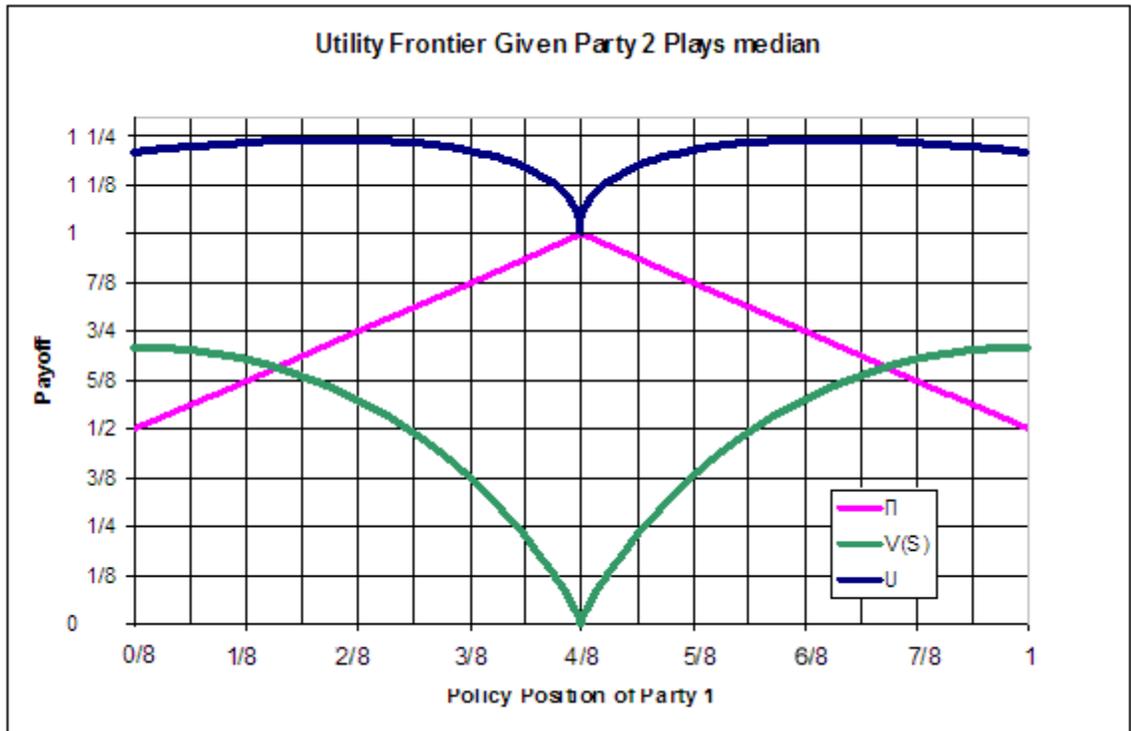


$$\Pi^1(4/8, 4/8) = (2)(0) + (1)(8/8) + (0)(0/8) = 8/8 \quad S^1 = 0/8 \quad V(S^1) = .000$$

$$U^1(4/8, 4/8) = 1.000$$

The payoffs and costs are symmetrical in this formulation, thus all the policies greater than  $p^m_{\text{national}}$  are not explicitly shown above. This information is graphed in Graph 3.3.1.

Graph 3.3.1



The utility function  $U$  can be seen to approximately reach a maximum at  $2/8$  in graph 3.3.1. A more precise way to find a solution is to differentiate the utility function with respect to the policy of party 1. It is possible to use definitions 6.1 and 6.5 in the appendix because  $c$  is sufficiently small. In this example so far  $p^1 < p^2$ , therefore there are two situations: that of competition due to both parties challenging elections ( $p^1 + 2c > p^2$ ) and that of non-competition ( $p^1 + 2c < p^2$ ). The case of non-competition is trivial, possible only at  $p^1 = 0/8$ , because the utility reached is significantly smaller than when both parties challenge. Therefore,  $\Pi^1 = p^1 + p^2$  and  $V(S^1) = (B - p^1 + p^2 - 2c)^{1/2}$ .

$$\Pi^1(p^1, 1/2) = p^1 + 1/2$$

$$V(S^1) = (1/2 - p^1 + 1/2 - 1/2)^{1/2} = (1/2 - p^1)^{1/2}$$

$$U^1 = p^1 + 1/2 + (1/2 - p^1)^{1/2}$$

To find a local maximum, set  $\delta U^1 / \delta p^1$  equal to zero:

$$\delta[p^1 + 1/2 + (1/2 - p^1)^{1/2}] / \delta p^1 = 0 \quad \rightarrow \quad 1 + 0 + 1/2(1/2 - p^1)^{-1/2}(-1) = 0$$

$$2 = - (1/2 - p^1)^{-1/2} \rightarrow 4 = 1/(1/2 - p^1) \rightarrow 1/4 = 1/2 - p^1 \quad \rightarrow \quad \underline{p^1 = 2/8}$$

Party 1 locates  $p^1$  at  $2/8$ , or symmetrically at  $6/8$ , on the national policy space. Either of these locations will yield the highest possible payoff given that  $p^2 = 4/8$ . The distance between  $p^1$  and  $p^2$  is  $1/4$ . This distance allows both parties to save a total of  $1/4$  each, which is exactly where the  $V$  function reaches a slope of  $-1$ . The point at which  $V$  has a slope of  $-1$  is important because  $\Pi$  has a slope of  $1$ , and thus the  $U$  function has slope  $= 0$ . To double check the accuracy of this  $1/4$  savings, I set  $\delta V(S^1) / \delta p^1$  to  $1$ .

$$\delta[(p^2 - p^1)^{1/2}] / \delta p^1 = -1 \rightarrow 1/2(p^2 - p^1)^{-1/2} = 1 \quad \rightarrow \quad (p^2 - p^1)^{1/2} = 1/2$$

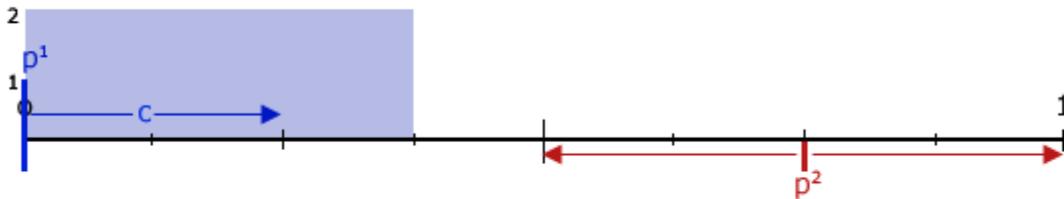
$$\underline{(p^2 - p^1) = 1/4}$$

In the case of  $p^1 > p^2$ ,  $V(S^1)$  equals  $(B - p^2 + p^1 - 2c)^{1/2}$ . This function has slope 1, counteracting slope -1 of  $\Pi^1$ , also when the distance between parties is  $1/4$ :

$$\delta[(-p^2 + p^1)^{1/2}]/\delta p^1 = 1 \rightarrow 1/2(-p^2 + p^1)^{-1/2} = 1 \rightarrow (-p^2 + p^1)^{1/2} = 1/2$$

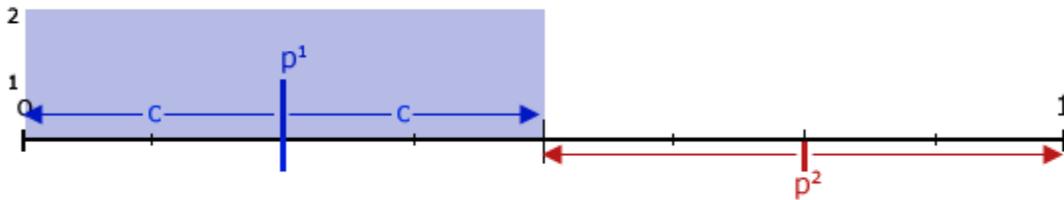
$$(p^1 - p^2) = 1/4$$

Given party 1's move, how does party 2 respond? I examine the response of the opponent to  $p^1 = 6/8$ , which is the utility maximizing position of party 1 in the previous example, hoping to find an equilibrium at policy pair  $(4/8, 6/8)$ . The locations of the parties are switched to keep the "fixed" party the same, thereby keeping the solution on the same side.



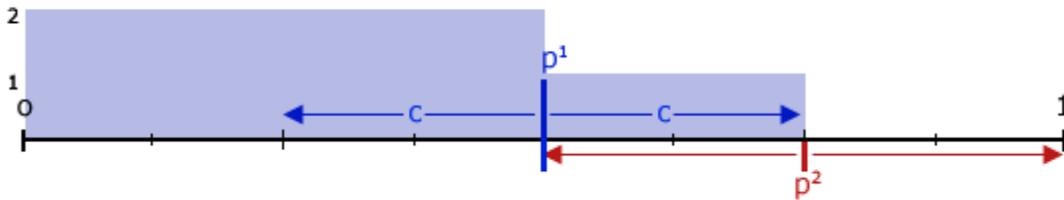
$$\begin{aligned} \Pi^1(0, 6/8) &= (2)(3/8) + (1)(0) + (0)(5/8) = 6/8 \\ U^1(0, 6/8) &= 1.457 \end{aligned}$$

$$S^1 = 4/8 \quad V(S^1) = .707$$



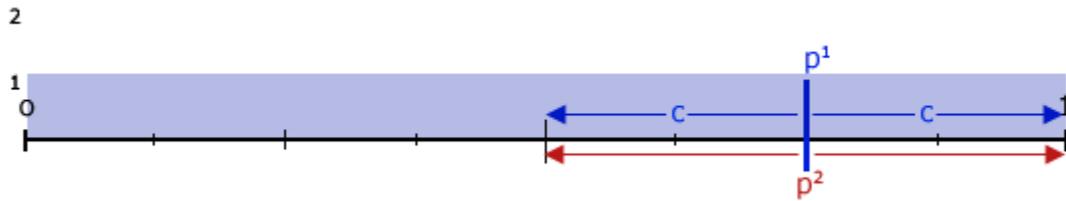
$$\begin{aligned} \Pi^1(2/8, 6/8) &= (2)(4/8) + (1)(0) + (0)(4/8) = 8/8 \\ U^1(2/8, 6/8) &= 1.707 \end{aligned}$$

$$S^1 = 4/8 \quad V(S^1) = .707$$



$$\begin{aligned} \Pi^1(4/8, 6/8) &= (2)(4/8) + (1)(2/8) + (0)(2/8) = 10/8 \\ U^1(4/8, 6/8) &= 1.750 \end{aligned}$$

$$S^1 = 2/8 \quad V(S^1) = .5$$



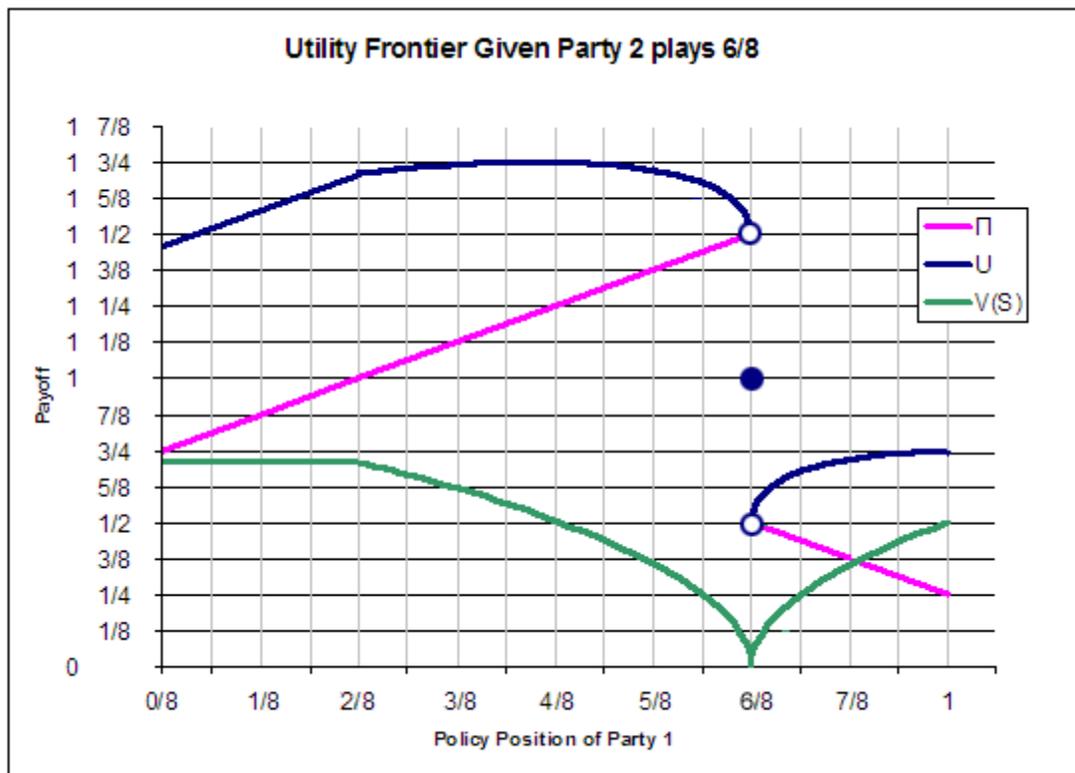
$$\Pi^1(6/8, 6/8) = (2)(0/8) + (1)(8/8) + (0)(0/8) = 8/8$$

$$U^1(4/8, 6/8) = 1.0000$$

$$S^1 = 0/8 \quad V(S^1) = 0$$

Graph 3.3.2 shows the utility frontier given the above information.  $\Pi^1$  increases linearly approaching  $12/8$ , but is discontinuous at  $p^1 = p^2$ , yielding  $8/8$  instead. Payoffs to the extreme right, where  $p^1 > p^2$ , are significantly smaller than those where  $p^1 < p^2$ .

**Graph 3.3.2**



On the relevant range function  $\Pi^1(p^1, 3/4)$  is  $(p^1 + 3/4)$  and  $V(S^1)$  is  $(B - p^1 + p^2 - 2c)^{1/2}$ .

The utility function adds them together:

$$\Pi^1(p^1, 3/4) = p^1 + 3/4 \quad V(S^1) = (1/2 - p^1 + 3/4 - 1/2)^{1/2} = (3/4 - p^1)^{1/2}$$

$$U^1 = p^1 + 3/4 + (3/4 - p^1)^{1/2}$$

To find the maximum, set  $\delta U^1 / \delta p^1$  equal to zero.

$$\delta[p^1 + 3/4 + (3/4 - p^1)^{1/2}] / \delta p^1 = 0 \quad \rightarrow \quad 1 + 0 + 1/2(3/4 - p^1)^{-1/2}(-1) = 0$$

$$2 = (3/4 - p^1)^{-1/2} \rightarrow 4 = 1/(3/4 - p^1) \rightarrow 1/4 = 3/4 - p^1 \rightarrow \quad \underline{p^1 = 4/8}$$

When party 2 plays  $p^2 = 6/8$ , party 1's best response is to play the median  $p^1 = 4/8$ . As before, parties wish to achieve  $S^1 = 1/4$ , moving apart a distance of  $1/4$ . The asymmetric policy pair  $(4/8, 6/8)$  is a stable equilibrium. Similarly, other equilibria with the distance of  $1/4$  between the two party policies are equilibria, including  $(2/8, 4/8)$ ,  $(6/8, 4/8)$  and the symmetric equilibrium  $(3/8, 5/8)$ .

The above example allowed us to find how much parties wish to save given  $x = 2$  and an unrestricted budget  $B = 2c$ . Below, I generalize the result for all values of  $x$ .

**If  $p^1 < p^2$**

$$\Pi^1(p^1, p^2) = p^1 + p^2$$

$$V(S^1) = ((B - 2c) + p^2 - p^1)^{1/x}$$

$$U^1 = p^1 + p^2 + (p^2 - p^1)^{1/x}$$

$$\delta [ p^1 + p^2 + (p^2 - p^1)^{1/x} ] / \delta p^1 = 0$$

$$1 + 0 + (1/x)(p^2 - p^1)^{(1/x - x/x)}(-1) = 0$$

$$1 = (1/x)(p^2 - p^1)^{(1/x - x/x)}$$

$$x = (p^2 - p^1)^{((1-x)/x)}$$

$$x^{(x/(1-x))} = (p^2 - p^1)$$

$$p^1 + x^{(x/(1-x))} = p^2$$

$$\underline{p^1 = p^2 - x^{(x/(1-x))}}$$

**If  $p^1 > p^2$**

$$\Pi^1(p^1, p^2) = 2 - (p^1 + p^2)$$

$$V(S^1) = ((B - 2c) + p^1 - p^2)^{1/x}$$

$$U^1 = 2 - (p^1 + p^2) + (p^1 - p^2)^{1/x}$$

$$\delta [ 2 - (p^1 + p^2) + (p^1 - p^2)^{1/x} ] / \delta p^1 = 0$$

$$0 - 1 - 0 + (1/x)(p^1 - p^2)^{(1/x - x/x)} = 0$$

$$1 = (1/x)(p^1 - p^2)^{(1/x - x/x)}$$

$$x = (p^1 - p^2)^{((1-x)/x)}$$

$$x^{(x/(1-x))} = (p^1 - p^2)$$

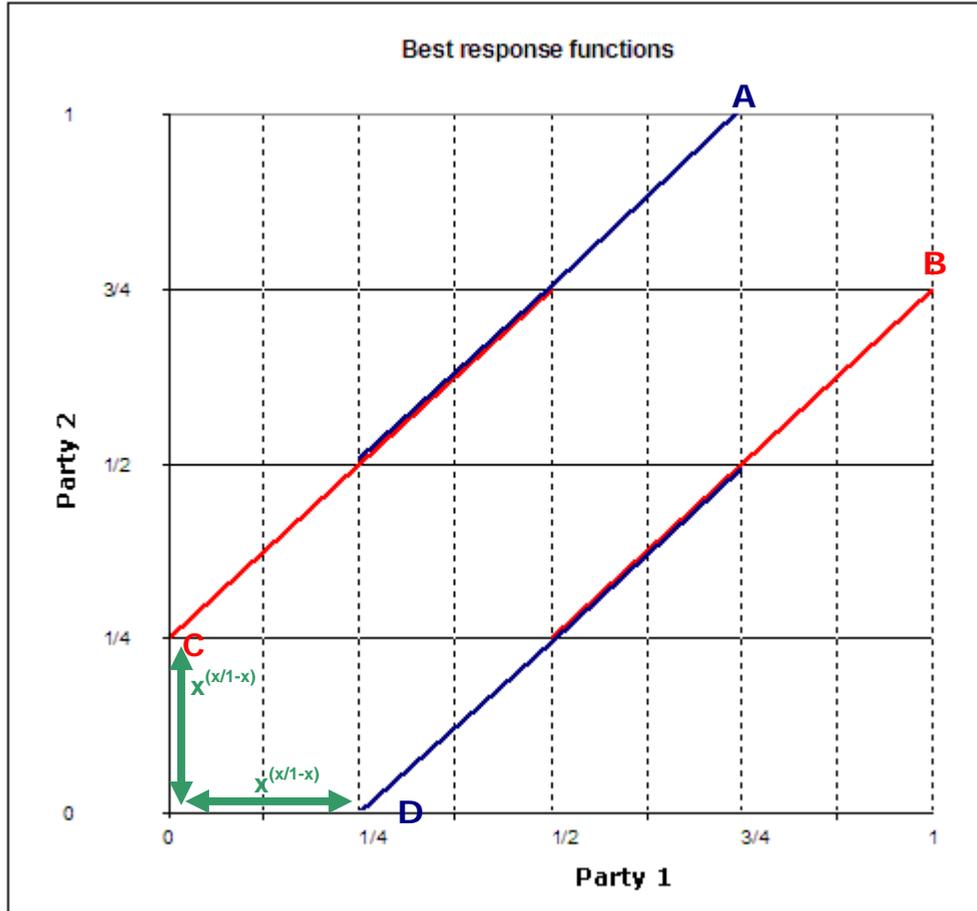
$$-p^1 + x^{(x/(1-x))} = -p^2$$

$$\underline{p^1 = p^2 + x^{(x/(1-x))}}$$

Therefore,  $|p^1 - p^2| = x^{(x/(1-x))}$ . As mentioned before, this distance is the result of a desire to save up to the point of where the slope of  $V(S)$  is 1 (or -1). A party will always try to save such that  $S^i = x^{(x/(1-x))}$  to maximize utility. If  $p^1 = p^2$ , then  $p^1$  can be on either side of the median given this

distance. It is important to note that *policy*  $p^{national} \in [p^1, p^2]$ . A party will never play a policy on the flank of its opponent if the median is not between them, because that party can easily switch to the opposite flank and achieve a higher  $\Pi$  without altering its savings. The best response functions of the two parties, given  $x = 2$ , are shown in graph 3.3.4.

Graph 3.3.4



Increasing the value of  $x$ , which inversely reflects the marginal utility of money, decreases the diagonal distance between A and D as well as C and B above. If the parties are closer together, they are saving less – this makes sense if the parties care less about savings with higher  $x$ . The two parties always diverge at  $x > 1$ , approaching full divergence at  $x = 1 + \epsilon$ . As  $x$  approaches infinity, the two parties approach convergence. Values of  $x \leq 1$  are trivial because they connote an increasing

marginal utility of money, and lead to a distance between parties that is either undefined or negative and large.

This section has shown that if parties care about savings, they will not want to spend their entire budget. Given a utility function that rewards parties for not financing all contestable elections, divergence becomes possible. Party  $i$  will always wish to be  $x^{x/(1-x)}$  away from party  $j$ , as long as the national median policy is between them. The next section explores the effect of a budget restriction on this conclusion.

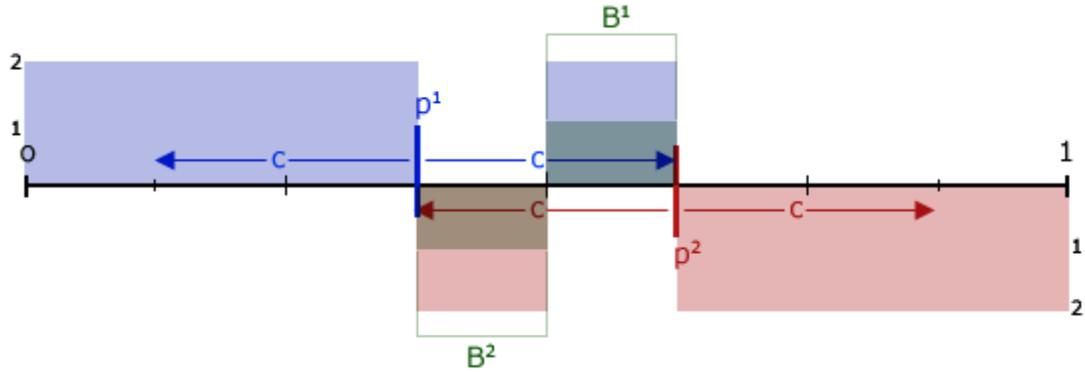
### 3.4 Two Party Model: Restricted Budget, Utility from Payoffs and Money

In section 3.2, it was established that restraining the budget leads to two results: initial *aggressive budgeting*, followed by convergence to the national median and indifference regarding where the budget is placed. Aggressive budgeting implies that party  $i$  places its budget  $B^i$ , or the largest part of  $B^i$ , facing the other party's position  $p^i$ . For simplicity, it will be assumed that parties pursue aggressive budgeting no matter what, and do not change to a different budgeting choice if they are indifferent between two options.

The last section also showed that a party will want to position itself such that  $x^{x/(1-x)}$  is saved. This is so because the slope of  $V(S^i)$  equals the negative slope of  $\Pi^i$  at that point, and is the maximum of the utility function. Thus if a party finances all the elections up to  $x^{x/(1-x)}$  away from the competing platform, the next election's marginal payoff from votes is higher than the marginal utility from savings. Conversely, the marginal gain in savings provides more utility than the marginal payoff from votes for all elections within the distance  $x^{x/(1-x)}$ . For  $x = 2$ ,  $x^{x/(1-x)}$  equals  $1/4$ . A party will always want to save  $1/4$  and spend the rest on financing elections, no more and no less. When the budget is unrestricted, or equal to  $1/2$ , the parties push apart in order to limit the overlap of contestable elections to  $1/4$ . In the case of a restricted budget, the situation is somewhat different. I first examine the situation with a budget constraint of  $B = 1/8$ . Next, I look at  $B = 2/8$  and then  $B =$

3/8. It is again assumed that  $c = 1/2$ ; this provides cleaner results because  $c$  is not “too large”, requiring the use of min/max functions.

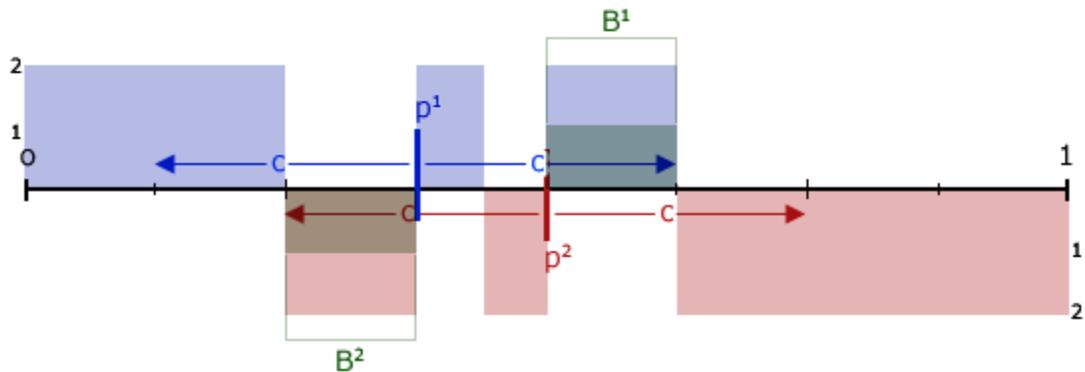
Below is equilibrium  $(3/8, 5/8)$  from the previous section, but the budget is restricted to  $B = 1/8$  and the two parties are budgeting aggressively.



$$\begin{aligned} \Pi^1(3/8, 5/8) &= 8/8; S^1 = 0, V(S^1) = 0 \\ U^1(3/8, 5/8) &= 1.000 \end{aligned}$$

$$\begin{aligned} \Pi^2(3/8, 5/8) &= 8/8; S^1 = 0, V(S^1) = 0 \\ U^2(3/8, 5/8) &= 1.000 \end{aligned}$$

Is there a beneficial deviation for party 2? Moving  $p^2$  to  $p^m_{\text{national}}$  yields higher  $U^2$ .



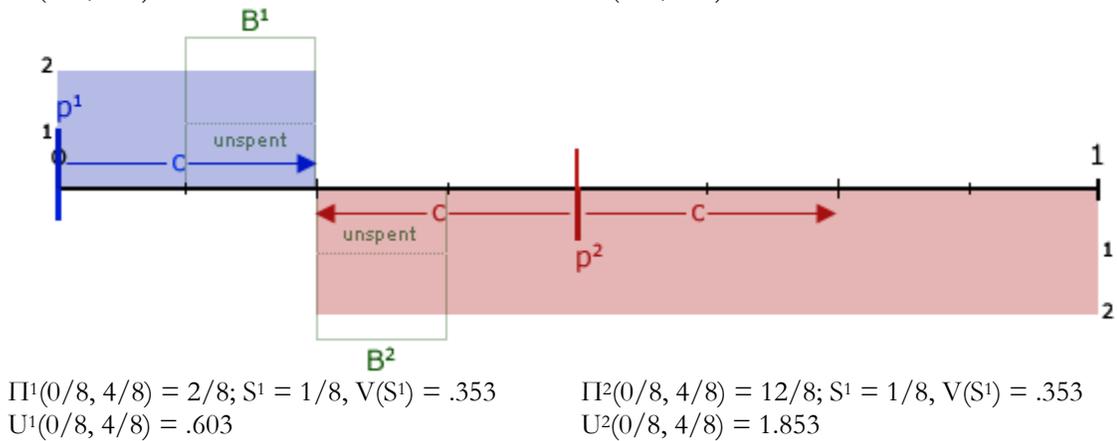
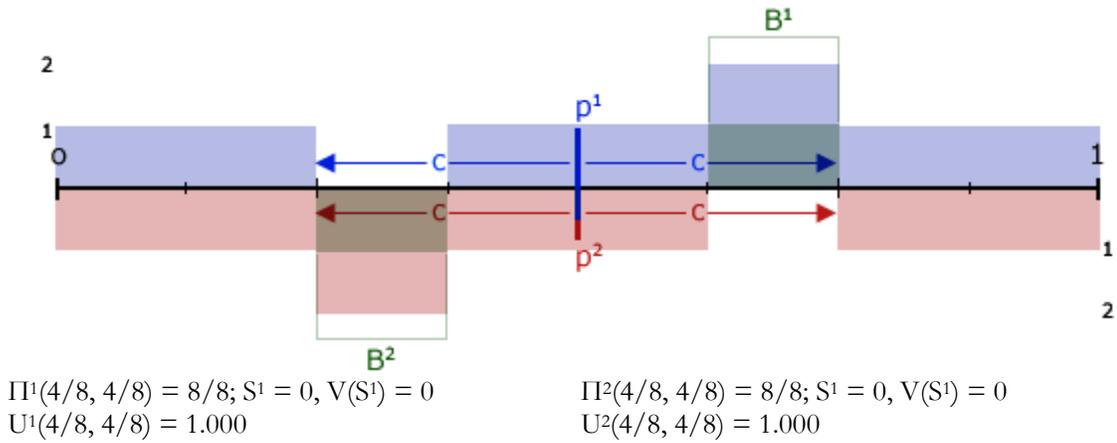
$$\begin{aligned} \Pi^1(3/8, 4/8) &= 7/8; S^1 = 0, V(S^1) = 0 \\ U^1(3/8, 4/8) &= .875 \end{aligned}$$

$$\begin{aligned} \Pi^2(3/8, 4/8) &= 9/8; S^1 = 0, V(S^1) = 0 \\ U^2(3/8, 4/8) &= 1.125 \end{aligned}$$

Party 1 has two potential responses in the above situation. It can converge to the median, thereby increasing  $\Pi^1$ , or it can attempt to increase utility from savings  $V(S^1)$  through divergence. The key difference in this case is that it is *difficult to save*. As long as there is an overlap in the credibility ranges of the two parties, that overlap has to be funded.<sup>12</sup> Moving  $p^1$  from  $3/8$  to  $2/8$  will

<sup>12</sup> This is an assumption that is necessary for divergence. As stated before, without this assumption convergence dominates since a party can play  $p^m_{\text{national}}$  and also choose to save the requisite part of its budget.

not increase savings by  $1/8$ ; instead it will just shift the budget from  $[4/8, 5/8]$  to  $[3/8, 4/8]$ . To save,  $p^1$  has to be on  $[0/8, 1/8)$ , or the symmetric  $(7/8, 8/8)$  if  $p^2 = 4/8$ .



In the case of convergence the utility of party 1 is equal to 1. In the situation where none of the budget is spent, utility equals .603. Clearly, it is preferable for the party to converge in this case.  $V(S^1)$  would have to be greater than  $6/8^{13}$  for the non-overlap of credibility ranges to be preferable to convergence; this could happen if  $x$  was  $> 7.23^{14}$ , which implies an improbably strong discount rate.<sup>15</sup> Nonetheless, when  $x = 2$  convergence is preferable to the gain from savings. The maximum of  $V(S^i)$  is smaller when the budget is restricted than when the budget is unrestricted. There is also a kind of

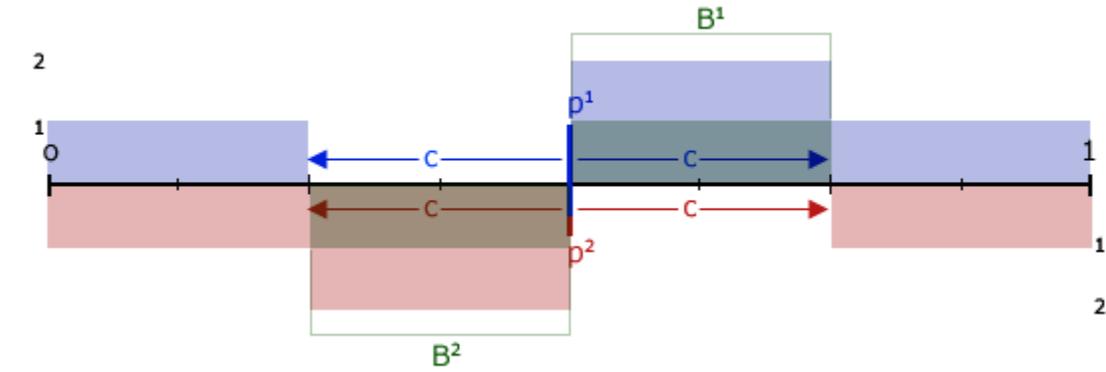
<sup>13</sup>  $[\Pi^1(4/8, 4/8) = 8/8] - [\Pi^1(0/8, 4/8) = 2/8] = 6/8$ . This difference can be called the opportunity cost of divergence.

<sup>14</sup> Set  $(1/8)^{1/x} = 6/8$  and solve for  $x$ .  $(1/8)$  is maximum savings,  $(6/8)$  is the difference in  $\Pi^1$  discussed in previous footnote.

<sup>15</sup> Alternately, it would be possible for the budget restriction of  $B = c/2$  to yield non-overlapping divergence if  $c$  was smaller than  $1/4$ . The reason for that is simple: as the credibility range shrinks for  $p^2$ ,  $p^1$  would be able to move closer to  $p^2$  on  $[0, 1]$  without incurring any loss in savings. As a result,  $\Pi^1$  increases and the opportunity cost of not challenging elections decreases.

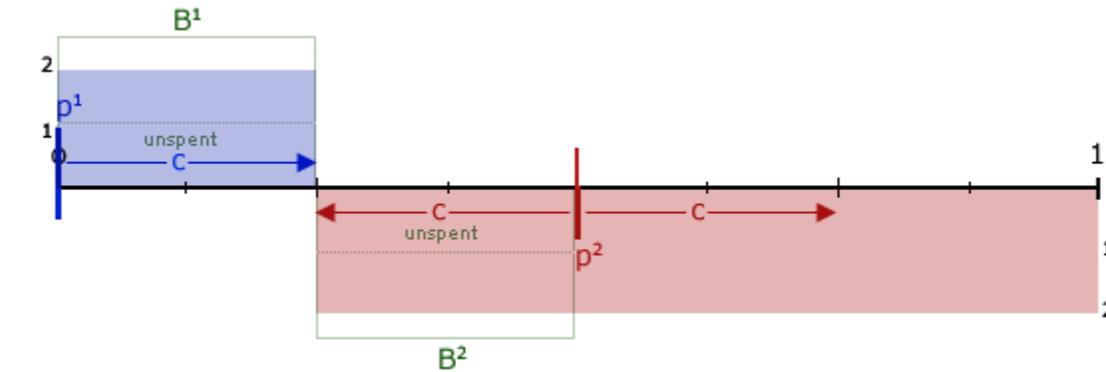
initial cost for savings to take effect, which equals  $(2c - B)$ . A party has to move  $(2c - B)$  away from the policy position of its opponent before saving can even start. Because it is each party's utility maximizing response,  $(4/8, 4/8)$  is the unique equilibrium.

To test a restriction of  $B = 1/2$ , I start with the equilibrium  $(4/8, 4/8)$  and examine the incentives of party 1.



$$\begin{aligned} \Pi^1(4/8, 4/8) &= 8/8; S^1 = 0, V(S^1) = 0 \\ U^1(4/8, 4/8) &= 1.000 \end{aligned}$$

$$\begin{aligned} \Pi^2(4/8, 4/8) &= 8/8; S^1 = 0, V(S^1) = 0 \\ U^2(4/8, 4/8) &= 1.000 \end{aligned}$$



$$\begin{aligned} \Pi^1(0/8, 4/8) &= 4/8; S^1 = 2/8, V(S^1) = 4/8 \\ U^1(0/8, 4/8) &= 1.000 \end{aligned}$$

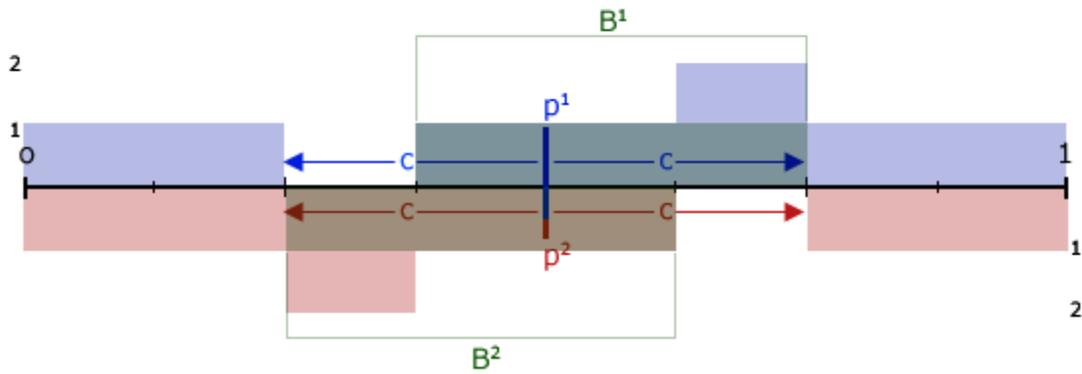
$$\begin{aligned} \Pi^2(0/8, 4/8) &= 12/8; S^1 = 2/8, V(S^1) = 4/8 \\ U^2(0/8, 4/8) &= 2.000 \end{aligned}$$

Utility from convergence is the same as that of non-overlap:  $\Pi^1(4/8, 4/8) = \Pi^1(0/8, 4/8) = 8/8$ . The value of  $x$  that yields the same total utility is precisely 2.<sup>16</sup> We know that  $x^{x/(1-x)}$  equals 1/4 and therefore all savings up to that amount yield more utility than the opportunity cost lost in payoffs from winning elections. As a result, moving  $p^1$  closer to the median short of convergence will

<sup>16</sup> Set  $(2/8)^{1/x} = 4/8$  and solve for  $x$ .  $(2/8)$  is maximum savings,  $(4/8)$  is the difference in  $\Pi^1$  between convergence and non-overlap divergence.

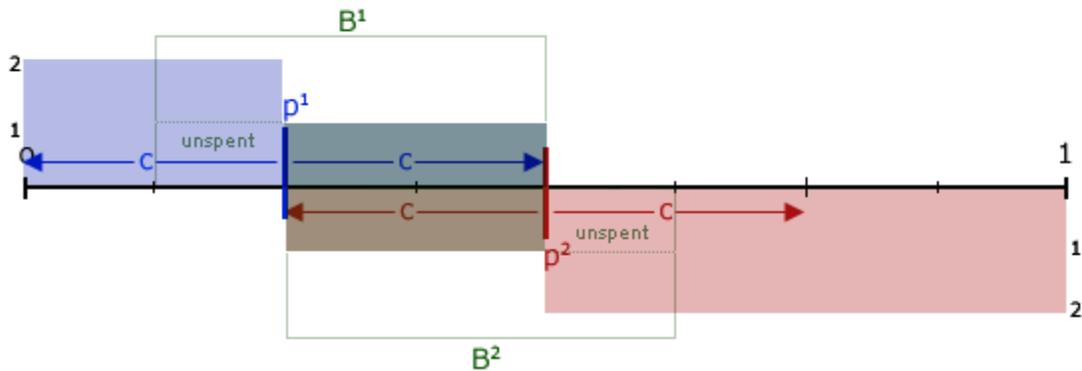
decrease total utility. If party 2 initially plays  $p^2 = 4/8$ , the possible equilibria are  $(0/8, 4/8)$ ,  $(4/8, 4/8)$  and  $(8/8, 4/8)$ . Otherwise, the parties will keep a distance of  $1/2$  to achieve the maximum payoff from savings, yielding a set of equilibria similar to that of section 3.3. Convergence should not occur at policy pairs other than  $(p^m_{\text{national}}, p^m_{\text{national}})$  – in policy pair  $(p^1, p^1)$  party 2 can always move to the national median and increase utility since  $U(p^1, p^1) < U(p^1, p^m_{\text{national}})$ .

The next specification involves a budget restriction of  $B = 3/8$ , meaning that the potential budget is larger than  $x^{(x/(1-x))}$ , which is the amount of the budget that provides higher utility from saving than from spending. The party should thus seek to challenge some elections. As seen in the unrestricted budget examples, parties position themselves such that  $x^{(x/(1-x))}$  remains unspent. Below, I start again with the convergent policy pair.



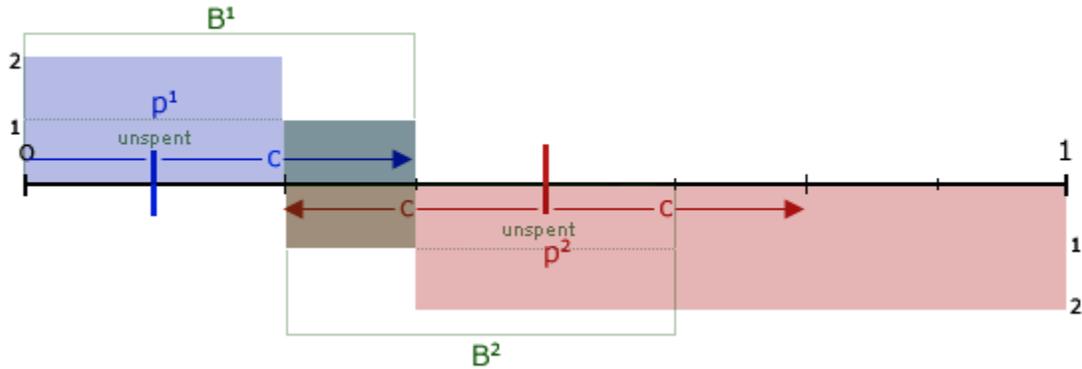
$$\begin{aligned} \Pi^1(4/8, 4/8) &= 8/8; S^1 = 0, V(S^1) = 0 \\ U^1(4/8, 4/8) &= 1.000 \end{aligned}$$

$$\begin{aligned} \Pi^2(p^1, p^2) &= 8/8; S^1 = 0, V(S^1) = 0 \\ U^2(4/8, 4/8) &= 1.000 \end{aligned}$$



$$\begin{aligned} \Pi^1(2/8, 4/8) &= 6/8; S^1 = 1/8, V(S^1) = .353 \\ U^1(2/8, 4/8) &= 1.103 \end{aligned}$$

$$\begin{aligned} \Pi^2(2/8, 4/8) &= 10/8; S^1 = 1/8, V(S^1) = .353 \\ U^2(2/8, 4/8) &= 1.603 \end{aligned}$$



$$\begin{aligned} \Pi^1(1/8, 4/8) &= 5/8; S^1 = 2/8, V(S^1) = 4/8 \\ U^1(1/8, 4/8) &= 1.125 \text{ (equilibrium position)} \end{aligned}$$

$$\begin{aligned} \Pi^2(1/8, 4/8) &= 11/8; S^1 = 2/8, V(S^1) = 4/8 \\ U^2(1/8, 4/8) &= 1.875 \text{ (equilibrium position)} \end{aligned}$$

The parties still maximize utility by saving up to  $x^{(x/(1-x))}$ , which means that the distance between parties is  $[x^{(x/(1-x))} + (2c - B)]$ . To quickly prove this, set the slope of  $V(S^1)$  to -1 for  $p^1 < p^2$ , as was done in the previous section, but without removing the budget notation. Function  $V(S^1)$  comes from Appendix Definition 6.5. Note that  $c$  is not “too large” and that the budget is bigger than  $x^{(x/(1-x))}$ .

$$V(S) = (B - 2c + p^2 - p^1)^{1/x}$$

$$\delta[(B - 2c + p^2 - p^1)^{1/x}] / \delta p^1 = -1$$

$$(1/x)(B - 2c + p^2 - p^1)^{(1/x - x/x)}(-1) = -1$$

$$(B - 2c + p^2 - p^1)^{(1/x - x/x)} = x$$

$$(B - 2c + p^2 - p^1) = x^{(x/(1-x))}$$

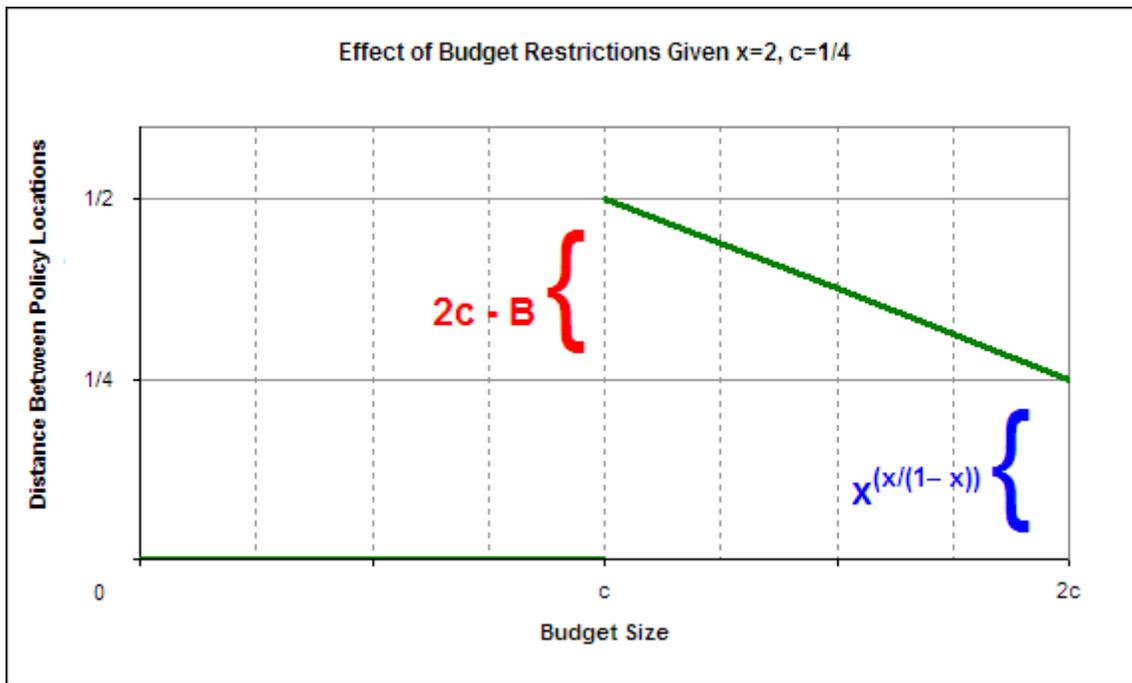
$$\underline{p^2 - p^1 = x^{(x/(1-x))} + 2C - B}$$

The result is the same for  $p^1 > p^2$ . In the unrestricted budget specification  $(2c - B)$  is 0 and so  $x^{(x/(1-x))}$  is the distance between policy locations. Here, the distance is increased by the additional cost<sup>17</sup>, which is a cost in the sense that it decreases  $\Pi^i$  without affecting  $V(S^i)$ . However, this cost is not

<sup>17</sup> This specification works out conveniently because  $x^{(x/(1-x))} = c$ . For cases where  $c$  takes on other values, the distance between parties is the following function:  $\text{Min}(x^{(x/(1-x))} + 2C - B, 2c)$  if the opportunity cost of divergence is less than the gain from  $V(S)$ , and 0 if that opportunity cost is greater than the gain in  $V(S)$ , on  $[0, 1]$ . The opportunity cost of divergence can be defined as the difference in  $\Pi$  that a party would earn from playing policies of convergence to the median versus saving up to  $x^{(x/(1-x))}$ .

great enough to make convergence a feasible equilibrium when  $B = 3/8$ . There are many equilibria here similar to those in section 3.3, with a distance of  $x^{x/(1-x)} + 2C - B$  between policies, and  $p^m_{\text{national}}$  located on  $[p^1, p^2]$ . I summarize the effect of a budget restriction on the distance between policy placement below:

Graph 3.4.1



Restricting the budget such that  $B < 1/4$  will lead to convergence to the median because parties face a high initial cost of saving and are unable to save the desired amount  $x^{x/(1-x)}$ . At  $B = 1/4$ , parties can either converge to the median or position themselves such that  $|p^1 - p^2|$  equals  $[x^{x/(1-x)} + (2c - B)]$ , which is  $1/2$ . At restrictions  $B > 1/4$ , convergence is no longer an option; the distance between parties decreases until it reaches  $x^{x/(1-x)}$ . This result provides a curious intuition regarding parties with differently sized budgeting. Poorer parties will want to be further away from the policy position of its opponent than richer parties. The richer party will chase around the weaker party indefinitely, as long as the poorer party is capable of switching flanks. If the poorer party is not

allowed to switch flanks, it is likely that the wealthier party will back it into a corner and capture a much larger payoff.

### 3.5 Three Party Model: Unrestricted Budget, Utility from Payoffs and Money

In this section, a third party considers entering the national electoral competition with an unrestricted budget and the amended utility function. I focus on the incentives of the new party, rather than looking for a three party equilibrium, because I am interested specifically in the issue of entry. The definitions are as follows:

$$U^i = U [\Pi^i(p^i, p^j, p^k)] = \Pi^i(p^i, p^j, p^k) + V(S^i)$$

$$\Pi^i(p^i, p^j, p^k) = (\text{payoff of local district})(\text{district portion})$$

$$\text{Unchallenged election payoff: victory} = 2, \text{ loss} = 0$$

$$\text{Challenged election payoff} = [2 / (\text{amount of parties challenging district})]$$

$$S^i = (B - C^i(p^i, p^j, p^k)); \quad B = 2c$$

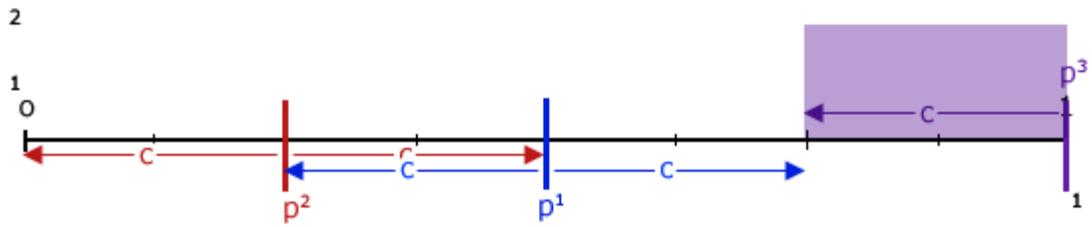
$$C^i(p^i, p^j, p^k) = (\text{cost of local district} = 1)(\text{district portion})$$

$$V(S^i) = (S^i)^{1/x}; \quad x = 2$$

There are multiple initial two party equilibria such that the parties are playing policies 1/4 apart. First, I examine policy set  $(4/8, 2/8, p^3)$ , fixing the location of the first two parties. Party 2 plays the most “extreme”<sup>18</sup> policy possible, allowing party 3 to achieve the highest utility, as will be shown. The example explored afterwards focuses on  $(5/8, 3/8, p^3)$ , which provides party 3 with the lowest utility. These two examples define the range of available  $U^3$  given different starting equilibria.

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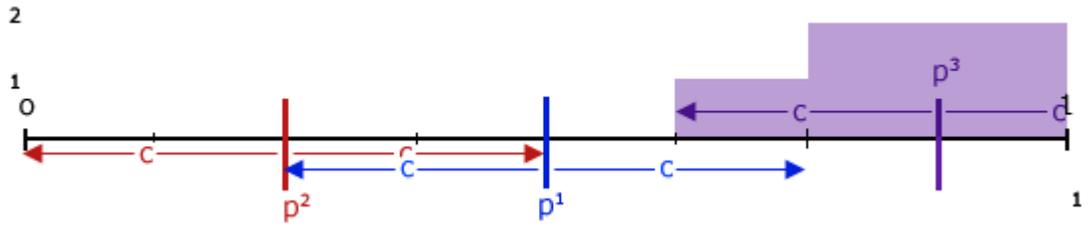
<sup>18</sup> Since  $p^1$  and  $p^2$  will never both be to the same side of the median,  $p^1 = 1/2$  creates the most extreme asymmetry.



$$\Pi^3(4/8, 2/8, 8/8) = (2)(2/8) = 4/8$$

$$U^3(4/8, 2/8, 8/8) = 1.207$$

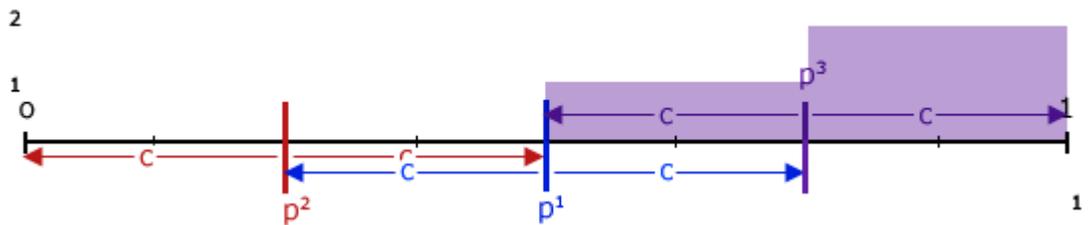
$$S^3 = 4/8 \quad V(S^3) = .707$$



$$\Pi^3(4/8, 2/8, 7/8) = (2)(2/8) + (1)(1/8) = 5/8$$

$$U^3(4/8, 2/8, 7/8) = 1.237$$

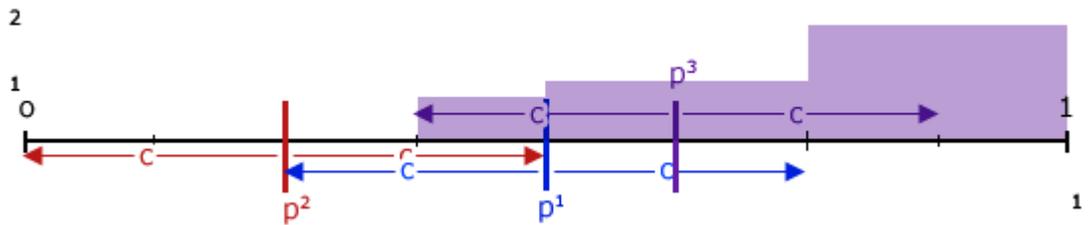
$$S^3 = 3/8 \quad V(S^3) = .612$$



$$\Pi^3(4/8, 2/8, 6/8) = (2)(2/8) + (1)(2/8) = 6/8$$

$$U^3(4/8, 2/8, 6/8) = 1.250$$

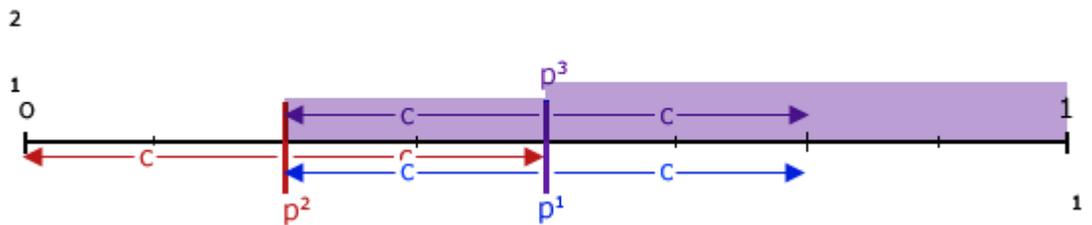
$$S^3 = 2/8 \quad V(S^3) = .5$$



$$\Pi^3(4/8, 2/8, 5/8) = (2)(2/8) + (1)(2/8) + (2/3)(1/8) = 6/8 + 2/24 = 20/24$$

$$U^3(4/8, 2/8, 5/8) = 1.186$$

$$S^3 = 1/8 \quad V(S^3) = .353$$



$$\Pi^3(4/8, 2/8, 4/8) = (1)(4/8) + (2/3)(2/8) = 16/24$$

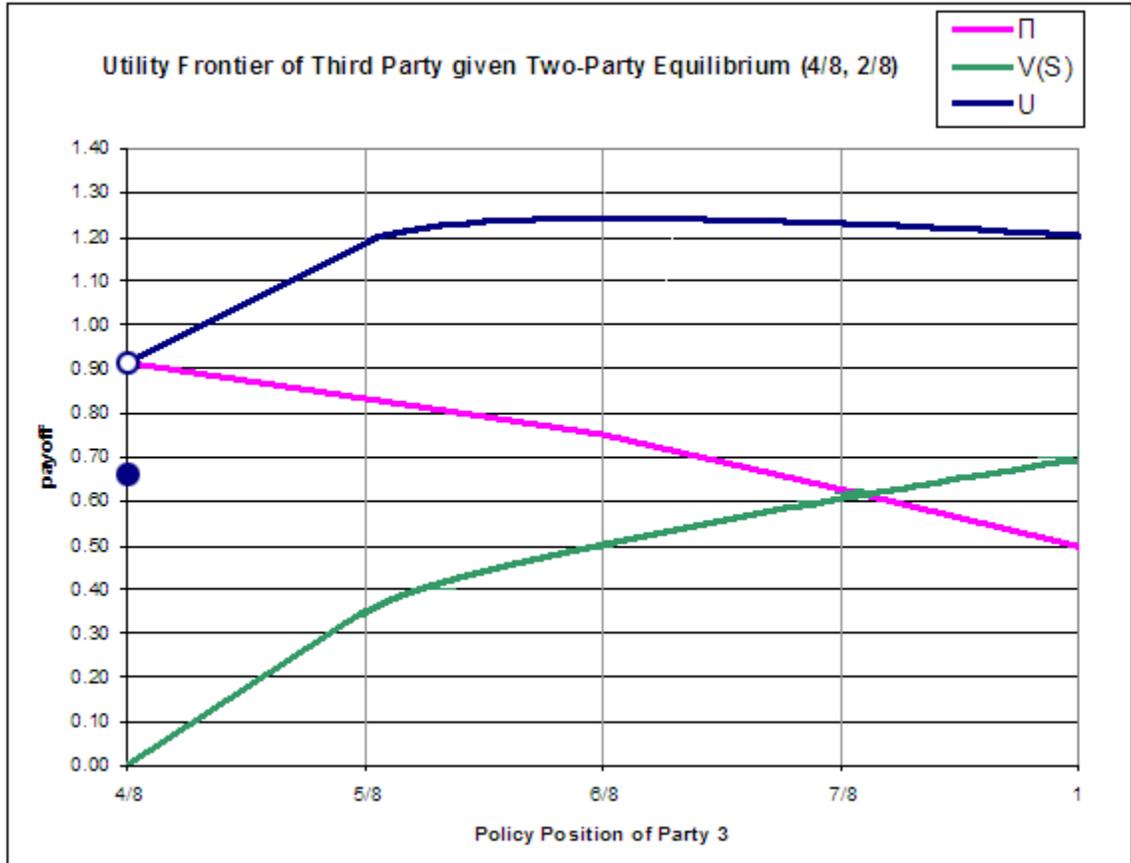
$$U^3(4/8, 2/8, 4/8) = .667$$

$$(approaches 22/24 from the left)$$

$$S^3 = 0/8 \quad V(S^3) = 0$$

Policies  $p^3 < p^1$  are not explored because their payoffs are significantly lower than those where  $p^3 \geq p^1$ . The new entrant will always want to play a policy on the most open flank. Graph 3.5.1 summarizes the utility frontier of the third party on  $[4/8, 8/8]$ .

Graph 3.5.1



In the diagram, the maximum of  $U^3$  is reached at  $6/8$ . The function  $\Pi^3$  has slope  $-1$  on  $[6/8, 8/8]$ , identical to Definition 6.1 in the appendix ( $p^1 > p^2$ ), replacing  $p^1$  with  $p^3$  and  $p^2$  with  $p^1$ . Thus,  $\Pi^3$  is  $[2 - (p^3 + p^1)]$  on  $[6/8, 8/8]$ . On  $[4/8, 6/8]$ , the slope of  $\Pi^3$  is  $-2/3$  because the third party is challenging elections that the two incumbents are already competing for and the local payoff is  $2/3$ . Therefore the maximum of  $U$  is either on  $[4/8, 6/8]$  such that  $V(S^3)$  has slope  $2/3$ , or is located on  $[6/8, 8/8]$  with  $\delta[V(S^3)]/\delta p^3$  equal to  $1$ .<sup>19</sup>

<sup>19</sup> I am equating slopes here instead of finding the maximum of function  $U$  from the start so as to not redefine the payoff function  $\Pi^1$ . While on  $[6/8, 8/8]$ ,  $\Pi^3$  looks like the payoff function used in previous sections; on other ranges, it has a different slope and is more difficult to track. Once I am sure that  $[6/8, 8/8]$  is the range where the maximum occurs, I can safely use the old definition of  $\Pi^1$  in  $U^3$  and find the maximum.

The slope of  $V^3$  equals  $2/3$  at  $p^3 = 17/16$ , found by setting the derivate of  $V^3$  with respect to  $p^3$  equal to  $2/3$ . This is outside the policy space. The other option for the maximum is on  $[6/8, 8/8]$ , where  $\Pi^3$  is regularly defined.

$$\Pi^3(4/8, 2/8, p^3) = 16/8 - (4/8 + p^3); \quad V(S^3) = (p^3 - 4/8)^{1/2}$$

$$U^3 = 12/8 - p^3 + (p^3 - 4/8)^{1/2}$$

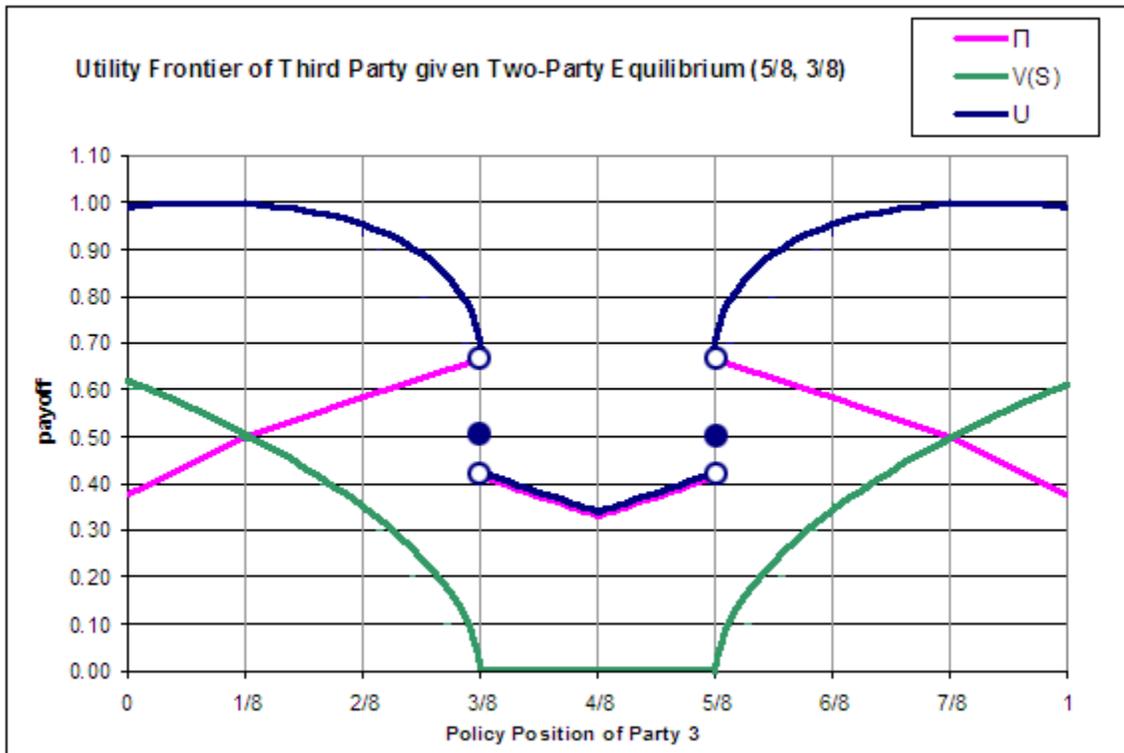
$$\delta[12/8 - p^3 + (p^3 - 4/8)^{1/2}]/\delta p^3 = 0 \quad \rightarrow \quad 0 + -1 + 1/2(p^3 - 4/8)^{-1/2} = 0$$

$$2 = (p^3 - 4/8)^{-1/2} \rightarrow 4 = 1/(p^3 - 4/8) \rightarrow 2/8 = p^3 - 4/8 \rightarrow \quad \underline{p^3 = 6/8}$$

The maximum utility is at policy position  $6/8$ , which is precisely  $1/4$  away from  $4/8$ , the position of party 1. It is anticipated that the new entrant will play a policy with distance  $x^{(x/(1-x))}$  from the party closest to the median. The highest utility available to party 3 is at  $(4/8, 2/8, 6/8)$ , where  $U^3(\Pi^3, V(S^3)) = 12/8 - 6/8 + (6/8 - 4/8)^{1/2} = 10/8$ .

Next, consider the single symmetric two-party equilibrium  $(3/8, 5/8)$ . By a now familiar process, the payoffs of party 3 are aggregated into a utility frontier in graph 3.5.2.

**Graph 3.5.2**



An important caveat is that party 3 would never play policy  $p^2 \geq p^3 \geq p^1$  because the payoffs are significantly lower in that range. The largest benefit is gained by entering from a side and capturing the flank: the payoffs are symmetric on either side. To find the preferred position for the third party, I focus on  $[4/8, 8/8]$  to be consistent with the previous example.

$$\begin{aligned} \Pi^3(5/8, 3/8, p^3) &= 16/8 - (5/8 + p^3); & V(S^3) &= (p^3 - 5/8)^{1/2} \\ U^3 &= 11/8 - p^3 + (p^3 - 5/8)^{1/2} \\ \delta [11/8 - p^3 + (p^3 - 5/8)^{1/2}] / \delta p^3 &= 0 & \rightarrow & 0 + -1 + 1/2(p^3 - 5/8)^{-1/2} = 0 \\ 2 &= (p^3 - 5/8)^{-1/2} \rightarrow 4 = 1/(p^3 - 5/8) \rightarrow 2/8 = p^3 - 5/8 & \rightarrow & \underline{p^3 = 7/8} \end{aligned}$$

The policy set  $(5/8, 3/8, 7/8)$  and the symmetric  $(5/8, 3/8, 1/8)$  provide the third party with the maximum utility. It again locates  $1/4$  away from the party closest to the national median. Below, I generalize the incentives of party 3 given the symmetric equilibrium of the incumbents.

If  $p^3 < p^2$

$$\begin{aligned} \Pi^3(p^1, p^2, p^3) &= p^2 + p^3 \\ V(S^3) &= ((B - 2c) + p^2 - p^3)^{1/x} \\ U^1 &= p^3 + p^2 + (p^2 - p^3)^{1/x} \\ \delta [p^3 + p^2 + (p^2 - p^3)^{1/x}] / \delta p^3 &= 0 \\ 1 + 0 + (1/x)(p^2 - p^3)^{(1/x - x/x)}(-1) &= 0 \\ 1 &= (1/x)(p^2 - p^3)^{(1/x - x/x)} \\ x &= (p^2 - p^3)^{((1-x)/x)} \\ x^{(x/(1-x))} &= (p^2 - p^3) \\ p^3 + x^{(x/(1-x))} &= p^2 \\ \underline{p^3 = p^2 - x^{(x/(1-x))}} \end{aligned}$$

If  $p^3 > p^1$

$$\begin{aligned} \Pi^3(p^1, p^2, p^3) &= 2 - (p^1 + p^3) \\ V(S^3) &= ((B - 2c) + p^3 - p^1)^{1/x} \\ U^1 &= 2 - (p^3 + p^1) + (p^3 - p^1)^{1/x} \\ \delta [2 - p^1 - p^3 + (p^3 - p^1)^{1/x}] / \delta p^3 &= 0 \\ (0) - 1 + (1/x)(p^3 - p^1)^{(1/x - x/x)}(1) &= 0 \\ 1 &= (1/x)(p^3 - p^1)^{(1/x - x/x)} \\ x &= (p^3 - p^1)^{((1-x)/x)} \\ x^{(x/(1-x))} &= (p^3 - p^1) \\ x^{(x/(1-x))} + p^1 &= p^3 \\ \underline{p^3 = p^1 + x^{(x/(1-x))}} \end{aligned}$$

Party 3 will always play  $p^3$  such that  $\text{Min}[|p^3 - p^2|, |p^3 - p^1|] = x^{(x/(1-x))}$ . It will not position itself between the two incumbents. At  $(5/8, 3/8, 7/8)$ , the third party achieves a maximum  $U^3 = 11/8 - 7/8 + (7/8 - 5/8)^{1/2} = 8/8$ . This is the minimum utility that parties 1 and 2 can force on the third party. The full range of  $U^3$  is  $[8/8, 10/8]$ , where  $U^3$  depends on the equilibrium of the first two players.

The possibility of entry should impact the behavior of the original two parties. A full exploration of that dynamic is beyond the scope of this paper<sup>20</sup>, but here is the intuition behind the problem. A third party has a significant negative effect on the utility of the party whose flank it consumes; the new competition drastically reduces the challenged incumbent's savings. If possible, the challenged incumbent will move to a location where it can again achieve a savings of  $x^{(x/(1-x))}$ . This is likely to be prevented by the party on the other flank, which will react by moving towards the extreme, keeping a fixed distance away from its opponent. Thus the central party has difficulty achieving a high utility. Notably, cooperatively merging with the other incumbent may be a good idea because competition is reduced and savings is again possible. If a merger happens, the new "incumbent" moves towards the new entrant until savings equals  $x^{(x/(1-x))}$ .

### 3.6 Three Party Model: Restricted Budget, Utility from Payoffs and Money

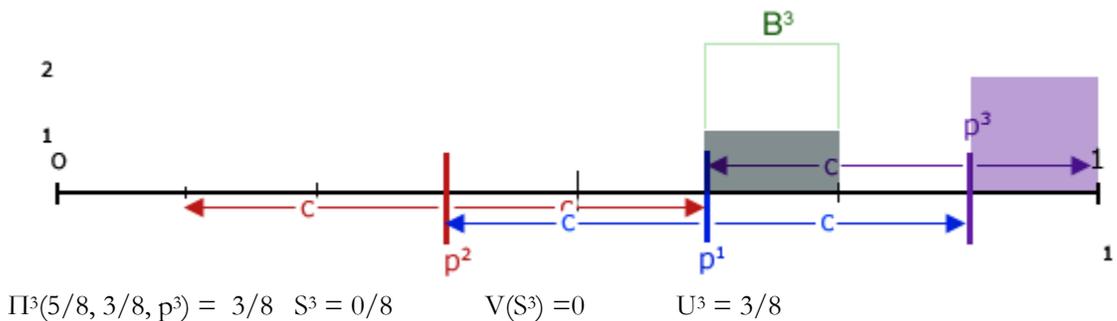
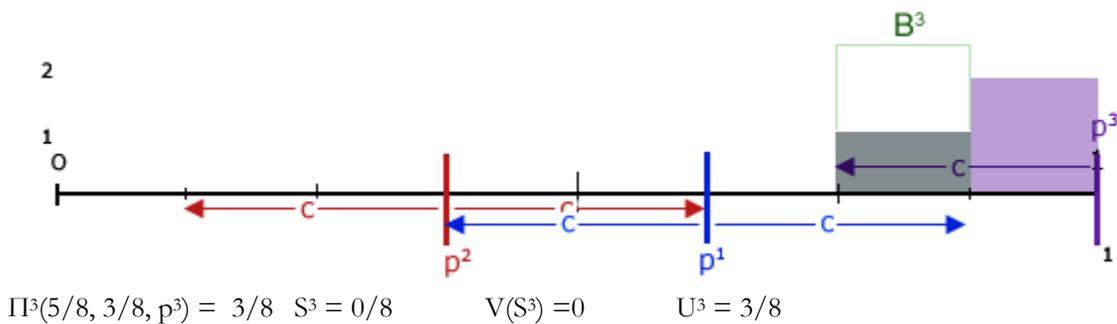
In reality, political parties come in different sizes. They certainly have different national budgets. Despite some variation, it can be said that the budgets of Republicans and Democrats are unrestricted in this model. Both parties are capable of raising a tremendous amount of money, funding contestable elections across the country up to the necessary limit. As in section 3.5, the two incumbent parties have a budget  $B = 2c$ . The third party, however, is an underdog. It is likely to have a much sharper budget constraint, because raising money for a political start-up is difficult. Even "established" third parties, like the Libertarians or the Green party, have tiny budgets compared to

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<sup>20</sup> In thinking about Duverger's Law, I am trying to show that entry is prohibitively difficult, rather than the full effects of entry on the incumbents.

that of the two incumbents. Furthermore, they challenge very few elections, instead serving a symbolic function of signaling to the larger parties preferences of the American populace.

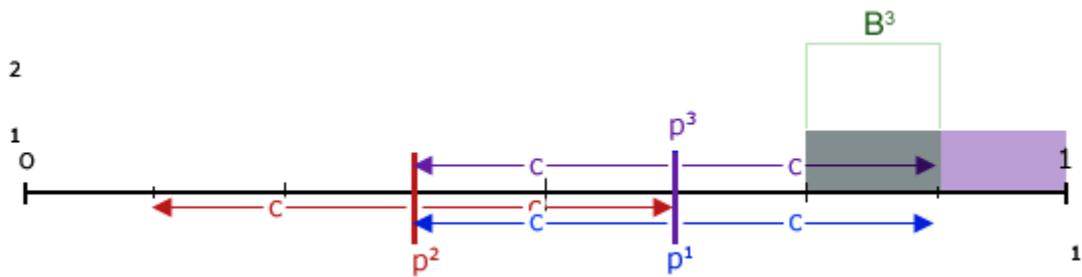
In section 3.5, an entrant is very dangerous to parties 1 and 2, capable of capturing an entire flank and severely crippling the incumbent caught in the middle. This result is based on the assumption that the new entrant instantaneously assumes national-party stature, with  $B^3 = 2c$ ,  $c = 1/2$ . However, it is easy to imagine that budget  $B^3$  is in fact significantly lower, by a very high magnitude in the United States<sup>21</sup>. Restricting  $B^3$  to  $c/2$  simulates the deep asymmetry in available financing between the two controlling parties and the new entrant. It is assumed that parties 1 and 2 locate at equilibrium  $(5/8, 3/8)$  in order to minimize the chance of entry.<sup>22</sup> The utility function is the same one used in section 3.5 and  $x = 2$  for  $V(S^i)$ .



<sup>21</sup> In the UK, 90% of campaign spending was done by the incumbents: Labor and Conservatives. Source: [<http://www.electoralcommission.org.uk/media-centre/newsreleasecorporate.cfm/news/139>]

<sup>22</sup> Positioning symmetrically at  $[p^m_{\text{national}} \pm (1/2)(x^{s/(1-s)})]$  minimizes party 3's potential utility and reduces the probability of a single party losing a flank from 100% to 50%. This may be preferable if parties are risk averse.

The prospects of the new entrant look pretty grim. The only secure payoff is on  $[7/8, 8/8]$  or  $[0/8, 1/8]$  because those ranges are not contestable by other parties. Otherwise, party 3 can only challenge a portion of  $1/8$  districts, and must let go its other contestable districts. It will prefer to challenge elections where there is only one other competitor, since the potential payoff is higher:  $1 > 2/3$ . It is clear that maximum utility equals a meager  $3/8$ , and there is no possibility of saving the budget because there is no situation of non-overlap. Moving to  $p^m_{\text{national}}$  yields a negligible utility of  $2/24$ . The option that worked in section 3.4, that of converging to the policy of the other party, fails here as well, yielding a utility of  $2/8$  as shown below.



$$\Pi^3(5/8, 3/8, p^3) = 2/8 \quad S^3 = 0/8 \quad V(S^3) = 0 \quad U^3 = 2/8$$

In the specification without a third-party budget restriction, given initial equilibrium  $(5/8, 3/8)$ , the third party could achieve a utility of  $8/8$ . Here, the maximum utility is  $3/8$ . The size of the budget can thus determine whether a party enters or not. In reality, if a party cannot raise enough money to meaningfully compete in the electoral competition, it will not spend the costs necessary for initial publicity. In other words, if there is some entry cost for the national competition, a party may be better off foregoing paying this cost and as a result staying out of the election. There is a useful element of uncertainty here: a party might not know how much money it can raise, and thus cannot accurately predict its payoffs and balance them against costs. The model of entry in the next section takes advantage of this uncertainty.

## 4 – Entry

The new entrant faces a lump-sum entry cost  $\alpha$ . This cost is the price paid for national advertising and other marketing. It establishes the credibility range by informing relevant districts about the new entrant's potential candidates: the cost of informing each district is 1, and the total cost is (local cost)(district portion informed). For consistency, that district portion is the incumbent credibility range  $2c$ .<sup>23</sup> A new party will enter if expected utility is greater than utility derived from saving the lump sum. Function  $V$  is used here again: the entry cost can be “saved” and the same utility derived as if the party were saving its budget. The third party, instead of spending  $\alpha$  on entry, could use that money to finance political seminars or impact policy through lobbying. To summarize:

**$\alpha$  is the entry cost for the national electoral competition,  $\alpha = 2c$**

$$V(\alpha) = (\alpha)^{1/x}, x = 2$$

**If  $V(\alpha) \geq U^k [\Pi^k, V(S^k)]$ , the party does not enter the electoral competition**

**If  $V(\alpha) < U^k [\Pi^k, V(S^k)]$ , the party enters the electoral competition**

**$P^k(V(\alpha) < U^k [\Pi^k, V(S^k)])$  is the probability with which the party enters the electoral competition**

The new entrant does not know how much money it will be able to raise: budget  $B$  is a random variable on  $[0, 2c]$  with mean  $c$ . Thus the expected budget of the third party is  $E(B) = c$ . The current utility threshold for entry, given  $2c = 1/2$ , is  $V(\alpha = 1/2) = .707$ . To calculate the expected utility of the new entrant, restrict the budget to  $1/4$  and consider the policy triple  $(5/8, 3/8, p^3)$ . Party 3 maximizes utility by playing either  $p^3 = 1/8$  or  $p^3 = 7/8$ , achieving  $U^3 = .500$ .<sup>24</sup> As a result,  $U^3 < V(\alpha)$  and the probability of entry is 0. If the initial incumbent equilibrium is symmetric around the national

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<sup>23</sup> This paper does not explore credibility ranges of different sizes, although arguably a new party will have a much smaller credibility range. Changing the credibility range assumption is certainly an interesting direction to explore, and is likely to allow more fringe parties to enter due to lowered entry costs. However, a smaller credibility range acts also as a constraint on the maximum, or “unrestricted” budget, while the marginal utility of money is likely to stay the same. Thus parties could want to save  $1/4$  while having a credibility range of  $1/8$ . The author hypothesizes that in a country like India, which has many different parties nationally, credibility ranges are relatively small throughout.

<sup>24</sup> The assumption of symmetric equilibria is meant to reflect risk-averse incumbents.

median, which should happen if parties 1 and 2 are moderately risk averse, the third party will not enter electoral competition. Entry is possible if the incumbents are located closer to  $(6/8, 4/8, p^3)$  because the third party can achieve a  $U^3 = .750$  in that situation. However, it is unlikely that the incumbent parties would open up their flanks in this manner, because losing one's flank reduces payoffs catastrophically.

This paper ends on the model of entry because entry is key to the main question I am addressing – why does Duverger's Law hold? United States electoral competitions have always been dominated by the two party system. In the entire history of the country, an incumbent party only once lost its position to a challenger: the Whigs disintegrated allowing for the rise of the Republicans. This two party equilibrium holds because entry is almost impossible. What new party can raise enough money for both the entry cost and the unrestricted budget? Thus third parties are almost non-existent on the national scale.

## 5 – Conclusion

On some level, the model raises as many questions as it answers. What are the effects of changing the size of the credibility range? What if parties have different marginal utilities of savings? How exactly do incumbents respond to entry, and should there be a model of exit? Do parties *have to* finance contestable elections, or can they simply position themselves anywhere and save the budget as desired? These are interesting questions to explore and limitations of this thesis. However, the assumptions in this paper are meant to reflect electoral competition in the United States and similar countries. In this situation, parties credibly compete over a large portion of the populace. Democrats and Republicans are seen as capable of representing voters in the majority of states – in 2004, there was only one Independent in the Senate. Additionally, campaign financing is not particularly restrictive and the incumbent parties have little trouble raising tremendous amounts of money, while new entrants face great uncertainty in their fundraising.

The logic of this model provides a number of separate useful results. First, despite electoral competition taking place on a continuum of districts, parties still converge if they are motivated only by winning elections. If parties care about savings and have a diminishing marginal utility of money, they will move apart their policies in order to save a certain amount of their budget. This amount depends essentially on greed. A third party entrant with similar preferences will want to also locate a distance away from the incumbents, taking the most advantageous flank. This entry is very damaging to the party caught in the middle. Budget restrictions, which reflect either legislation that limits fundraising or simply a lesser ability to acquire money, have complicated effects. Despite occasionally leading to convergence, restrictions primarily increase the distance between party policies by creating an additional opportunity cost to savings in terms of election victories. A third party entrant with a restricted budget faces grim circumstances – a very low utility. Using a simple, although somewhat arbitrary, model of entry I show that uncertainty regarding an entrant's potential budget translates into a low expected utility, and therefore precludes entry. The two-party equilibrium remains unchallenged and Duverger's Law holds.

## 6 – Appendix

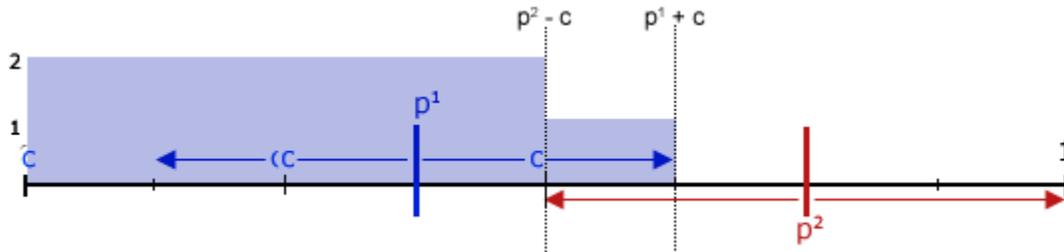
### 6.0 Definition: Local Payoffs of Party 1 in a given district

District type	Contestable and Challenged	Contestable and not Challenged	Uncontestable and closer to $p^1$ than $p^2$	Uncontestable and closer to $p^2$ than $p^1$
Local Payoff	2 / number of parties challenging district	0 if another party challenges, otherwise as uncontestable	2	0

### 6.1 Definition: $\Pi^1(p^1, p^2) = (\text{payoff of local district})(\text{district portion})$

If  $p^1 < p^2$  then  $(p^2 - c) > 0$  and  $(p^1 + c) < 1$

If  $p^1 > p^2$  then  $(p^1 - c) > 0$  and  $(p^2 + c) < 1$



$$p^1 < p^2: \quad \Pi^1 = 2[p^2 - c] + 1[(p^1 + c) - (p^2 - c)]$$

$$\Pi^1 = 2p^2 - 2c + 1p^1 + 1c - 1p^2 + 1c$$

$$\underline{\Pi^1 = p^1 + p^2}$$

$$p^1 = p^2: \quad \Pi^1 = 2[0] + 1[1] + 0[0]$$

$$\underline{\Pi^1 = 8/8}$$

$$p^1 > p^2: \quad \Pi^1 = 2[1 - (p^2 + c)] + 1[(p^2 + c) - (p^1 - c)]$$

$$\Pi^1 = 2 - 2p^2 - 2c + 1p^2 + 1c - 1p^1 + 1c$$

$$\Pi^1 = 2 - p^2 - p^1$$

$$\underline{\Pi^1 = 2 - (p^1 + p^2)}$$

The above definition is tractable, but does not accommodate large values of  $c$ , such as  $c = 1$ . When  $p^1 < p^2$ , and  $(p^2 - c) < 0$  or  $(p^1 + c) > 1$ , the payoffs are different than stated above. In order to prevent  $(p^i \pm c)$  from exiting  $[0, 1]$ , maximum and minimum functions are used below.

**6.2 Definition:**  $\Pi^1(p^1, p^2) = (\text{payoff of local district})(\text{district portion})$

$$p^1 < p^2: \quad \Pi^1 = 2(\text{Max}[(p^2 - c), 0]) + 1(\text{Min}[(p^1 + c), 1] - \text{Max}[(p^2 - c), 0])$$

$$p^1 = p^2: \quad \Pi^1 = 1$$

$$p^1 > p^2: \quad \Pi^1 = 2([1 - \text{Min}[(p^2 + c), 1]]) + 1(\text{Min}[(p^2 + c), 1] - \text{Max}[(p^1 - c), 0])$$

**6.3 Definition:**  $C^1(p^1, p^2) = (\text{cost of local district})(\text{district portion})$

**If  $p^1 < p^2$  then  $(p^2 - c) > 0$  and  $(p^1 + c) < 1$**

**If  $p^1 > p^2$  then  $(p^1 - c) > 0$  and  $(p^2 + c) < 1$**

$$p^1 < p^2 \text{ AND } p^1 + 2c > p^2: \quad C^1 = 1[(p^1 + c) - (p^2 - c)] = p^1 - p^2 + 2c$$

$$p^1 < p^2 \text{ AND } p^1 + 2c \leq p^2; \quad C^1 = 0$$

$$p^1 = p^2: \quad C^1 = 2c$$

$$p^1 > p^2 \text{ AND } p^1 - 2c < p^2: \quad C^1 = 1[(p^2 + c) - (p^1 - c)] = p^2 - p^1 + 2c$$

$$p^1 > p^2 \text{ AND } p^1 - 2c \geq p^2: \quad C^1 = 0$$

**6.4 Definition:**  $C^1(p^1, p^2) = (\text{cost of local district})(\text{district portion})$

$$p^1 < p^2 \text{ AND } p^1 + 2c > p^2: \quad C^1 = \text{Min}[1(\text{Min}[(p^1 + c), 1] - \text{Max}[(p^2 - c), 0]), 1]$$

$$p^1 < p^2 \text{ AND } p^1 + 2c \leq p^2; \quad C^1 = 0$$

$$p^1 = p^2: \quad C^1 = \text{Min}[2c, 1]$$

$$p^1 > p^2 \text{ AND } p^1 - 2c < p^2: \quad C^1 = \text{Min}[1(\text{Min}[(p^2 + c), 1] - \text{Max}[(p^1 - c), 0]), 1]$$

$$p^1 > p^2 \text{ AND } p^1 - 2c \geq p^2: \quad C^1 = 0$$

6.5 Definition:

$$V(S^1) = [B - C^1(p^1, p^2)]^{1/x}$$

If  $p^1 < p^2$  then  $(p^2 - c) > 0$  and  $(p^1 + c) < 1$

If  $p^1 > p^2$  then  $(p^1 - c) > 0$  and  $(p^2 + c) < 1$

$$p^1 < p^2 \text{ AND } p^1 + 2c > p^2:$$

$$V(S^1) = (B - p^1 + p^2 - 2c)^{1/2}$$

$$p^1 < p^2 \text{ AND } p^1 + 2c \leq p^2;$$

$$V(S^1) = B^{1/2}$$

$$p^1 = p^2:$$

$$V(S^1) = 0$$

$$p^1 > p^2 \text{ AND } p^1 - 2c < p^2:$$

$$V(S^1) = (B - p^2 + p^1 - 2c)^{1/2}$$

$$p^1 > p^2 \text{ AND } p^1 - 2c \geq p^2:$$

$$V(S^1) = B^{1/2}$$

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