Electoral Rules and Income Tax Progressivity

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Abstract

This paper analyzes the effects of two stylized electoral procedures on the progressivity of income tax schedules: single-district majoritarian rule and proportional rule. The analysis is undertaken assuming an endogenous labor supply and that tax revenues are raised to produce a public good that provides different levels of utility to different income groups. We use a probabilistic voting model to capture the main features of political competition under majoritarian electoral rule and an adapted version of the Baron and Ferejohn (1989) legislative bargaining model for proportional rule. We find sufficient conditions to ensure a progressive income tax schedule under both the majoritarian and the proportional systems and then investigate how various parameters of the model influence the size of the tax rates and the degree of progressivity.

We consider the effects of three economic variables: the elasticity of the labor supply, the preference over the public good and income mobility. We find that under both rules the optimal taxation result holds qualitatively: groups with larger labor supply elasticity are taxed at a smaller marginal rate. However, the models predict that changes in public good preferences and population mobility may affect tax progressivity differently under the two electoral rules. As the income distribution becomes more skewed to the right, equilibrium tax progressivity decreases under a majoritarian system and may increase under proportional representation. Finally, our model suggests that preference diversity has a stronger effect on tax progressivity under the proportional rule.

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1 Introduction

How do electoral systems influence income tax schedules? Do certain economic variables affect tax progressivity differently depending on the electoral rule in place? In this paper we study the effects of two different electoral systems on the progressivity of income tax schedules. The first is the single-district majoritarian rule, under which the party with the highest number of votes wins the election and implements its desired policy. The second is the proportional rule, under which each party running in an election receives a percentage of seats in the legislature equal to its share of the national vote. Although highly simplified compared to the electoral systems implemented in practice, these two theoretical frameworks allow us to study some of the channels through which electoral systems can influence policy.

The role of institutional features, including electoral systems, in determining the shape of income tax schedules has been analyzed widely in the public finance and political economy literature. As opposed to the normative perspective on optimal taxation, which studies the shape of the optimal tax function in the social planner problem[1], the positive approach starts from the assumption that, in democratic systems, tax schedules are the result of some collective choice mechanism and thus reflect the preferences of self-interested voters. A quick look at the shape of tax schedules in most developed democracies reveals that statutory income taxation is typically progressive, meaning that the proportion of income paid as taxes increases with income. We can observe a large cross-country variation in the degree of progressivity, measured by the difference between the top and the bottom statutory marginal tax rates[2]. For example, income tax progressivity in the OECD area varies from 0% in the Slovak Republic to 50% in France[3] (OECD 2007). Therefore, whether progressivity emerges

[1] See **?**, Weymark (1987), Kanbur and Tuomala (1994), **?** for an overview of this literature.

[2] The measurement of the degree of progressivity varies in the literature, with many authors using the top statutory rate as a proxy for the difference between the top rate and bottom rate (generally assumed to be 0).

[3] These percentages represent the difference between the highest and the lowest marginal income tax rates.

as a voting outcome and whether specific institutional features can affect the degree of tax progressivity achieved in equilibrium become interesting questions from the perspective of tax design, redistribution and tax reform.

The early contributions to studying the effects of voting rules on income tax schedules were limited to modeling majority voting over linear tax schedules. In this unidimensional policy space, a Condorcet winner[4] can emerge under a limited set of assumptions, leading to a unique pure strategy Nash equilibrium of the electoral competition. Using the median-voter theorem, the tax equilibrium is given by the preferred policy of the voter with median productivity (Romer 1975, Roberts 1977) or with median income (Meltzer and Richard 1981). The restriction to linear taxes is made for simplicity purposes and it departs from the reality of tax design, since most real world income tax schedules are non-linear, with both average and marginal tax rates raising as income increases. However, the study of voting over non-linear tax schedules moves the game into a multi-dimensional policy space, where it encounters the well-known problem of cycling. This results from the fact that aggregation by majority rule of transitive individual preferences does not necessarily yield a transitive social preference function. Generally, this means the absence of a Condorcet winner in the multi-dimensional policy space and thus, nonexistence of a pure strategy Nash equilibrium (Plott 1967). Although various assumptions have been used to restrict the possible shapes of tax schedules and to reduce the dimensionality of the policy space, voting cycles remain unavoidable when studying progressive tax schedules under direct democracy (Carbonell-Nicolau and Klor 2003). This is not necessarily the case under representative democracies, where the decision over tax policy is delegated to a set of elected representatives, who then choose and implement policies (Persson and Tabellini 2000).

In this paper, we model a representative democracy under two idealized electoral proce-

[4] A policy that is preferred under the collective choice mechanism to any other policy available in the policy space.

dures: the majoritarian rule and the proportional rule. We use a probabilistic voting model to capture the main features of the electoral competition under the majoritarian system. We then turn to electoral competition under the proportional rule to consider legislative bargaining by using an adapted version of the Baron and Ferejohn (1989) legislative bargaining model. We find that progressive income tax schedules can emerge under both majoritarian and proportional systems and we derive sufficient conditions under which progressivity exists in each case. Then, we study and compare the effects of changes in the model's exogenous parameters on the size of the tax rates, on public good provision and on the degree of tax progressivity under the two electoral systems.

Although by no means exhaustive, the simple models presented in this paper provide a framework for studying the link between electoral institutions and the progressivity of the statutory income tax schedule. The rest of the paper is organized as follows. Section 2 provides a brief overview of the related literature. Section 3 presents the model, characterizes the equilibria under each two electoral rule and compares the effects of economic variables on the size of tax rates, on public good provision and on the degree of tax progressivity. Section 4 concludes.

2 Related Literature

We study the effects of electoral rules on tax progressivity by building on a couple of different strands of literature. First, the positive approach to optimal taxation provides a long line of research into the existence of Nash equilibria in voting games over tax schedules. As discussed in the introduction, various approaches have been used in order to reduce the dimensionality of the policy space and to obtain a Condorcet winner. Initial work in this area has focused on restricting the shape of tax schedules to obtain a unidimensional policy space. Romer (1975) only looks at choices over linear tax schedules and finds that, with single-peaked preferences and work disincentives, a Condorcet winner exists and it involves tax progressivity. Using similar assumptions, Roberts (1977) and Meltzer and Richard (1981) also arrive at the median voter result. Cukierman and Meltzer (1991), Roemer (1999 and 2001) and De Donder and Hindriks (2003) confine their analysis to quadratic tax functions, allowing for only convex, concave or linear tax functions. Berliant and Gouveia (1994) reduce the dimensionality of the space by introducing uncertainty over the distribution of wages and abilities and by requiring ex-ante feasibility of any proposed tax schedule. Snyder and Kramer (1988) and Röell (1997) assume that parties can only propose schedules that are optimal for some voter. Carbonell-Nicolau and Klor (2003) allow any increasing tax function but assume that parties prefer simple schedules, with few brackets. Finally, Hettich and Winer (1988 and 1999) assume that there exist finitely many types of voters, which differ in terms of income, valuation of public services and loss of income due to taxation. In this paper, we adopt a set of restrictions similar to the ones used by Hettich and Winer (1999) and assume that the population can be divided into a discrete number of groups that differ along three economic dimensions: labor supply elasticity, preference for the public good and ability (as measured by wage).

Even after applying the restrictive assumptions that reduce the dimensionality of the policy space, the existence of a Nash equilibrium is not guaranteed without rather strong additional constraints (De Donder and Hindriks 2003). Thus, various approaches have been explored for finding equilibria under electoral competition. In the case of two-party competition, several papers adopted the Downsian model[5] but looked at other solution concepts than pure-strategy Nash equilibrium. De Donder and Hindriks (2003) showed that an equilibrium tax schedule can be found by eliminating weakly dominated strategies or allowing for mixed-strategy equilibria, a solution also found in Carbonell-Nicolau and Ok (2007). Roemer

[5]The Downsian model assumes a one-shot electoral game where the two competing parties have perfect information about voter preferences and are able to commit to their announced policy.

(1999 and 2001) introduced the Party Unanimity Nash Equilibrium (PUNE), which assumes that a party's platform in an electoral competition is the result of intra-party negotiations between three factions: reformers, opportunists and militants. A Nash equilibrium between party proposals arises because changing the proposed policy would be too costly, given the necessity to negotiate among the three factions every time a change is sought. Moreover, all parties propose progressive schedules. Bohn and Stuart (2005) show that, in a model of voting over non-linear tax schedules with parties lacking commitment power,[6] the resulting tax schedule would have high marginal tax rates at the extremes and low (negative) marginal tax rates for the median voter. Casamatta et al. (2008) model an infinitely repeated voting game over tax schedules and show that all equilibria involve piecewise linear tax-functions. Roemer (2008) models an electoral competition in the infinite dimensional policy space, but assumes that politicians concentrate on swing and core voters. He also derives the result that equilibrium tax schedules are piecewise linear.

Another avenue for modelling two-party competition is to introduce uncertainly over voter preferences by using a probabilistic voting framework, under which a Nash equilibrium is known to exist, as shown in Coughlin and Nitzan (1981). Hettich and Winer (1999) adopt a probabilistic voting model to model a two-party competition over non-linear tax rates when there are administrative costs to taxation and voters differ in their economic abilities. They show that the winning tax policy would offer a different tax rate to each voter (or group of voters), so no party would ever find it optimal to offer a flat tax. In this paper, we adopt a model similar to the of Hettich and Winer (1999) for studying the tax equilibrium reached under the majoritarian system. However, we do not consider administrative costs and we explicitly model labor supply disincentives through the elasticity of labor supply.

Our model of decision-making under proportional rule is related to the large literature on legislative bargaining. In a seminal paper, Baron and Ferejohn (1989) extend the Stahl-

[6] Meaning that they can only commit to implementing the policy that best serves their own interests.

Rubinstein non-cooperative two-person bargaining game and model inter-party negotiation as a multi-agent non-cooperative bargaining game. They prove that this game always has equilibria, even when the number of proposal rounds allowed is infinite. Moreover, the game allows for equilibria to emerge even when the policy space is multidimensional, which makes this model attractive for studying decisions over non-linear tax schedules. Battaglini and Coate (2007) use a legislative bargaining model along the lines of Baron and Ferejohn (1989) in an infinite-horizon model in which a legislature decides over linear taxes and government investment in a public good. Similarly, Persson, Roland and Tabellini (2000) use a legislative bargaining model to derive the equilibrium public spending and linear tax schedules chosen by a legislature. In this paper, we adopt a simple version of the Baron and Ferejohn (1989) legislative bargaining model in order to derive the equilibrium policy under proportional representation. Unlike Battaglini and Coate (2007) and Persson, Roland and Tabellini (2000), we use a more simple version of the bargaining game and focus on policies involving non-linear tax schedules.

This paper is also related to the vast strand of theoretical research on the effects of electoral rules on fiscal policy. Lizzeri and Persico (2001) look at the effects of electoral rules on the composition of government spending. They find that, under proportional rule, a larger number of parties reduces the support base of each party and creates incentives for parties to propose targeted transfers to specific groups rather than broad public goods. Milesi-Ferretti et al. (2002) study the effects of electoral rules on the size of government spending and find that proportional rule induces a larger size of government. Austen-Smith (2000) compares redistribution and linear tax rates under proportional representation versus majoritarian systems and shows that proportional representation leads to higher tax rates and more redistribution than majoritarian systems. Persson et al. (2007) focus on studying the particular link between electoral rules and legislative decision-making. They argue that, by limiting the number of parties in the government, the electoral rule can reduce the incidence of coalition governments (which have been shown to run greater government expenditures that single party governments). The model predicts that majoritarian elections reduce the number of parties entering the legislature, and so they lead to less government spending, while proportional elections have the opposite effect. We develop a similar comparative perspective on the effects of electoral rules, but we focus on the issue of progressivity, which has not been addressed in the above mentioned studies.

3 The Model

We study the conditions under which progressive income tax schedules emerge when tax revenues are used to provide a public good that gives different levels of utility to different income groups. Moreover, we explore the effects of certain economic variables on the progressivity of equilibrium tax schedules and how these effects may vary depending on the electoral system in place.

Under each electoral rule, we study the effects of three exogenous economic variables on the degree of progressivity achieved in equilibrium. The first variable is the elasticity of the labor supply, the key determinant of the distortionary effect of taxation in our model. We find that the elasticity of the labor supply is a variable whose effects are not significantly influenced by electoral rules. Regardless of the electoral rule in place, the qualitative optimal taxation result holds: groups with larger labor supply elasticity should receive a smaller marginal tax rate. Our second variable of interest is the preference over the public good provided with the tax revenues. We find that the electoral system can lead to qualitatively different effects for the public good preference on tax progressivity. Another interesting result is that the role of preference diversity in determining tax progressivity increases significantly when operating under a proportional system as opposed to a majoritarian system. Significant preference differences between groups make the formation of coalitions between these groups more costly, affecting the set of coalitions that can be formed under legislative bargaining. Finally, we focus on the income distribution. We find that income mobility can have different effects on tax progressivity depending on the electoral rule. The effects of population mobility towards the middle income group change when moving from a majoritarian rule to a proportional rule. As the income distribution becomes more skewed to the left (there is downward mobility), progressivity may increase under a majoritarian system (if labor supply elasticity is large enough), while it decreases under proportional representation.

3.1 The Environment

We consider a 3-good economy. The goods are: labor (l), private consumption (x) and a government provided public good (g). The public good is provided from tax revenues at the per-unit cost c. Without loss of generality, assume $c = 1$, so the amount of public good provided equals the tax revenue. The population of this economy consists of 3 economic groups denoted as L , M and H. The groups are indexed by J. The groups are differentiated by a group-specific wage (ability) w_J, preference for the public good, θ_J and labor supply elasticity, ε_J. We assume throughout the analysis that $w_L < w_M < w_H$. The mass of the population is normalized to one and and each group J has size α_J[7]. We assume the government can identify each individual's group (for example, by the type of economic activity that the group is engaged in). Each citizen in group J earns before-tax income $y_J = w_J l_J$ and pays a total income tax $\tau_J y_J$, where τ_J is the average (and marginal) income tax rate for group J. An individual's consumption equals that person's after tax income: $x_J = y_J(1 - \tau_J)$. The government collects revenue equal to $\sum\limits_{J=L,M,H} \alpha_J \tau_J y_J$, which is used to

[7] So it consists of a share α_J of the total population.

provide public good g. Since the per-unit cost of producing the public good equals 1,

$$g = \sum_{J=L,M,H} \alpha_J \tau_J w_J l_J$$

Each citizen derives utility from private consumption, utility from the public good and disutility from labor. For simplicity, we adopt a well-behaved quasi-linear utility function:

$$U_J = x_J + \theta_J g - h(l_J) \tag{3.1}$$

As mentioned above, θ_J represents the preference of group the public good; $h(l_J)$ is the disutility from labor, described by the following function:

$$h(l) = \frac{l^{1+\frac{1}{\varepsilon_J}}}{\varepsilon_J + 1}, \tag{3.2}$$

where ε_J is the elasticity of the labor supply for group J. The utility function has the standard properties: the marginal utility with respect to private consumption and the public good is positive; the marginal disutility is positive. Furthermore, $h(l)$ is increasing, convex and continuously differentiable.Thus, U is well-behaved.

The utility maximizing choice for individual labor supply and private consumption is obtained from the maximization problem:

$$\max_{x_J, l_J} x_J + \theta_J g - h(l_J)$$
$$s.t.\ x_J \leq w_J l_J (1 - \tau_J)$$

The optimal labor supply would then be

$$l_J = (\varepsilon_J w_J (1 - \tau_J))^{\varepsilon_J} \tag{3.3}$$

Notice that all individuals supply positive units of labor in equilibrium. Given the optimal choice of labor supply, the associated individual indirect utility function is given by

$$V_J(\tau_L, \tau_M, \tau_H) = \frac{\varepsilon_J^{\varepsilon_J} w_J^{\varepsilon_J+1}(1-\tau_J)^{\varepsilon_J+1}}{\varepsilon_J+1} + \theta_J g \tag{3.4}$$

where $g = \sum_{I=L,M,H} \alpha_I \tau_I w_I l_I$.

To sum up, notice that, in the environment described above, the three groups differ with respect to four exogenous factors: wage, preference for the public good, elasticity of the labor supply and share of the population. The wage and share of the population variables allow us to study variations in the income distribution and inequality. The differences in labor supply elasticity introduce differentiated labor supply disincentives and allows us to study the taxation problem in the presence of these disincentives. Finally, the preference for the public good allows us to study the effects of preference diversity on the equilibrium tax schedules. Individual utility maximization coupled with the political process determine three endogenous variables for each group: the tax rate, the private consumption and the labor units supplied.

3.2 Political Equilibrium under the Single District Majoritarian Rule

The majoritarian electoral rule specifies that the party with the highest number of votes wins the election and implements its desired policy. A long line of research in political science (Cox 1997, Laver and Schofield 1990, Lijphart 1994) has studied the influence of electoral rules on party formation. According to these models, the majoritarian rule leads in most cases to the emergence of a two-party system (due to the electoral advantage that large parties tend to have). Thus, we model the political competition under a single-district majoritarian rule as a two-party competition. For this, we will use a probabilistic voting model of political competition.

3.2.1 Introduction to Probabilistic Voting

To motivate our choice of the probabilistic voting system, we start by giving a brief overview of a related modelling approach, deterministic voting (Mueller 2003: 230-257).The pioneering model and the most well-known framework is the Hotelling-Downs model, which fulminates in the well-known Median Voter Theorem. In the basic version of this model, the political space can be represented along a left-right continuum, a one-dimensional space. Political parties can position themselves at any point in this space. If individuals have single-peaked preferences and vote for the party which positions itself closest to their most preferred point, then the winning platform will be the one most preferred by the median voter. The existence of an equilibrium in the deterministic models hinges, generally, on the limiting assumption that the policy space is unidimensional. If the number of policy dimensions increases, the deterministic models will typically fail to deliver an equilibrium. To appreciate this, assume we have three equally sized groups of voters, similar to the ones described above (L, M and H), and only two policy dimensions (say, the tax rates on groups M and H).[8] The winning policy is elected by majority rule. For simplicity, assume the indifference curves of the groups are circles whose centers lie at their respective ideal points. Figure 3.1 shows the ideal points of the three groups: it can be easily seen that L would prefer to maximize the provision of the public good by imposing high tax rates on both M and H. M would prefer a high tax on H and a low (possibly 0) own-group tax. Similarly, H would prefer a low (possibly 0) own-group tax rate and a high tax rate for M.

Now consider Figure 3.1a and policy proposal A. Both groups L and H prefer any point in the shaded convex hall to policy A. Hence, policy B can defeat policy A in an election. Next, consider Figure 3.1b and policy B. Both groups L and M prefer any policy in the

[8] We assume that the tax rate for group L is exogenously determined, so that, for purposes of illustration, 2-dimensional figures can be used instead of 3-dimensional ones.

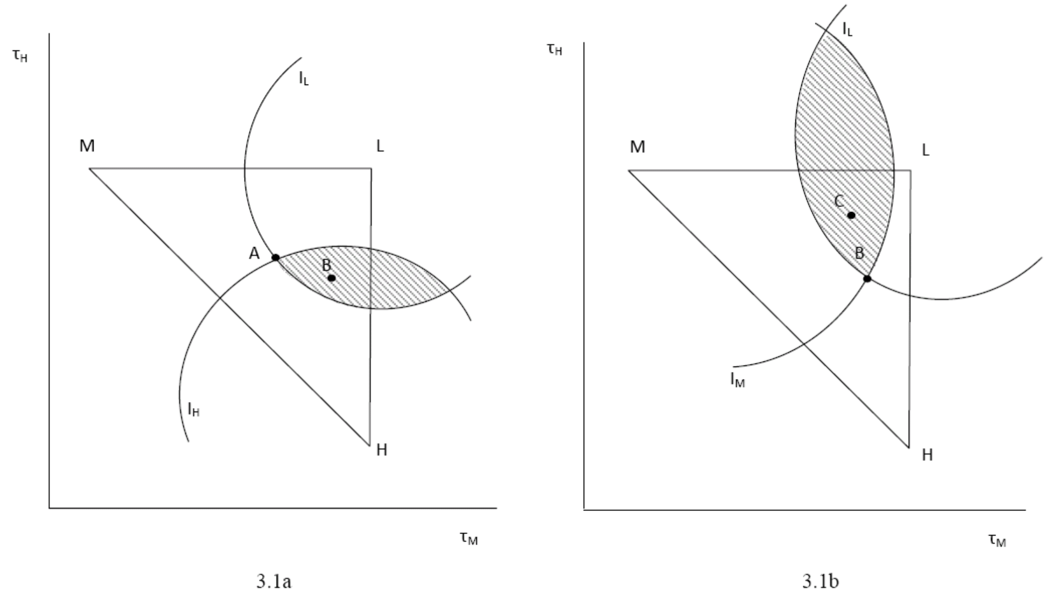


Figure 3.1: Ideal Points and Non-Existence of an Equilibrium

shaded convex hall to policy B. Hence policy B can be beaten by policy C in an election. In general, any policy can be defeated, an equilibrium does not exist, and cycling results.

The non-existence of an equilibrium results from the discontinuity in voter behavior. To appreciate this, consider the situation depicted in Figure 3.2. Begin by focusing on Figure 3.2a and policy A. We assume that policy A and policy P_0 lie on the same indifference curve of group M, so this group is indifferent between the two options. Now consider all policies that lie on the line segment connecting policies P_1 and P_2. Note that this segment passes through P_0. Then any policy on the line segment between P_1 and P_0 is preferred to policy A and policy A is preferred to any policy that lies between P_0 and P_2. Figure 3.2b describes the probability that a member of group M votes for policy A when it is pitted against a policy from the line segment P_1P_2. Clearly, there is a discontinuity at τ_0, the tax rate for group M under policy P_0.

To avoid this pitfall, the probabilistic voting model "smooths" the voters' preferences by introducing uncertainty. For example, a voter's preference might depend on other factors

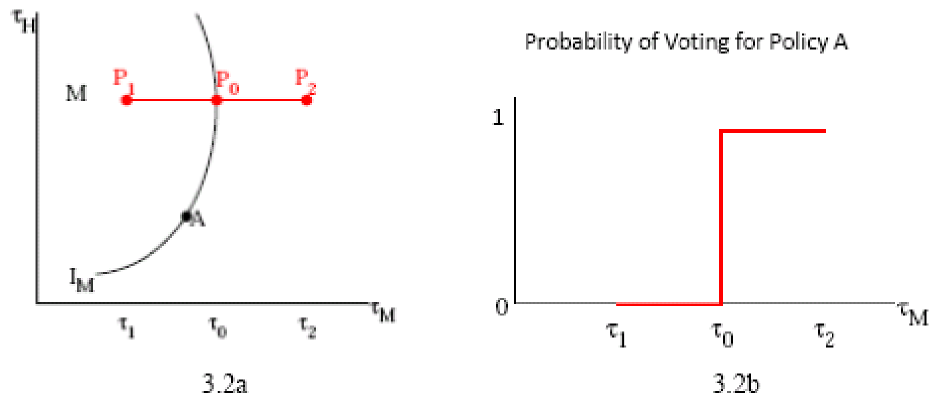


Figure 3.2: Deterministic Voting

than just the platforms offered. This other factor could be party ideology, personal characteristics of the candidates or any other characteristic that makes individuals not vote entirely following their "economic" self-interest.[9] This "ideological" factor cannot be modified as part of the electoral platform. Therefore, the introduction of the ideological dimension creates an element of uncertainty into voting behavior. Figure 3.3 illustrates how probabilistic voting can eliminate the discontinuity in voter preferences, by adding the ideological dimension. Consider group M and the policies described in Figure 3.3a. As we move along the line segment P_1P_2, members of group M become more likely to vote for policy A. Figure 3.3b illustrates the probability of voting for Policy A. Note that the discontinuity that used to exist at τ_0 under deterministic voting has disappeared. A Nash equilibrium in the multi-dimensional policy space now exists (Coughlin and Nitzan 1981). Notice that probabilistic voting implies that a party has an incentive to modify its proposed policy in the direction of any given group of voters, since every group of voters could potentially vote for it. The extent to which any group of voters is favored by the proposed policy will depend on the expected electoral gains from moving the policy towards that group's ideal point relative

[9] See Persson and Tabellini (2000) and Mueller (2003) for an extensive discussion of the probabilistic voting model.

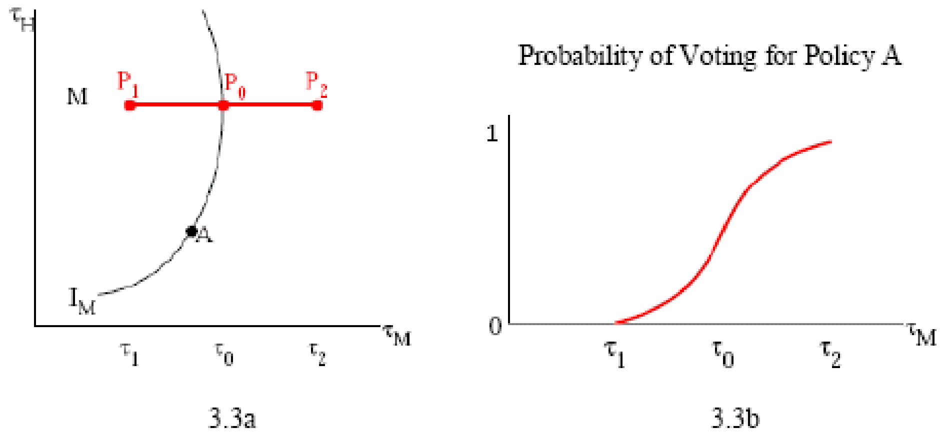


Figure 3.3: Probabilistic Voting

to the cost of moving the policy away from the other groups' ideal points (measured by the decreased probability that members of other groups will vote for the respective party). Thus, in equilibrium, each party's proposed policy will depend on the weighted utilities of all groups of voters. Moreover, just like in the deterministic case, it can be shown that both parties propose the same policy in equilibrium (Persson and Tabellini 2000: 54).

3.2.2 The Formal Model and Characterization of Equilibria

Formally, the game has two parties, A and B. Each party attempts to maximize the expected value of the votes received. The game proceeds as follows: (1) Before voting takes place, parties present their platforms to the electorate, simultaneously and non-cooperatively. The winner is then committed to implementing the platform previously proposed. The parties make their choices knowing voters' preferences for taxation and the probability distribution of ideological biases. (2) Voters make choices based both on a self-interested welfare calculation and an idiosyncratic ideological inclination for one of the parties. (3) The winning policy is implemented.

As stated above, the main implication of the ideological bias is to introduce a probabilistic element into the voting process. In what follows, we build on the probabilistic voting game described by Persson and Tabellini (2000). Following their lead, we assume that an individual i belonging to group J chooses to vote for party A if:

$$V_J^A > V_J^B + \sigma_{iJ}^{BA} \tag{3.5}$$

where V_J^A and V_J^B are the indirect utilities from policy A and policy B, and σ_{iJ}^{BA} is the ideological bias of person i in group J for party B relative to party A, σ_{iJ}^{BA} is a random variable that can take both positive and negative values. We assume σ_{iJ}^{BA} is distributed uniformly on $[-\frac{1}{2\phi_J}, \frac{1}{2\phi_J}]$ for each group J. Thus, the density of each distribution is $\phi_J > 0$, where ϕ_J can take a different value for each group J. Since ϕ_J reflects the spread of the distribution around 0, it can be thought of as a measure of the ideological dispersion of the voters in a group. As illustrated in Figure 3.4, if ϕ_J is large, then the distribution is concentrated around 0 and voters of group J are less strongly inclined towards one party or the other (the ideological factor plays a smaller role in their choice). Therefore, voters of group J can be more easily swayed by a change in their expected utility from the tax policy. In other words, if ϕ_J is large, group J has a larger political influence, because a small change in a party's platform towards group J can potentially yield a large number of additional votes. Thus, the magnitude of ϕ_J reflects the political influence of group J.

In any given group denote the "swing voter" as the voter for whom $V_J^A = V_J^B + \sigma_{iJ}^{BA*}$. The swing voter's ideological bias, σ_{iJ}^{BA*}, is such that he is indifferent between the policy offered by party A and the policy offered by party B[10]. Then, all voters of group J for whom σ_{iJ}^{BA} is smaller than the swing voter's bias σ_{iJ}^{BA*} will vote for party A, while the ones for whom

[10] We implicitly assume that $|\sigma_{iJ}^{BA*}| < \frac{1}{\phi_J}$

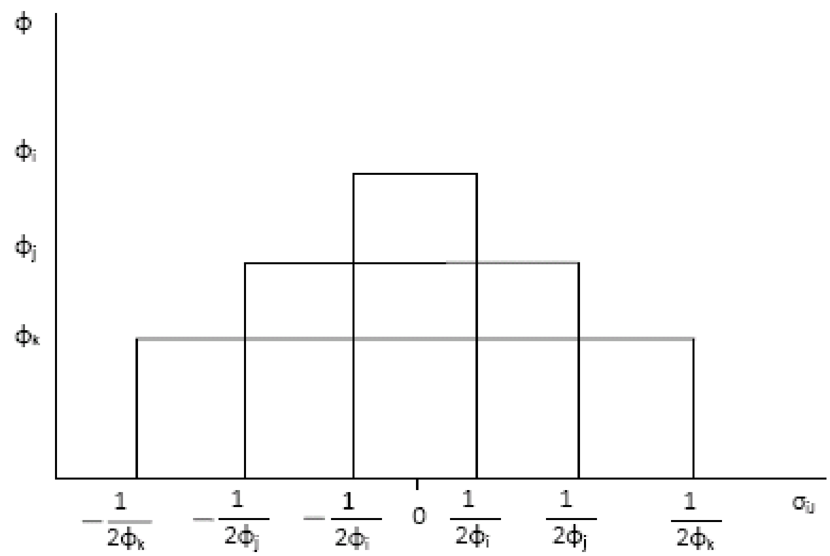


Figure 3.4: A possible voter ideological dispersion

σ_{iJ}^{BA} is larger than σ_{iJ}^{BA*} will vote for party B. Therefore, the expected share of votes won by party A from group J will be given by $\pi_J^A = F_J[\sigma_{iJ}^{BA*}] = F_J[V_J^A - V_J^B]$, where F_J is the CDF of the uniform distribution. Over the entire population the expected share of votes won by A will be:

$$\pi^A = \sum_{J=L,M,H} \alpha_J F_J[V_J^A - V_J^B] = \sum_{J=L,M,H} \alpha_J \phi_J((V_J^A - V_J^B) + \frac{1}{2\phi_J}) = \frac{1}{2} + \sum_{J=L,M,H} \alpha_J \phi_J (V_J^A - V_J^B) \tag{3.6}$$

Similarly, the share of party B would be $\pi^B = \sum_J \alpha_J F_J[V_J^B - V_J^A]$. Notice that, unlike deterministic voting, in the probabilistic model the probability of winning is a continuous function of the parties' electoral platforms. Since winning the election (and implementing the desired policy) involves winning the largest share of votes, each party will try to maximize their expected share. Since π_A and π_B are concave, there exists a unique solution (τ_L, τ_M, τ_H) to the maximization problem, and the game has a unique Nash equilibrium where both parties propose the same policy, and this policy is a weighted sum of voter utilities (Persson and Tabellini 2000: 54). To arrive at the equilibrium policy, we differentiate π_A with respect to τ_L, τ_M and τ_H, to obtain the first order conditions[11]:

$$\alpha_L \phi_L \frac{\partial V_L^A}{\partial \tau_J} + \alpha_M \phi_M \frac{\partial V_M^A}{\partial \tau_J} + \alpha_H \phi_H \frac{\partial V_H^A}{\partial \tau_J} = 0 \tag{3.7}$$

for $\tau_J \in \{\tau_L, \tau_M, \tau\}$.

Writing out V_L^A, V_M^A, and V_H^A in the form given by Equations 3.4 and 3.3[12] and taking the derivative with respect to τ_J, we obtain the the following first order condition for $J \in \{L, M, H\}$

$$\alpha_J \phi_J (w_J \frac{\partial l_J}{\partial \tau_J} - w_J \tau_J \frac{\partial l_J}{\partial \tau_J} - w_J l_J - \frac{\partial h}{\partial l_J} \frac{\partial l_J}{\partial \tau_J}) + (\alpha_J w_J l_J + \alpha_J w_J \tau_J \frac{\partial l_J}{\partial \tau_J}) \sum_{i=L,M,H} \alpha_i \phi_i \theta_i = 0 \tag{3.8}$$

This simplifies to:

$$(\alpha_L \phi_L \theta_L + \alpha_M \phi_M \theta_M + \alpha_H \phi_H \theta_H)(1 - \frac{\tau_J \varepsilon_J}{1 - \tau_J}) - \phi_J = 0 \tag{3.9}$$

We denote $\Lambda_J = 1 - \frac{\phi_J}{\alpha_L \phi_L \theta_L + \alpha_M \phi_M \theta_M + \alpha_H \phi_H \theta_H}$ to be the index of group J's political influence relative to the average political demand for the public good (denoted by $\alpha_L \phi_L \theta_L + \alpha_M \phi_M \theta_M + \alpha_H \phi_H \theta_H$). The index is lower if group J is more politically influent (i.e. ϕ_J is higher) compared to the other groups. Then, rearranging the terms of Equation 3.9, we obtain the simple form:[13]

$$\tau_J = \frac{1}{1 + \frac{\varepsilon_J}{\Lambda_J}} \tag{3.10}$$

[11]Since both A and B propose the same policy, we denote the equilibrium policy by $(\tau_{L,}\tau_M, \tau_H)$, omitting the superscripts A or B.

[12]$V_J(\tau_L, \tau_M, \tau_H) = \frac{\varepsilon_J^{\varepsilon_J} w_J^{\varepsilon_J+1}(1-\tau_J)^{\varepsilon_J+1}}{\varepsilon_J+1} + \theta_J \sum_{I=L,M,H} \alpha_I \tau_I w_I l_I$ where $l_J = (\varepsilon_J w_J (1-\tau_J))^{\varepsilon_J}$.

[13]For ϕ_L, ϕ_M, ϕ_H and ε_J such that $\frac{\partial^2 \pi_A}{\partial \tau_J^2} = \alpha_J \varepsilon_J^{\varepsilon_J+1} w_J^{\varepsilon_J+1}(1-\tau_J)^{\varepsilon_J-2}(\phi_J(1-\tau_J) + (\sum_J \alpha_J \phi_J \theta_J)(-2 + \tau_J(1+\varepsilon_J)) < 0$, so π_A satisfies the second-order conditions.

Equation 3.10[14] shows that the equilibrium tax rate for each group is the result of a trade-off between a couple of key economic and political costs and benefits, captured by the $\frac{\varepsilon_J}{\Lambda_J}$ term. The $\frac{\varepsilon_J}{\Lambda_J}$ ratio provides a measure of the average economic and political costs relative to the benefits of taxation and, as expected, a higher relative cost of taxation for a group J (a higher $\frac{\varepsilon_J}{\Lambda_J}$) means a lower tax rate on that group. In fact, if $\frac{\varepsilon_J}{\Lambda_J} > 1$, then $\tau_J < \frac{1}{2}$, so τ_J is relatively low. If instead $\frac{\varepsilon_J}{\Lambda_J}$ is lower, $\frac{\varepsilon_J}{\Lambda_J} < 1$, then the tax rate τ_J will be high ($\tau_J > \frac{1}{2}$).

To see how $\frac{\varepsilon_J}{\Lambda_J}$ captures the costs of taxation, first consider the ε_J term. The extent of the labor supply distortion caused by taxation depends on each group's labor supply elasticity, ε_J. Thus, Equation 3.10 qualitatively illustrates the well-known result from the optimal taxation literature that as ε_J is higher, the relative cost of taxation, $\frac{\varepsilon_J}{\Lambda_J}$, is higher, so the tax rate paid by group J decreases. Then, consider the term Λ_J, which captures two important characteristics of group J, the strength of the preference for the public good and the extent of J's political influence. Thus, Λ_J captures the benefits of taxation (the public good provision) relative to the political costs of taxation (the loss in vote share). The preference for the public good is reflected by θ_J. A higher θ_J increases the benefits from the public good, hence we expect a higher tax rate for group J as θ_J increases. Indeed, a higher θ_J translates into a higher Λ_J and thus a lower cost of taxation relative to its benefits. So, from Equation 3.10, it can be seen that this results in a higher tax rate. The extent of the loss in vote share is given by the degree to which a group of voters cares about economic policy as opposed to ideology. When the ideological dispersion of the group is low (ϕ_J is high), voters are most sensitive to the parties' economic platform and the vote share lost as a result of increases in taxation is higher. Λ_J is an index of the political influence of group J (namely ϕ_J) weighted by the "average political demand" for the public good (namely $\alpha_L\phi_L\theta_L + \alpha_M\phi_M\theta_M + \alpha_H\phi_H\theta_H$). From Equation 3.10 we obtain the intuitive result that if

[14] We assume $\frac{\varepsilon_J}{\Lambda_J} > 0$ and $0 < \Lambda_J < 1$. Thus, we are assuming that the demand for the public good (θ_J) is high enough to ensure that the tax revenue will be raised.

group J is relatively more politically influent, Λ_J is lower, the relative cost of taxation, $\frac{\varepsilon_J}{\Lambda_J}$, is higher and tax rate τ_J is lower.

We shall now investigate the model by following a two step strategy:

- First, in Propositions 1 and 2, we articulate two sets of sufficient conditions that ensure tax progressivity. These propositions focus on the supply elasticity of labor and on political influence.

- Second, in Remarks 1 through 4, we investigate how the various parameters of the model influence the tax rates on each group and the degree of progressivity. The degree of progressivity is measured by the difference between the top and bottom tax rates.

We start by assuming that the labor supply elasticity is identical for all groups. Then the condition for the equilibrium tax schedule to be progressive is given in the following proposition:

Proposition 1 *If all groups have the same supply elasticity of labor, $\varepsilon_L = \varepsilon_M = \varepsilon_H = \varepsilon$, and $\phi_L > \phi_M > \phi_H$, then the tax schedule is progressive ($\tau_L < \tau_M < \tau_H$).*

Proof. Since $\varepsilon_L = \varepsilon_M = \varepsilon_H = \varepsilon$, Equation 3.10 implies $\tau_J = \frac{1}{1+\frac{\varepsilon}{\Lambda_J}}$. Also, since $\Lambda_J = 1 - \frac{\phi_J}{\alpha_L \phi_L \theta_L + \alpha_M \phi_M \theta_M + \alpha_H \phi_H \theta_H}$, $\phi_L > \phi_M > \phi_H$ implies $\Lambda_L < \Lambda_M < \Lambda_H$. Hence, $\frac{1}{1+\frac{\varepsilon}{\Lambda_L}} < \frac{1}{1+\frac{\varepsilon}{\Lambda_M}} < \frac{1}{1+\frac{\varepsilon}{\Lambda_H}}$, and the tax schedule is progressive. ■

Intuitively, as discussed above, each party is equating the political cost of taxing a group to the electoral benefit obtained from that group. Recall that ϕ_J measures the political influence of group J. A group whose ϕ_J is large has much political influence and hence can exploit its influence to be taxed at a lower rate.

We are now interested in the situation where all groups have the same ideological dispersion, but varying labor supply elasticities. Then, the progressivity condition is given by the following proposition:

Proposition 2 *If all groups have the same political influence, $\phi_L = \phi_M = \phi_H = \phi$, and $\varepsilon_L > \varepsilon_M > \varepsilon_H$, then the tax schedule is progressive ($\tau_L < \tau_M < \tau_H$).*

Proof. Since $\phi_L = \phi_M = \phi_H$ and $\Lambda_J = 1 - \frac{\phi_J}{\alpha_L \phi_L \theta_L + \alpha_M \phi_M \theta_M + \alpha_H \phi_H \theta_H}$, this implies that $\Lambda_L = \Lambda_M = \Lambda_H = \Lambda$ and Equation 3.10 becomes $\tau_J = \frac{1}{1+\frac{\varepsilon_J}{\Lambda}}$. Also, since $\varepsilon_L > \varepsilon_M > \varepsilon_H$, it follows that $\frac{1}{1+\frac{\varepsilon_L}{\Lambda}} < \frac{1}{1+\frac{\varepsilon_M}{\Lambda}} < \frac{1}{1+\frac{\varepsilon_H}{\Lambda}}$, and the tax schedule is progressive. ■

The intuition behind this result links back to the above discussion about the costs and benefits of taxation. As mentioned above, the "economic" costs of taxation depend on the elasticity of the labor supply, since this measures the extent to which a change in the tax rate will lead to a decrease in the labor supplied by voters (and consequently in their income). A higher labor supply elasticity translates into a higher labor supply distortion when a tax is imposed. Thus, a group with a higher elasticity is taxed at a lower rate.

Last, notice that the wage gap and the preference for the public good play no role in establishing whether a tax schedule is progressive. As it will be shown in the next subsection, the ordering of the preferences for the public good and wage inequality become significant in many of the equilibria reached under the proportional system.

3.2.3 Comparative Statics

The results of the previous section outline sufficient conditions under which the equilibrium tax schedule is progressive. We shall now turn to the question of how the exogenous economic variables influence the degree of progressivity (measured as the difference between the highest and lowest tax rate). As mentioned in the introduction, the labor supply elasticity allows us to account for labor supply disincentives into the taxation problem. The share of the population variables allow us to study variations in income distribution and mobility. We use variations in the preference for the public good to study the effects of preference diversity on progressivity. Lastly, we look at the role of political variables, by studying the effect of political influence within groups and between groups.

Labor Supply Elasticities We start by looking at the effects of variations in the labor supply elasticity on tax progressivity. As in the previous section, we look first at the case when $\varepsilon_L = \varepsilon_M = \varepsilon_H = \varepsilon$.

Remark 1 *Assume that the tax schedule is progressive under the conditions of Proposition 1: $\varepsilon_L = \varepsilon_M = \varepsilon_H = \varepsilon$ and $\phi_L > \phi_M > \phi_H$. Suppose that the supply elasticity of labor decreases. Then the tax rate increases for each group, more public good is provided and:*

- *if $\frac{\varepsilon^2}{\Lambda_H \Lambda_L} > 1$, then progressivity increases.*
- *if $\frac{\varepsilon^2}{\Lambda_H \Lambda_L} < 1$, then progressivity decreases.*

Proof. See the Appendix. ■

Notice that the case when elasticities differ but the ideological dispersion is the same for all groups (when the tax schedule is progressive under the conditions of Proposition 2) has fairly straightforward implications regarding the effects of changes in elasticities: the degree of progressivity increases when ε_H decreases or when ε_L increases.

Income Distribution Next, we consider the role of α_J, the proportion of the total population that is in group J. Each political party maximizes a sum of indirect utilities weighted by α_J. Therefore, as can be seen from equation 3.10,[15] a change in α_J would have a similar effect on all three tax rates: all tax rates would either increase or decrease. To study the

[15] Recall this is: $\tau_J = \frac{1}{1+\frac{\varepsilon_J}{\Lambda_J}}$, where $\Lambda_J = 1 - \frac{\phi_J}{\alpha_L \phi_L \theta_L + \alpha_M \phi_M \theta_M + \alpha_H \phi_H \theta_H}$

effects of income distribution on progressivity, we look at a couple of possible changes in the income distribution. First, we reduce the complexity of the problem, to allow us to derive the intended comparative statics. The problem becomes too complex when both labor supply elasticity and ideological dispersion vary across groups, so we reduce our analysis to the case when the tax schedule is progressive under the conditions of Proposition 2 (when the political influence of each group is identical). Moreover, we assume that the preference for the public good is decreasing as income increases (that is, $\theta_L > \theta_M > \theta_H$). We make this assumption because we want to focus on the situations when the equilibrium tax schedules are progressive, and a redistributive public good provides favorable conditions for the emergence of tax progressivity. Also, a redistributive public good allows us to capture the redistributive nature of government spending, which is an important aspect of tax policy.

Remark 2 *Assume that $\theta_L > \theta_M > \theta_H$ and the tax schedule is progressive under the conditions of Proposition 2: $\varepsilon_L > \varepsilon_M > \varepsilon_H$ and $\phi_L = \phi_M = \phi_H = \phi$. If there is downward population mobility, from a higher income group to a lower income group (i.e. some members of group M now become members of group L or some members of H move to either M or L), then*

- *if $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} > 1$, the tax rate paid by each group increases, more public good is produced, and progressivity increases.*
- *if $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} < 1$, the tax rate paid by each group decreases, less public good is produced, and progressivity decreases.*

Proof. See the Appendix. ■

Intuitively, the above result illustrates the close dependence of political platforms on the preference of the electorate (if $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} > 1$). If voters move from a group A to a group B, then the platform offered by the political parties will adjust to put more weight on the preferences of group B, which now has a higher importance in the electoral process (since it has more voters) and less weight on the preferences of group A, which has become less important. This will affect progressivity since the tax burden on B decreases while the tax burden on A increases. The role of the $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2}$ ratio is discussed in more detail towards the end of this section.

Public Good Preferences The parameter θ_J captures group J's preference for the public good. We thus investigate how an increase in the preference for the public good for one of the groups affects equilibrium progressivity.

Remark 3 *Assume that the tax schedule is progressive under the conditions of Proposition 2: $\varepsilon_L > \varepsilon_M > \varepsilon_H$ and $\phi_L = \phi_M = \phi_H = \phi$. If the preference for the public good increases (for any of the groups), then the tax rate paid by each group increases and*

- *if $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} > 1$, progressivity increases.*
- *if $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} < 1$, progressivity decreases.*

Proof. See the Appendix. ■

Notice that Remarks 1, 2 and 3 share two common elements. First, in each case, the change in the exogenous parameters results in a higher demand for the public good, which requires additional tax revenue to be raised by increasing all three tax rates. Secondly, in each case, the effect on progressivity depends on a ratio involving ε_J and Λ_J. How can we explain this? Focus on Remark 2, in which $\varepsilon_L > \varepsilon_M > \varepsilon_H$ and $\phi_L = \phi_M = \phi_H = \phi$. When $\frac{\varepsilon_J}{\Lambda}$ is high ($\frac{\varepsilon_J}{\Lambda} > 1$), it follows from Equation 3.10 that τ_J is relatively low. Then, the necessary increase in taxes can be achieved in a distortion minimizing way by taxing more heavily the groups with lower distortions of taxation, namely the higher income groups, which have the lower labor supply elasticities. On the other hand, when $\frac{\varepsilon_J}{\Lambda}$ is low ($\frac{\varepsilon_J}{\Lambda} < 1$), the tax rate on each group is high and the need to raise tax revenue requires imposing even higher tax rates. The already large size of the tax rates limits the degree to which parties can place tax burdens onto groups. Since the high income group is already taxed at a very high rate (because the tax schedule is already progressive), adding on an additional high tax burden would involve significant distortionary effects. Thus, efficiency considerations make it necessary to sacrifice some progressivity in order to raise more tax revenue.

Political Influence Lastly, we are also interested in the effects of variations in the key political variable of the model Thus, we study the effect of a change in a group's political influence on the progressivity of the tax schedule.

Remark 4 *The progressivity of the tax schedule increases as the low income group becomes more politically influent. Similarly, the progressivity of that tax schedule decreases if the high income group becomes more politically influent.*

Proof. See the Appendix. ■

This conclusion follows easily from the result derived earlier that as a group becomes more politically influent, it can use its influence to obtain a lower tax rate and to shift the tax burden on to the other groups.

3.3 Political Equilibrium under the Proportional Electoral Rule

We now turn to the second electoral system of our study, the proportional electoral system. Under proportional elections all voters belong to a single (national) district and each party running in the election receives a percentage of seats in the legislature equal to its share of the national vote[16]. If one single party wins the majority of seats in the legislature, then

[16] This analysis can also be applied to multiple-district majoritarian systems, like the system used in the U.K.

that party will be able to implement its desired policy. However, if no single party holds a majority of seats, then a majority coalition would have to be formed by bargaining among the groups represented in the legislature. In this section, we use a simple model of legislative bargaining to analyze the effects of policy-making on the progressivity of the equilibrium tax schedules, under the proportional rule, when no party holds an absolute majority of seats.

3.3.1 An Adapted Legislative Bargaining Model

In modelling the proportional system, we assume that there exist three parties, L, M and H, each corresponding to one of the three groups described in Section 3.1. Each of the parties is a perfect representative of a particular group, so no other party has the incentive to enter the electoral race as a fourth candidate, since members of each group are already represented perfectly by one of the existing parties. In line with the citizen-candidate literature (Besley and Coate 1997; Osborne and Slivinski 1996), we assume that each party can only credibly commit to preferred policies of the group that it represents. In the case of coalitions, we assume, as in Levy (2004), that the members can only credibly commit to policies that lie in the Pareto set of the members of the coalition. Since we are interested in characterizing the non-trivial equilibria, in which no single party holds a majority of seats in the legislature, it follows that any two parties can form a majority.

The most used proportional rule legislature model is the Baron and Ferejohn (1989) model of legislative bargaining. It depicts the inter-party negotiation process as a multi-agent non-cooperative bargaining game with decisions being made by a (qualified) majority rule in the legislature.[17] The sequence of events is as follows:

1. Nature randomly chooses one legislator to be the formateur.

2. This legislator then proposes a policy vector from the available policy space and the legislature votes on the proposal. A legislator is assumed to vote for the proposal if it gives

[17] The term "qualified" refers to the requirement of more than a simple majority for a policy to be approved.

him a higher utility than his continuation value in the event that the proposal is rejected.

3. If a (qualified) majority of the representatives are in favor of the proposal, then the proposal is implemented and the legislature adjourns. Otherwise, nature randomly chooses another legislator to make a proposal and steps 1-3 are repeated until a proposal is accepted or until a maximum number of rounds has been reached.[18] If the proposal is rejected in the last possible round, then a default policy is implemented. The default policy could be the current status-quo or some other policy prescribed in the constitution for cases when the legislature cannot agree on policy.

Baron and Ferejohn (1989) proved that this game always has equilibria, even when the number of proposal rounds allowed is infinite. If policy outcomes in future periods are discounted at some factor $\beta \leq 1$,then it can be shown that is never optimal for any formateur to make an unacceptable proposal in the first round.

In our model, we will use a simple version of the Baron and Ferejohn (1989) model with 3 parties. As it follows from the above discussion, a proposal under this game is implemented if it is supported by one of the three possible coalitions: ML, MH and HL.[19] Moreover, the range of equilibrium policies will differ depending on which party is selected to make the proposal. In this aspect, we depart slightly from the original Baron and Ferejohn (1989) model, and we assume that, once a party is recognized as the formateur, this party is fixed as the formateur for the entire duration of the negotiations (i.e. if M is chosen as the formateur, L and H have a 0 probability of being selected as formateurs as any stage of the game). We motivate this change by the observation that, in most proportional systems, government formation is not achieved through the random selection of a legislator in each negotiation round. Instead, a party is recognized as the formateur and is in charge of carrying

[18] The maximum number of rounds is known from the beginning.

[19] A grand coalition LMH is also possible, but notice that the Pareto space of the grand coalition is a subset of one of the 2-party coalitions; as it is well-known from "dividing the dollar"-type of games, since only 2 parties are needed to form a majority, it is always (weakly) optimal for any 2 parties to deviate from the grand coalition. Thus, a grand coalition will never be a stable equilibrium.

out negotiations with any other party in the legislature in order to form a government.[20]. Therefore, once a formateur is selected, our game will focus on the negotiation over policy between 2 parties, one of which is the formateur.

The Stahl-Rubinstein bargaining game provides a useful framework for modelling a 2-party negotiation. This game yields a unique sub-game perfect Nash equilibrium in the infinite period game. The Stahl-Rubinstein game is as follows. In period 1, the first player makes a proposal to the second player for splitting a pie. The second player then has the option to either accept or reject the proposal. If he accepts the proposal, then the players receive the payoffs proposed by player 1 and the game ends. If he rejects the proposal, then the game moves on the period 2 and player 2 gets to make a proposal to player 1. Then player 1 can accept or reject. If he accepts, they both receive the payoffs proposed by player 2 discounted by the discount factor of each player. If he rejects, then the game moves on to period 3 and player 1 gets to make a proposal. The game continues in this way until an offer is accepted, or until the maximum number of periods is reached (if only a finite number of periods are allowed). Rubinstein (1982) showed that the infinite-horizon game has a unique sub-game perfect Nash equilibrium (with a fixed optimal allocation for player 1 and a range of allocations that would be accepted by player 2). Moreover, the first player has an advantage (will receive more of the pie) even when discount factors are equal. However, if players are equally patient (have the same discount factor) and the time between offers tends to 0, then the first-mover advantage disappears and the unique subgame perfect Nash equilibrium is that the pie will be split evenly between the 2 players.

To sum up, in our model we use a Baron and Ferejohn (1989) bargaining game with 3 parties, an infinite number of negotiation rounds and a simple majority requirement.[21] We

[20] The formateur is chosen based on constitutionally mandated criteria like being the party that holds the most seats in the legislature, being located more towards the center of the Pareto space, being nominated by another institutional player etc.

[21] Since we are allowing for an infinite number of negotiation rounds, an explicit default policy is not necessary.

make the assumption described above regarding the persistence of the formateur. Thus the Baron and Ferejohn (1989) 3-party game that we start with is reduced to a Stahl-Rubinstein non-cooperative two-person bargaining game. We model the 2 party coalition negotiations as a Stahl-Rubinstein infinite game with equal discount factors for all parties [22] and time between offers tending to 0.[23] This model will provide a simple and tractable framework for analyzing the progressivity question.

3.3.2 The Bargaining Model and Characterization of Equilibria

The Equilibrium We analyze the equilibria reached when each of the three parties is the formateur. We begin by considering the case in which M is recognized as the formateur. Since M alone does not hold a majority of seats, M faces the choice whether to negotiate with L or with H.[24] Negotiations are assumed to be carried out over each group's post-tax utility, since this is the ultimate measure of each group's gains or losses from taxation. We thus assume that the platforms proposed during a negotiation between M and some other party involve maximizing a weighted sum of the utilities of these two parties. The discount factor β is the same for all parties and we assume that the time period between offers goes to 0, so in the spirit of Stahl-Rubinstein, we consider equilibria in which the utilities of the two negotiation partners are weighted equally.

As the formateur, M can compare its expected utility from negotiating with L versus its expected utility from negotiating with H and can form the coalition that would offer it the highest utility. First, consider the ML coalition. The equilibrium tax vector $t = (\tau_L, \tau_M, \tau_H)$ of this coalition will maximize $S_{ML}(t) = V_M(t) + V_L(t)$, the sum of the indirect utilities of

[22] This assumption relies on the idea that not forming a government results in similar loses of rents or political capital for all players.

[23] A similar approach was followed by Iversen and Soskice (2006) in their study of the effects of partisanship on redistribution.

[24] Unlike the standard Stahl-Rubinstein game, the negotiations between M and another party involve not only dividing a pie (establishing the tax rates for each group), but also agreeing on how large the total pie should be (how much tax revenue is collected).

L and M. As derived in Section 3.1, $V_J(t) = \frac{\varepsilon_J^{\varepsilon_J} w_J^{\varepsilon_J+1}(1-\tau_J)^{\varepsilon_J+1}}{\varepsilon_J+1} + \theta_J \sum_{J=L,M,H} \alpha_J \tau_J w_J l_J$, thus the above maximization problem becomes:

$$\max_t S_{ML}(t) = \sum_{J=L,M} \frac{\varepsilon_J^{\varepsilon_J} w_J^{\varepsilon_J+1}(1-\tau_J)^{\varepsilon_J+1}}{\varepsilon_J+1} + (\theta_M + \theta_L) \sum_{J=L,M,H} \alpha_J \varepsilon_J^{\varepsilon_J} w_J^{\varepsilon_J+1}(1-\tau_J)^{\varepsilon_J}\tau_J \tag{3.11}$$

The solution to the above maximization problem is the tax vector $t = (\tau_L^{ML}, \tau_M^{ML}, \frac{1}{1+\varepsilon_H})$. $\frac{1}{1+\varepsilon_H}$ is the maximum tax rate on the Laffer curve of group H. This happens because H's utility is not taken into account inside the ML coalition, so both M and L want to extract as much revenue as possible from group H in order to finance the public good. The tax rate for the other two groups, L and M, can be either:

- $\tau_J^{ML} = 0$ if $\Omega_{ML} > 1$, or
- $\tau_J^{ML} = \frac{1-\Omega_{ML}}{1+\varepsilon_J-\Omega_{ML}}$ if $\Omega_{ML} < 1$, $J = L, M$.

where $\Omega_{JK} = \frac{1}{\theta_J\alpha_J+\theta_K\alpha_K}$ is the reciprocal of the weighted average preference for the public good between coalition members. A higher Ω_{ML} indicates a lower average coalition preference for the public good, so a higher "dislike" for the public good. If the weighted average intensity of preference for the public good between M and L is large enough, specifically if $\Omega_{ML} < 1$, then $S_{ML}(t)$ is concave in τ_J^{ML} $\left(\frac{\partial^2 S_{ML}(t)}{\partial \tau_J^2} < 0\right)$ and the maximization problem yields the positive solution $\tau_J^{ML} = \frac{1-\Omega_{ML}}{1+\varepsilon_J-\Omega_{ML}}$. Conversely, if the weighted average intensity of the preferences for the public good between M and L is small enough, such that $\Omega_{ML} > 1$, then the public good yields less utility than the utility lost by paying taxes, so $\tau_J^{ML} = 0$.

If instead M negotiates with H, the tax vector will maximize $S_{MH}(t)$, along the lines of Equation 3.11. The analysis of the maximization problem is identical to the one outlined in the ML case presented above. The equilibrium tax vector is $t = (\frac{1}{1+\varepsilon_L}, \tau_M^{MH}, \tau_H^{MH})$, where $\tau_J^{MH} = \frac{1-\Omega_{MH}}{1+\varepsilon_J-\Omega_{MH}}$ (if $\Omega_{MH} < 1$) or $\tau_J^{MH} = 0$ (if $\Omega_{MH} > 1$), for $J = M, H$ and

$\Omega_{MH} = \frac{1}{\theta_M \alpha_M + \theta_H \alpha_H}$. The group excluded from the coalition, group L, is taxed at the highest point on its Laffer curve, $\frac{1}{1+\varepsilon_L}$.

Given the equilibrium tax rates that could be obtained through negotiations with L and H, party M will choose to form the coalition that offers it the highest utility. This choice will depend on the size of the coalition tax rates and on the amount of public good provided (by the total tax revenue). As can be easily observed from the above equilibrium formulas, our three economic variables of interest (θ_J, ε_J and α_J) will be crucial determinants of the coalition tax rates and, consequently, of M's coalition choice.

The general analysis when L or H are chosen as formateurs is identical to the one above. Due to the symmetry of the game at the 2-party negotiation stage, the equilibrium tax rates of a coalition will be the same no matter which of the two parties is the formateur. Thus there are six possible equilibria of this game:

1. An ML coalition with equilibrium tax rates: τ_J^{ML} equal to $\frac{1-\Omega_{ML}}{1+\varepsilon_J-\Omega_{ML}}$ (if $\Omega_{ML} < 1$) or 0 (if $\Omega_{ML} > 1$) and $\tau_H = \frac{1}{1+\varepsilon_H}$, $J = L, M$.

2. An MH coalition with equilibrium tax rates: τ_J^{MH} equal to $\frac{1-\Omega_{MH}}{1+\varepsilon_J-\Omega_{MH}}$ (if $\Omega_{MH} < 1$) or 0 (if $\Omega_{MH} > 1$) and $\tau_L = \frac{1}{1+\varepsilon_L}$, $J = M, H$.

3. An LH coalition with equilibrium tax rates: τ_J^{LH} equal to $\frac{1-\Omega_{LH}}{1+\varepsilon_J-\Omega_{LH}}$ (if $\Omega_{LH} < 1$) or 0 (if $\Omega_{LH} > 1$) and $\tau_M = \frac{1}{1+\varepsilon_M}$, $J = L, H$.

Focusing on the non-zero solutions described above, we can derive a couple of results regarding how a change in the exogenous parameters affects the tax rate of each group:

- First, it follows immediately from the tax formulas that a decrease in ε_J increases group J's tax rate.

- Secondly, if a group is not a member of the coalition, its preference for the public good (θ_J) and its share in the population (α_J) play no role in the solution (since the non-coalition member's tax rate is always $\frac{1}{1+\varepsilon_J}$), so a change in these parameters does not affect the equilibrium tax rates.

- Lastly, if J is a member of the coalition, then an increase in θ_J or α_J leads to an increase in τ_J.[25]

The last two implications mark a difference between the proportional and the majoritarian systems. In the latter system, a change in θ_J or α_J for any of the groups resulted in changes in all three tax rates. Under the proportional system, coalition members do not take into account the preferences of non-coalition members, and this leads to an asymmetry between the effects of changes in θ_J or α_J when J is a coalition member versus when J is outside the coalition.

Labor supply elasticities and tax progressivity We can now derive the general conditions under which equilibrium tax schedules are progressive. First, notice that the case when the coalition tax rate is 0 is very predictable (progressivity emerges only under the ML coalition). Thus, in this and most of the following parts, we will focus our attention on the more complex solution, the one involving non-zero tax rates. In this case, a progressive tax schedule can emerge under any coalition. The conditions for progressivity are given in the following proposition.

Proposition 3 *Under the ML coalition, the equilibrium tax schedule is progressive if $\varepsilon_L > \varepsilon_M > \varepsilon_H(1 - \Omega_{ML})$.*

Under the MH coalition, the equilibrium tax schedule is progressive if $\varepsilon_L(1 - \Omega_{MH}) > \varepsilon_M > \varepsilon_H$ and $\Omega_{MH} < 1$ (the coalition weighted average preference for the public good is high enough).

[25] since $\frac{\partial \tau_J}{\partial \theta_J} = \frac{\varepsilon_J \alpha_J \Omega_{JK}^2}{(1+\varepsilon_J - \Omega_{JK})^2} > 0$ and $\frac{\partial \tau_J}{\partial \alpha_J} = \frac{\varepsilon_J \theta_J \Omega_{JK}^2}{(1+\varepsilon_J - \Omega_{JK})^2} > 0$

Under the LH coalition, the equilibrium tax schedule is progressive if $\varepsilon_L > \varepsilon_M(1-\Omega_{MH}) > \varepsilon_H$ and $\Omega_{MH} < 1$ (the coalition weighted average preference for the public good is high enough).

Proof. See the Appendix. ■

The progressivity conditions derived above are similar to the ones in Proposition 2 ($\varepsilon_L > \varepsilon_M > \varepsilon_H$). There are two important differences, however. First, if the group outside the coalition is H, then progressivity can be obtained even for higher values of ε_H. Secondly, the group outside the coalition receives a higher tax rate than under majoritarian rule. So, if the group outside the coalition is either L or M, it becomes more difficult to obtain progressivity. Therefore, under the MH or LH coalitions, the range of labor supply elasticities for which the tax schedule is progressive is smaller than under the majoritarian system.

Public Good Preference and Tax Progressivity Next, we focus on the role of the intensity of preferences for the public good in determining a formateur's coalition choice. For simplicity and to keep the analysis parallel to the one in Section 3.2, we assume that $\varepsilon_L = \varepsilon_M = \varepsilon_H = \varepsilon$. We also assume that θ_J is positive for all groups, so that each group derives some utility from the public good. Moreover, to parallel our assumption from the analysis of the majoritarian system, we assume that the public good is redistributive, so $\theta_L > \theta_M > \theta_H$.

As stated above, we focus on the non-zero solutions, since the implications for the case of 0 coalition tax rates are straightforward. We thus assume that group L has a high enough preference for the public good to rend $\Omega_{LJ} < 1$, $J = M, H$. To keep the analysis tractable while still obtaining that same qualitative results as in the general case, we assume that the public good preferences of the other two members are lower, such that $\Omega_{MH} > 1$. In this case, the coalition tax rate of any coalition containing group L is positive while the tax rate of the coalition excluding L is 0. More specifically, an ML coalition would yield the tax vector $t_{ML} = \left(\frac{1-\Omega_{ML}}{1+\varepsilon-\Omega_{ML}}, \frac{1-\Omega_{ML}}{1+\varepsilon-\Omega_{ML}}, \frac{1}{1+\varepsilon} \right)$, an LH coalition would yield the tax vector $t_{LH} = \left(\frac{1-\Omega_{LH}}{1+\varepsilon-\Omega_{LH}}, \frac{1}{1+\varepsilon}, \frac{1-\Omega_{LH}}{1+\varepsilon-\Omega_{LH}} \right)$ and, finally, an MH coalition would yield the tax vector $t_{MH} = \left(\frac{1}{1+\varepsilon}, 0, 0 \right)$. Notice that $\tau_J^{ML} = \frac{1-\Omega_{ML}}{1+\varepsilon-\Omega_{ML}}$, $J = L, M$, will be larger than the tax rate that L would implement if it had absolute majority. In that case, L would maximize its indirect utility V_L, which implies taxing the other groups at their maximal "Laffer rates" and setting its own tax rate to $\tau_L^* = \frac{\frac{1}{\theta_L \alpha_L}}{(1+\varepsilon)-\frac{1}{\theta_L \alpha_L}}$.Since $\Omega_{ML} < \frac{1}{\theta_L \alpha_L}$ $(\theta_M > 0)$ and τ_J^{ML} is decreasing in Ω_{ML}, we can see that $\tau_J^{ML} > \tau_L^*$. Also, $\tau_J^{ML} < \frac{1}{1+\varepsilon}$, since each group prefers their own tax rate to be lower than the maximal Laffer rate and the other group's tax rate to be equal to the maximal Laffer rate. Since the negotiation is carried out with a positive weight on each group's preference, the solution must be below the maximal Laffer rate. We obtain a similar condition for τ_J^{LH}. Thus,

$$\tau_L^* < \frac{1 - \Omega_{JL}}{1 + \varepsilon - \Omega_{JL}} < \frac{1}{1 + \varepsilon}, \;\; J = M, H \tag{3.12}$$

Given these possible tax vectors, each formateur faces a trade-off between bearing a higher tax burden and receiving more of the public good. For example, if M is the formateur, a coalition with L would yield a higher tax rate for group M than a coalition with H. On the other hand, a coalition with L would also yield more public good provision since taxing H at its maximum capacity would result in more tax revenues than when L is taxed at its maximum capacity. Thus, M will form a coalition with L if the utility from the higher public good provision is larger than the disutility from the higher coalition tax rate. The analysis is similar for the case when H is the formateur. We formalize these remarks in the following Proposition. Notice, however, that these conclusions are not meant to be general, as they are derived under specific assumptions about the exogenous parameters, especially the magnitude and ordering of the preferences for the public good ($\theta_L > \theta_M > \theta_H$). They are primarily meant to illustrate the significant role that is played by the preferences for the public good under the proportional system as opposed to the majoritarian system.

Proposition 4 *Assume that* $\varepsilon_L = \varepsilon_M = \varepsilon_H = \varepsilon$*,* $\theta_L > \theta_M > \theta_H$*,* $\Omega_{MH} > 1$*,* $\Omega_{ML} < 1$*,* $\Omega_{LH} < 1$*, and the high income group represents a large enough proportion of the voters* ($\alpha_H \geq \alpha_L \left(\frac{w_L}{w_H}\right)^{\varepsilon+1}$).

- *If* M *is the formateur, there exists* $\overline{\theta_M} \geq 0$*, a threshold value of* θ_M*, such that* M *forms a coalition with* L *whenever* $\theta_M \geq \overline{\theta_M}$*,and* M *forms a coalition with* H *whenever* $\theta_M < \overline{\theta_M}$*.*

- *If* H *is the formateur, there exists* $\overline{\theta_H} \geq 0$*, a threshold value of* θ_H*, such that* H *forms a coalition with* L *whenever* $\theta_H \geq \overline{\theta_H}$*,and* H *forms a coalition with* M *whenever* $\theta_H < \overline{\theta_H}$*.*

Proof. See the Appendix. ■

Intuitively, the above result means that for either M (or H) to want a more redistributive tax schedule, M (or H) must receive a high enough benefit from redistribution. In our set-up, this means M (or H) must derive enough utility from the public good to be worth it to pay a higher tax.

When L is the formateur and $\theta_M > \theta_H$, L faces the same trade-off between disutility from taxation and higher utility from the public good provision. If the utility derived from the public good is large enough (so if θ_L is large) to compensate for the disutility from increased taxation, then L will prefer the LM coalition. Otherwise, L will prefer the LH coalition.

3.3.3 Comparative Statics

As in Section 3.2.3, we proceed to ask how the variation in the exogenous economic variables influences the degree of progressivity, if the electoral rule is proportional. We will focus on three exogenous economic variables: labor supply elasticity, preference for the public good, and share of the population occupied by each group.

Labor Supply Elasticities We start by looking at the effects of variations in the labor supply elasticity on tax progressivity. Following a structure similar to the one in Section 3.2.3, we begin by assuming that labor supply elasticity is the same across groups.

Remark 5 *Assume ML is the ruling coalition, $\varepsilon_L = \varepsilon_M = \varepsilon_H = \varepsilon$ and $\Omega_{ML} < 1$. Assume ε decreases. Then all tax rates increase, more public good is provided and*

- *if $\frac{\varepsilon^2}{1-\Omega_{ML}} > 1$, the progressivity of the tax schedule increases*
- *if $\frac{\varepsilon^2}{1-\Omega_{ML}} < 1$, the progressivity of the tax schedule decreases.*

Proof. See the Appendix. ■

The above result links back to the qualitative result from the majoritarian case. The decrease in labor supply elasticity requires an increase in tax rates. However, the change in progressivity is dependent on a ratio of the labor supply elasticity and a measure of the average preference for the public good (within the coalition). The intuition for this result is similar to the one outlined in the discussion following Remark 3. If $\frac{\varepsilon^2}{1-\Omega_{ML}} > 1$, then tax rates are small and the necessary increase in taxes will be made in the cost minimizing way, by taxing group H more, since the political cost of taxing this group is 0.[26] However, if $\frac{\varepsilon^2}{1-\Omega_{ML}} < 1$, then the tax rates are already high and a further increase comes at high distortionary costs. Thus, progressivity must be sacrificed in order to obtain more tax revenues.

The case when labor supply elasticities vary across groups was discussed above and the results of changes in elasticities are similar to the ones from Section 3.2.3. The implications on progressivity follow directly from the result that a decrease in ε_J increases group J's tax rate: the progressivity of the equilibrium tax schedule increases if ε_H decreases or ε_L increases.

[26] Because no weight is placed on H's preferences if the ruling coalition is ML

Income distribution and Wage Inequality Another important determinant of coalition preferences is the difference between group income levels. The pre-tax income inequality (measured by the difference between group wages) affects the relative share of the cost of producing the public good borne by each group and thus, the utility cost in forgone public good consumption when a group pays a lower tax.

The following remark analyses the effects of an increase in the wage gap between the middle and high groups, assuming M is the formateur. Intuitively, as w_H increases, more tax revenue can be extracted from group H to provide the public good. Thus, $M's$ expected utility from the ML coalition increases and the option of forming a coalition with L becomes relatively more attractive. Since the ML coalition implements a progressive tax schedule (when $\varepsilon_L = \varepsilon_M = \varepsilon_H = \varepsilon$), it becomes more likely to see a progressive tax schedule in equilibrium.

Remark 6 *Assume M is the formateur, $\varepsilon_L = \varepsilon_M = \varepsilon_H = \varepsilon$, $\Omega_{MH} > 1$, $\Omega_{ML} < 1$ and $\Omega_{LH} < 1$. Then, as income inequality between the high and the middle group $(w_H - w_M)$ increases, the probability that an ML coalition (rather than an MH coalition) will emerge in equilibrium increases. Since the ML coalition delivers a progressive tax schedule under equal labor supply elasticities, the progressivity of the tax schedule weakly increases.*

Proof. See the Appendix. ■

The above result is based on the possibility of a switch in coalition partners when the expected utilities from the possible coalitions change as a result of the change in wages. In this sense, this remark links back to the results described in Proposition 4, about the existence of a threshold $\overline{\theta_M}$ above which M prefers L to H. As the high income group becomes richer compared to the middle income group, the tax income that can be extracted from H at any positive tax rate is higher and can produce more public good. Thus, intuitively, taxing the high income group as much as possible (at the maximum of their Laffer curve) should be more advantageous for M and L as opposed to making a coalition with H, which would mean a minimum level of taxation on the high income group.

In the next remark, we investigate how the income distribution affects the degree of progressivity. Since the coalition tax rates are functions of α_J (where J is a member of the coalition), changes in the size of α_J, achieved when members of one group move to another group, will affect the equilibrium coalition tax rates and, implicitly, tax progressivity.

Remark 7 *Assume the ruling coalition is ML, $\varepsilon_L > \varepsilon_M > \varepsilon_H(1-\Omega_{ML})$, $\Omega_{ML} < 1$, $\Omega_{LH} < 1, \Omega_{MH} > 1$ and $\theta_L > \theta_M > \theta_H$. If there is downward population mobility from M to L or from H to either L or M, then τ_L and τ_M increase while τ_H stays the same, more public good is produced, and the progressivity of the tax schedule decreases.*

Proof. See the Appendix. ■

The above remark shows that changes to the income distribution inside the ruling coalition affect the balance of power within the coalition. As M becomes a larger group relative to L, it can tilt the equilibrium policy more in its desired direction (in this case, towards a lower coalition tax rate). However, if M receives an influx of population from outside the coalition, we now have a larger part of the voter population preferring the public good. The demand for the public good in the whole population is thus higher, leading to more public good provision through a higher coalition tax rate (since H is already taxed at its maximal Laffer rate). Linking back to Remark 2, which looks at the population mobility question under the majoritarian system, notice that the qualitative changes in tax rates for the coalition members are the same (and nothing changes for H), and the direction of the change in public good provision is the same. However, the changes in progressivity are the exact opposites of the changes derived in Remark 2.

Public Good Preferences As it was discussed above, each group's preference for the public good (θ_J) is an important factor in determining which one of the two tax equilibria emerges in equilibrium. Assume, for instance, that M is the formateur. Group M's preference for the public good was shown to determine which coalition is formed in equilibrium (Proposition 4). The following two remarks parallel the remark made in the majoritarian case about the effects of changes in θ_J on tax progressivity (Remark 3). However, due to the multiple equilibria that can emerge under the proportional system, the remarks and their proofs use simplifying assumptions that limit the set of possible solutions. Nevertheless, we obtain the same qualitative result as in the majoritarian case, namely that an increase in θ_J increases public good provision (if J is in the ruling coalition) and that the effects on progressivity depend on the magnitude of a taxation cost-benefit ratio.

Remark 8 *Assume the equilibrium tax schedule is progressive,* $\Omega_{ML} < 1, \Omega_{LH} < 1, \Omega_{MH} > 1$ *and* $\varepsilon_L > \varepsilon_H$. *If the low income group's preference for the public good* (θ_L) *increases then more public good is provided when* L *is a coalition member, the coalition tax rates increase, and*

- *under the* ML *coalition, progressivity decreases;*
- *under the* LH *coalition, if* $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} > 1$, *progressivity increases;*

  if $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} < 1$, *progressivity decreases.*

Moreover, the formation of an ML *coalition requires a (weakly) higher preference for the public good from group* M, *when* M *is the formateur (i.e.* $\overline{\theta_M}$ *increases) and the wage difference between the middle and the low group is sufficiently high.*

Proof. See the Appendix. ■

Regarding the formation of the ML coalition, notice that an increase in θ_L forces an increase in the coalition tax rate. The low income group benefits from the coalition by increasing M's contribution to the provision of the public good, but M is forced to increase its payment and further distort its labor supply without the extra benefit from the public good (since θ_M does not change) to compensate for the costs of providing it. Thus, a coalition with H becomes relatively more desirable for M and so, the threshold $\overline{\theta_M}$ is higher, since M has fewer incentives to want to form a coalition with L. The changes in the preference for the public good for the other two groups have similar implications:

Remark 9 *If the middle income group's preference for the public good (θ_M) increases then coalition tax rates increase and more public good is provided. Progressivity increases under the MH coalition and decreases under the ML coalition.*

If the high income group's preference for the public good (θ_H) increases then coalition tax rates increase, more public good is provided and:

- *under the MH coalition, progressivity increases.*
- *under the LH coalition, if $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} > 1$, progressivity increases, and*

  if $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} < 1$, progressivity decreases.

Proof. See the Appendix. ■

The results of the last two remarks show that, as long as a group has some political power (i.e. is not outside the ruling coalition), a higher preference for the public good is translated into a higher provision of the public good. Intuitively, as the public good is more demanded by a member of the coalition, then there is more pressure for a higher amount to be provided. Since the group outside the coalition is already taxed at the maximum efficient level (at the peak of their Laffer curve), the only other source of funding is through higher tax rates on the members of the coalition. However, compared to the majoritarian system (Remark 3), the effects of changes in the preference for the public good vary depending on the ruling coalition and the group whose preference changes. This is because a group's preference matters only when the group is inside the ruling coalition, while under the majoritarian system, there is always some positive weight placed on every group's preferences.

Notice that, as in the majoritarian system, $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2}$ is a measure of the cost of taxation relative to the benefits of it. The elasticities of labor supply give the economic cost of taxation for the members of the coalition, while the term $(1-\Omega_{LH})^2$ proxies for the benefits of taxation for the coalition, since it increases as the average coalition preference for the public good increases. Under the LH coalition, if $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} > 1$, the result follows, yet again, from the qualitative implication of the optimal taxation literature: the group on which taxation has the highest distortionary effect receives the lowest increase in taxes. In this case, group L's labor supply elasticity is higher than group H's, so the latter will suffer a higher increase in its tax rate. However, if $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} < 1$, we get a result similar to the one obtained under the majoritarian system. If the elasticities of the labor supply are low, the equilibrium tax rate is already high and even higher for H since $\varepsilon_L > \varepsilon_H$. Thus, imposing a higher tax burden on H would involve high distortionary effects and therefore, progressivity must be reduced in order to obtain the desired increase in tax revenues.

Another observation is that the effects of preference diversity on progressivity are more complex under the proportional system compared to the majoritarian system, due to the role θ_J plays in coalition formation. More diversity in preferences, represented by a larger difference between one group's preference θ_J and another group's preference θ_K leads to a stronger coalition polarization. As in comes out of Remark 8, more variation in preferences between members of the ML coalition pushes the formateur more strongly towards a coalition with H. Thus, the formateur is more strongly inclined to ally against a group whose preferences are further away from its own.

3.4 Discussion

Using the simple models outlined above, we derived sufficient conditions under which tax schedules are progressive given two different electoral rules: two-party competition under the majoritarian system and multi-party competition under the proportional system. Moreover, we examined the implications of variations in labor supply elasticity, preference diversity and income distribution on the size of tax rates and the degree of tax progressivity. Throughout the analysis, we assumed that the public good is redistributive in nature (i.e. $\theta_L > \theta_M > \theta_H$). This assumption helped us to better capture the redistribute function of taxation as well as to restrict our attention to the equilibria involving progressive tax schedules

We start by discussing and contrasting the effects on the size of group tax rates. Table 3.1 below presents a summary of the effects of changes in the model's economic variables on group tax rates. First, we observe that labor supply elasticity, income distribution and the preference for the public good have similar qualitative effects under both electoral systems. A decrease in labor supply elasticity, downward mobility and increased preference for the public good generally lead to higher tax rates. A difference between the two electoral systems emerges when looking at the effects of the last two rows of the table, namely at the effects of group size and preference for the public good. Under the majoritarian system, there is a positive weight placed on every group's preferences, which means changes in the size or preferences of any groups affect the tax rates of all other groups. On the other hand, under the proportional system, group size and preference for the public good affect a group's tax rate only when that group is a member of the coalition; tax rates for non-coalition members are always set at the maximal Laffer rates, and hence and unaffected by changes in these two variables.

Effects of Changes in Variables on Group Tax Rates			
Variable	Change	Majoritarian System	Proportional System
ε_J	Down	$\tau_J \uparrow$	$\tau_J \uparrow$
α_J	Downward Mobility	$\tau_L,\ \tau_M, \tau_H \uparrow$	$\tau_J \uparrow$, if J is in the coalition
θ_J	Up	$\tau_L,\ \tau_M, \tau_H \uparrow$	$\tau_J \uparrow$, if J is in the coalition

Table 3.1: Effects of Changes in Variables on Group Tax Rates

We now turn to examining the effects of changes in economic variables on the degree of tax progressivity, measured as the difference between the top and bottom marginal tax rates ($\tau_H - \tau_L$). Table 3.2 presents a summary of these effects. The results of the proportional system are especially difficult to summarize because they often depend on the ruling coalition's composition. Consequently, only a subset of the results will be presented here to illustrate some of the similarities and some of the differences in the effects of the two electoral systems.

Summary of Effects on the Degree of Tax Progressivity				
Variable	Change	Conditions	Majoritarian	Proportional
ε_H	Down	$\varepsilon_L > \varepsilon_H$	up	up
ε_L	Down	$\varepsilon_L > \varepsilon_H$	down	down
ε	Down	$\varepsilon_L = \varepsilon_H$; $\frac{\varepsilon^2}{\Lambda_H \Lambda_L} > 1$; $\frac{\varepsilon^2}{1-\Omega_{ML}} > 1$	up	up if ML
ε	Down	$\varepsilon_L = \varepsilon_H$; $\frac{\varepsilon^2}{\Lambda_H \Lambda_L} < 1$; $\frac{\varepsilon^2}{1-\Omega_{ML}} < 1$	down	down if ML
θ_L	Up	$\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} > 1$; $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} > 1$ if LH	up	$\uparrow$ if LH; $\downarrow$ if ML
θ_L	Up	$\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} < 1$; $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} < 1$ if LH	down	down
α_J	Downward mobility	$\varepsilon_L > \varepsilon_H$, $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} > 1$	up	down if ML
α_J	Downward mobility	$\varepsilon_L > \varepsilon_H$, $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} < 1$	down	down if ML

Table 3.2: Effects of Changes in Variables on the Degree of Tax Progressivity

Electoral rules do not alter the qualitative effects of changes in the labor supply elasticity on tax progressivity. The direction of change in progressivity when ε_J changes is dependent on the magnitude of the costs of taxation relative to its benefits (denoted by $\frac{\varepsilon^2}{\Lambda_H \Lambda_L}$ under the majoritarian system and $\frac{\varepsilon^2}{1-\Omega_{ML}}$ under the proportional system) in both systems. If this cost ratio is large enough ($\frac{\varepsilon^2}{\Lambda_H \Lambda_L}$ or $\frac{\varepsilon^2}{1-\Omega_{ML}}$ are larger than 1), then progressivity increases as ε decreases. However, if the cost ratio is small, then group tax rates are already high and a higher tax burden on the high income groups would come at high distortionary costs. Therefore, progressivity must be sacrificed if tax rates must be further increased in order to produce more public good.

Our analysis showed that the electoral system can lead to qualitatively different effects for the preference for the public good, θ_J. Under the majoritarian system, an increase in the preference for the public good results in an increase in progressivity if $\frac{\varepsilon^2}{\Lambda_H \Lambda_L} > 1$ and a decrease in progressivity if $\frac{\varepsilon^2}{\Lambda_H \Lambda_L} < 1$, where the dependence on the $\frac{\varepsilon^2}{\Lambda_H \Lambda_L}$ ratio appears for the same reasons as those discussed above. Under the proportional system, however, the changes in progressivity will depend both on the proportional system cost ratio, $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2}$, and on the composition of the ruling coalition. Since the members outside the coalition are already taxed at the Laffer rate, a higher demand for the public good from one of the coalition members can only be satisfied if the coalition tax rates increase. Thus, the direction of the change in progressivity will depend on which members are in the coalition (if $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} > 1$). If the cost ratio is small, then it sets a constraint on how much taxes can increase for the high income group, and an increase in θ_J always leads to a decrease in progressivity.[27]

As shown in Table 3.2, changes in the shape of the income distribution can affect progressivity differently depending on the electoral system. Interestingly, the effects of income mobility are sometimes reversed when moving from a majoritarian rule to a proportional rule. Under a majoritarian rule, downward mobility leads to an increase in progressivity if $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} > 1$ and to a decrease in progressivity if $\frac{\varepsilon_H \varepsilon_L}{\Lambda^2} < 1$. When labor supply elasticity is high enough, the result that progressivity increases is consistent with the empirical evidence provided by Jacobs and Waldman (1983), who find that greater income inequality leads to more progressive tax systems in US states. Under the same preference constraints, a downward movement from a higher income group to a lower income group results in a decrease in tax progressivity under the proportional system (under the ML coalition). Thus, as the income distribution becomes more skewed to the left, the effects on tax progressivity depend on the electoral system. This result illustrates one possible channel through which differences in electoral procedures can lead to systemic differences in the effects that economic variables have on tax progressivity.

Finally, we can derive some implications about the weights placed on the preferences of each group under the two electoral rules. As can be observed from the two models of political

[27] Of course, as long as J is a coalition member.

competition, electoral rules play a significant role in determining the weight that each group will receive in the function that will be maximized as part of the political process. The majoritarian system forces parties to take into account, albeit with different weights, the preferences of all groups. In fact, if the ideological dispersion, ϕ_J, is the same for all groups, then the majoritarian rule sets the same weight on each group. Unlike the majoritarian system result, the legislative bargaining solution does not generally place positive weights on all individual utilities. The coalition behavior engendered by the proportional electoral system results in some groups being left outside the decision making process. Their preferences receive no weight and therefore they have to bear a much larger tax burden compared to the members of the ruling coalition. Therefore, in the issue of tax design, the electoral rule creates a paradox, since a more precise political representation of group preferences can lead to a higher disparity in tax burdens. The majoritarian voting rule encourages the emergence of two large parties and limits the exact representation of group preferences through representative parties. Proportional representation, on the other hand, allows parties to represent very closely the preferences of their constituent groups. However, a closer representation of group interests through a party does not translate into a reflection of these interests in the structure of the tax system.

4 Conclusion

In this paper we studied the effects of two stylized electoral procedures, the majoritarian electoral system and the proportional electoral system, on the size of group tax rates and on the progressivity of income tax schedules in a simple 3-good economy. We derived the conditions under which progressive income tax schedules emerge in equilibrium, assuming that labor supply decisions are endogenous and the tax revenue is used to provide a public good that gives different levels of utility to different income groups.

These two simple models of electoral competition allowed us to study certain channels through which electoral systems can affect the size of tax rates and tax schedule progressivity. First, we found that the electoral system does not significantly influence the effects of changes in labor supply elasticity on the qualitative changes of tax rate magnitudes and on tax progressivity. Regardless of the electoral rule in place, we arrived at the qualitative optimal taxation result that groups with larger labor supply elasticity receive a smaller tax rate and decreases in labor supply elasticity lead to higher tax rates. It would be interesting to examine whether countries with larger variations in labor supply elasticity across income groups also display more progressive tax schedules, and whether reductions in the differences between labor supply elasticities across groups (for example, by increased possibilities of labor force migration) have led to a reduction in progressivity.

Our model suggests that electoral rules do influence the qualitative effects that certain variables have on the degree of tax progressivity. The model predicts that changes in income distribution could have different effects on tax progressivity under different electoral rules. Further exploration of this result might offer some insight into the different effects that income distribution is shown to have on redistribution (which is partially achieved through tax progressivity) in different empirical studies (Persson and Tabellini 1994, Perotti 1996, Bassett et al. 1999).

We also found that changes in the preference for the public good have qualitatively similar effects on the size of tax rates under the two electoral rules, but may have qualitatively different effects on tax progressivity. Moreover, preference diversity was shown to have a more complex role in determining the equilibrium tax schedule under the proportional rule, due to its effect on coalition choices. This last implication could be tested empirically by looking at the composition of government spending and testing whether legislature in states which provide more targeted public goods (for which variations in preferences are higher across groups) display the polarization suggested by our model.

In future research, it would be interesting to extend this simple model to endogenize the determining features of electoral institutions. One interesting extension would be to endogenize the number of parties engaged in electoral competition and also to link this number to the characteristics of the groups of voters by endogenizing party formation. This would shed more light on the specific mechanisms that link electoral rules to policy outcomes. Another interesting avenue for further extensions would be to explore the role of preference diversity by allowing for multiple public goods to be provided by the government. The possibility to target certain groups through the public good provision could be an important mitigating factor for the degree of progressivity achieved in equilibrium.

A Appendix

Proof of Remark 1. Notice that $\phi_L > \phi_M > \phi_H$ implies $\Lambda_L < \Lambda_M < \Lambda_H$. Moreover, ε is always positive, by assumption. From Equation 3.10 we can derive $\tau_H - \tau_L = \frac{1}{1+\frac{\varepsilon}{\Lambda_H}} - \frac{1}{1+\frac{\varepsilon}{\Lambda_L}}$. Thus, $\frac{\partial(\tau_H - \tau_L)}{\partial \varepsilon} = \frac{(\Lambda_H - \Lambda_L)\left(\Lambda_H \Lambda_L - \varepsilon^2\right)}{(\Lambda_H + \varepsilon)^2 (\Lambda_L + \varepsilon)^2} < 0$ if $\frac{\varepsilon^2}{\Lambda_H \Lambda_L} > 1$. So tax progressivity increases. If $\frac{\varepsilon^2}{\Lambda_H \Lambda_L} < 1$, then $\frac{\partial(\tau_H - \tau_L)}{\partial \varepsilon} > 0$ and progressivity decreases as ε decreases. ■

Proof of Remark 2. First notice that $\phi_L = \phi_M = \phi_H$ implies $\Lambda_L = \Lambda_M = \Lambda_H = \Lambda$. Assume then that the proportion of middle-income voters, α_M, decreases by a small ϵ while the proportion of low income voters, $\alpha_{L,}$ increases by ϵ. If the middle income group has a lower preference for the public good relative to the low income group, such that $\epsilon\theta_M < \epsilon\theta_L$ (i.e. $\epsilon\phi_M\theta_M < \epsilon\phi_L\theta_L$ since $\phi_M = \phi_L$), then all Λ_J's in equation 3.10 increase, which implies that all tax rates τ_J must also increase, $J = L, M, H$. Moreover, $\tau_H - \tau_L = \frac{1}{1+\frac{\varepsilon_H}{\Lambda}} - \frac{1}{1+\frac{\varepsilon_L}{\Lambda}}$. Thus, $\frac{\partial(\tau_H - \tau_L)}{\partial \Lambda} = \frac{(\varepsilon_L - \varepsilon_H)(\varepsilon_L \varepsilon_H - \Lambda^2)}{(\varepsilon_L + \Lambda)^2 (\varepsilon_H + \Lambda)^2} > 0$ whenever $\varepsilon_L \varepsilon_H > \Lambda^2$. Thus, $\tau_H - \tau_L$ increases as Λ increases, which means that progressivity increases. The converse is true if $\varepsilon_L \varepsilon_H < \Lambda^2$. The analysis for H to M or from H to L is identical to the one above. ■

Proof of Remark 3. Assume θ_J increases, where J denotes one the three income groups. As it can be seen from Equation 3.9, the supply of public good in equilibrium increases regardless of the relative preferences for the public good across groups. Thus, $\Lambda_J = 1 - \phi_J/(\alpha_L\phi_L\theta_L + \alpha_M\phi_M\theta_M + \alpha_H\phi_H\theta_H)$ increases, and from Equation 3.10, the tax rate for each group must also increase in equilibrium. The marginal effect of the increase in Λ on $\tau_H - \tau_L$ is positive whenever $\varepsilon_H\varepsilon_L > \Lambda^2$, as shown in the proof to Remark 2. Thus, progressivity increases in this case. Otherwise, if $\varepsilon_H\varepsilon_L < \Lambda^2$, $\tau_H - \tau_L$ is lower and progressivity decreases. ■

Proof of Remark 4. Assume ϕ_L increases. From equation 3.10, applying the implicit function theorem we obtain the marginal changes in τ_L and τ_H when ϕ_L increases. Then $\frac{\partial\tau_L}{\partial\phi_L} = \frac{-\varepsilon_L(\alpha_M\phi_M\theta_M+\alpha_H\phi_H\theta_H)}{(\alpha_L\phi_L\theta_L+\alpha_M\phi_M\theta_M+\alpha_H\phi_H\theta_H)^2(\Lambda_L+\varepsilon_L)^2} < 0$ and $\frac{\partial\tau_H}{\partial\phi_L} = \frac{\varepsilon_H\phi_H\theta_L\alpha_L}{(\alpha_L\phi_L\theta_L+\alpha_M\phi_M\theta_M+\alpha_H\phi_H\theta_H)^2(\Lambda_H+\varepsilon_H)^2} > 0$. Thus, τ_L decreases and τ_H increases leading to a higher difference between τ_H and τ_L. It follows that overall progressivity increases. Taking the derivatives of $\tau_L$and τ_H with respect to ϕ_H: $\frac{\partial\tau_L}{\partial\phi_H} = \frac{\varepsilon_L\phi_L\theta_H\alpha_H}{(\alpha_L\phi_L\theta_L+\alpha_M\phi_M\theta_M+\alpha_H\phi_H\theta_H)^2(\Lambda_L+\varepsilon_L)^2} > 0$ and $\frac{\partial\tau_H}{\partial\phi_H} = \frac{-\varepsilon_H(\alpha_M\phi_M\theta_M+\alpha_L\phi_L\theta_L)}{(\alpha_L\phi_L\theta_L+\alpha_M\phi_M\theta_M+\alpha_H\phi_H\theta_H)^2(\Lambda_H+\varepsilon_H)^2} < 0$. An increase in ϕ_H leads to a decrease in $\tau_H - \tau_L$. Thus, progressivity decreases. ■

Proof of Proposition 3. Under the ML coalition, we have $\tau_J^{ML} = \frac{1-\Omega_{ML}}{1+\varepsilon_J-\Omega_{ML}}$ for $J = M, L$ and $\tau_H = \frac{1}{1+\varepsilon_H}$. $\frac{\partial\tau_J}{\partial\varepsilon_J} < 0$, $J = L, M, H$.[28] For strict progressivity we need $\tau_L^{ML} < \tau_M^{ML} < \tau_H$. First, notice that $\tau_L^{ML} < \tau_M^{ML}$ whenever $\varepsilon_L > \varepsilon_M$. The, $\tau_M^{ML} < \tau_H$ whenever

$$\tau_M^{ML} = \frac{1-\Omega_{ML}}{1+\varepsilon_M-\Omega_{ML}} < \frac{1}{\varepsilon_H+1} = \tau_H$$

This reduces to $\varepsilon_M > \varepsilon_H(1-\Omega_{ML})$. Therefore, the equilibrium tax schedule is progressive if $\varepsilon_L > \varepsilon_M > \varepsilon_H(1-\Omega_{ML})$.

Under the MH coalition, Ω_{MH} must be less than 1 in order to obtain non-zero coalition tax rates. If this is the case, then $\tau_J^{MH} = \frac{1-\Omega_{MH}}{1+\varepsilon_J-\Omega_{MH}}$ and we follow the same steps as above

[28]If the preference for the average public good is low enough such that $\tau_J^{ML} = 0$ ($J = M, L$), then the equilibrium tax schedule is always progressive. Thus, this situation is not particularly interesting, so we are not focusing on it.

to derive the condition for strict progressivity: $\varepsilon_L(1-\Omega_{MH}) > \varepsilon_M > \varepsilon_H$.

Finally, the equilibrium tax schedule can be progressive under the LH coalition only if $\Omega_{LH} < 1$. Following the same steps as above, we derive the condition for progressivity: $\varepsilon_L > \varepsilon_M(1-\Omega_{LH}) > \varepsilon_H$. ■

Proof of Proposition 4. For M to prefer a coalition with L, the utility derived by M under the ML coalition must be larger than the utility derived by M under the MH coalition. Since we assumed $\Omega_{ML} < 1$ and $\Omega_{MH} > 1$, the tax vector under the ML coalition is $t_{ML} = (\frac{1-\Omega_{ML}}{1+\varepsilon-\Omega_{ML}}, \frac{1-\Omega_{ML}}{1+\varepsilon-\Omega_{ML}}, \frac{1}{1+\varepsilon})$, while the tax vector under the MH coalition is $t_{MH} = (\frac{1}{1+\varepsilon}, 0, 0)$. Thus, M will form a coalition with L whenever $V_M(t_{ML}) \geq V_M(t_{MH})$. Thus, whenever $V_M(\tau_L^{ML}, \tau_M^{ML}, \frac{1}{1+\varepsilon}) \geq V_M(\frac{1}{1+\varepsilon}, 0, 0)$.

After applying Equation 3.4, we obtain the following and performing the necessary reductions, we obtain the following condition for an ML coalition to emerge:

$$\frac{\varepsilon^{2\varepsilon+1} w_M^{\varepsilon+1}}{(\varepsilon+1)(1+\varepsilon-\Omega_{ML})^{\varepsilon+1}} + \theta_M \varepsilon^{2\varepsilon} \left(\frac{(\alpha_L w_L^{\varepsilon+1} + \alpha_M w_M^{\varepsilon+1})(1-\Omega_{ML})}{(1+\varepsilon-\Omega_{ML})^{\varepsilon+1}} + \frac{\alpha_H w_H^{\varepsilon+1}}{(1+\varepsilon)^{\varepsilon+1}} \right) \geq \frac{\varepsilon^{\varepsilon} w_M^{\varepsilon+1}}{\varepsilon+1} + \theta_M \frac{\alpha_L \varepsilon^{2\varepsilon} w_L^{\varepsilon+1}}{(1+\varepsilon)^{\varepsilon+1}} \quad \text{(A.1)}$$

First, observe that both $V_M(t_{ML})$ and $V_M(t_{MH})$ are continuous since they are combinations of elementary functions and arithmetic operations. Also, both functions are monotonically increasing. For the MH coalition, $\frac{\partial V_M(t_{MH})}{\partial \theta_M} = \alpha_L w_L^{\varepsilon+1} \frac{\varepsilon^{2\varepsilon}}{(1+\varepsilon)^{\varepsilon+1}} > 0$, for all $\theta_M > 0$. For the ML coalition, $\frac{\partial V_M(t_{ML})}{\partial \theta_M} > 0$. To see this, notice that an increase in θ_M has 2 effects on $V_M(t_{ML})$. On the one hand, as θ_M increases, Ω_{ML} decreases, so the equilibrium coalition tax rate τ_J^{ML} increases, and this has a downwards effect on M's indirect utility. By taking the derivative of $\frac{\varepsilon^{\varepsilon} w_M^{\varepsilon+1} \varepsilon^{\varepsilon+1}}{(\varepsilon+1)(1+\varepsilon-\Omega_{ML})^{\varepsilon+1}}$ with respect to θ_M, we find that this effect equals $-\frac{\alpha_M \varepsilon^{2\varepsilon+1} w_M^{\varepsilon+1}}{(1+\varepsilon-\Omega_{ML})^{\varepsilon+2}}$. On the other hand, as θ_M increases, there is higher public good provision, which has an upwards effect on $V_M(t_{ML})$.[29] The positive marginal effect of higher public good provision exists as long as $\theta_M > 0$ and $\alpha_L \theta_L - 1 > 0$.[30] Adding these two marginal ef-

[29]Notice that, since $\tau_J^{ML} < \frac{1}{1+\varepsilon}$ (as shown in the text of the paper), we are not at the peak of M's or L's Laffer curves, so total tax revenue increases when τ_J^{ML} increases. This confirms that indeed, as θ_M increases, tax revenue will increase, so there will be more public good provision

[30]This effect is obtained by taking the derivative of $\theta_M g$ in $V_M(t_{ML})$ with respect to θ_M. This derivative is:

fects together, we find that the marginal increase in utility dominates the marginal decreases whenever θ_L is sufficiently large.[31] Thus, we have shown that $\frac{\partial V_M(t_{ML})}{\partial \theta_M} > 0$.

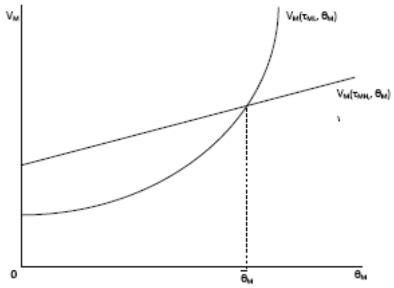
We now look at M's coalition choice for 2 different values of θ_M. First, when $\theta_M = 0$, $V_M(t_{ML}) = \frac{\varepsilon^{2\varepsilon+1} w_M^{\varepsilon+1}}{(\varepsilon+1)(1+\varepsilon-\Omega_{ML})^{\varepsilon+1}}$ and $V_M(t_{MH}) = \frac{\varepsilon^{\varepsilon} w_M^{\varepsilon+1}}{\varepsilon+1}$. Thus, $V_M(t_{MH}) > V_M(t_{ML})$ (since θ_L is positive). Then, we look at the difference between $V_M(t_{MH})$ and $V_M(t_{ML})$. Rearranging the terms of the expression, it can be shown that at $\theta_M = \frac{w_M^{\varepsilon+1}\varepsilon^{\varepsilon}}{\left(\alpha_H w_H^{\varepsilon+1} + \alpha_L w_L^{\varepsilon+1}\right)}((1+\varepsilon)^{\varepsilon+1} - \varepsilon^{\varepsilon})$, $V_M(t_{ML}) > V_M(t_{MH})$.[32] Thus, $V_M(t_{MH})$ and $V_M(t_{ML})$ are two continuous, monotonically increasing functions, at $\theta_M = 0$, $V_M(t_{MH}) > V_M(t_{ML})$ and at $\theta_M = \frac{w_M^{\varepsilon+1}\varepsilon^{\varepsilon}}{\left(\alpha_H w_H^{\varepsilon+1} + \alpha_L w_L^{\varepsilon+1}\right)}((1+\varepsilon)^{\varepsilon+1} - \varepsilon^{\varepsilon}) > 0$, $V_M(t_{ML}) > V_M(t_{MH})$. It follows that there exists a value $\overline{\theta_M} > 0$ at which M's indirect utility from the ML coalition becomes larger than M's indirect utility from the MH coalition. So, $\forall \theta_M \geq \overline{\theta_M}$, inequality (A.1) holds and M prefers to form a coalition with L. The analysis when H is the formateur is similar. Thus, we obtain the conclusion that there exists a value $\overline{\theta_H} > 0$ at which the H's indirect utility from the HL coalition becomes larger than H's indirect utility from the MH coalition$\forall \theta_H \geq \overline{\theta_H}$, the above inequality holds and H will prefer to form an alliance with L.

$\frac{\varepsilon^{2\varepsilon}(\alpha_L w_L^{\varepsilon+1} + \alpha_M w_M^{\varepsilon+1})}{(1+\varepsilon-\Omega_{ML})^{\varepsilon+2}} \begin{bmatrix} \alpha_L\theta_L(-1+\alpha_L\theta_L)(-1+\alpha_L\theta_L(1+\varepsilon)) + \alpha_M\theta_M(1+\varepsilon-2\alpha_L\theta_L(2+\varepsilon)+ \\ 3\alpha_L^2\theta_L^2(1+\varepsilon)) + \alpha_M^2\theta_M^2(-2-\varepsilon+3\alpha_L\theta_L(1+\varepsilon)) + \alpha_M^3\theta_M^3(1+\varepsilon) \end{bmatrix} > 0$ for all $\theta_M > 0$, as long as $\alpha_L\theta_L - 1 > 0$ (which is true given our assumption that $\Omega_{ML} < 1$)

[31] The marginal utility is increasing with θ_M whenever $\theta_L > \frac{2+\varepsilon}{2\alpha_L(1+\varepsilon)} + \frac{1}{2\alpha_L(1+\varepsilon)}\sqrt{4\frac{w_L}{w_M}\varepsilon(1+\varepsilon)+\varepsilon^2}$.

[32] $V_M^{ML} - V_M^{MH} = \frac{\varepsilon^{\varepsilon} w_M^{\varepsilon+1}}{\varepsilon+1}\left(-1 + \varepsilon^{\varepsilon+1}(1-\Omega_{ML})\frac{1+\theta_M\left(\alpha_L w_L^{\varepsilon+1} + \alpha_M w_M^{\varepsilon+1}\right)}{(1+\varepsilon-\Omega_{ML})^{\varepsilon+1}}\right) + \theta_M \frac{(\alpha_H w_H^{\varepsilon+1} - \alpha_L w_L^{\varepsilon+1})}{(1+\varepsilon)^{1+\varepsilon}}$

It follows after a series of simplifications that $V_M^{ML} - V_M^{MH} > \varepsilon^{\varepsilon} w_M^{\varepsilon+1}(-(\varepsilon+1)^{\varepsilon+1} + \varepsilon^{\varepsilon+1}(1+$
$+\theta_M\left(\alpha_L w_L^{\varepsilon+1} + \alpha_M w_M^{\varepsilon+1}\right))(1-\Omega_{ML}) + \theta_M(\alpha_H w_H^{\varepsilon+1} - \alpha_L w_L^{\varepsilon+1})(1+\varepsilon)$. This last expression is greater than 0 for $\theta_M > \frac{\varepsilon^{\varepsilon} w_M^{\varepsilon+1}((\varepsilon+1)^{\varepsilon+1} - \varepsilon^{\varepsilon+1})}{(\varepsilon+1)(\alpha_H w_H^{\varepsilon+1} - \alpha_L w_L^{\varepsilon+1})}$. Thus, for our chosen value of θ_M, $V_M(t_{ML}) - V_M(t_{MH})$ is positive, whenever $\alpha_H \geq \alpha_L\left(\frac{w_L}{w_H}\right)^{\varepsilon+1}$.



Sketch of $V_M(t_{ML})$ and

$V_M(t_{MH})$ as functions of θ_M.

■

Proof of Remark 5. If $\Omega_{ML} < 1$, then the equilibrium tax vector is $\left(\frac{1-\Omega_{ML}}{1+\varepsilon-\Omega_{ML}}, \frac{1-\Omega_{ML}}{1+\varepsilon-\Omega_{ML}}, \frac{1}{1+\varepsilon}\right)$. The marginal effect of a change in ε on τ_J^{ML} is $-\frac{1-\Omega_{ML}}{(1+\varepsilon-\Omega_{ML})^2}$ and on τ_H is $-\frac{1}{(1+\varepsilon)^2}$. Therefore, all tax rates increase as ε decreases, more tax revenue is raised, which results in higher public good provision. Also, $\frac{\partial(\tau_H-\tau_L^{ML})}{\partial\varepsilon} = \frac{-\Omega_{ML}(\varepsilon^2-1+\Omega_{ML})}{(1+\varepsilon-\Omega_{ML})^2(1+\varepsilon)^2} < 0$ whenever $\varepsilon^2 > 1-\Omega_{ML}$ and progressivity increases as ε decreases. If $\varepsilon^2 < 1-\Omega_{ML}$, $\frac{\partial(\tau_H-\tau_L^{ML})}{\partial\varepsilon} > 0$ and progressivity decreases as ε decreases. ■

Proof of Remark 6. Recall that $w_H > w_M > w_L$ and the income inequality between the high and the middle groups is given by $w_H - w_M$. Let w_H increase by some $\delta > 0$, so that the income inequality between H and M increases. Notice that the tax rates on H and M under either coalition do not depend directly on the income of these groups. Thus, after the change in w_H, H and M will have the same tax rates, the government income will (weakly) increase, and the quantity of public good provided will increase by $\alpha_H\varepsilon^\varepsilon(1-\tau_H)^\varepsilon\tau_H\left[(w_H+\delta)^{\varepsilon+1} - w_H^{\varepsilon+1}\right]$.

The utility of M from the ML coalition will increase by $\theta_M\alpha_H\varepsilon^\varepsilon(1-\frac{1}{1+\varepsilon})^\varepsilon\frac{1}{1+\varepsilon}\left[(w_H+\delta)^{\varepsilon+1} - w_H^{\varepsilon+1}\right]$. The utility of M from the MH coalition will be unchanged since $\tau_J^{MH} = 0$, so H's higher income does not result in higher public good provision. A coalition with L will therefore be more attractive for M. From Proposition 4, this means that $\overline{\theta_M}$ will be lower when w_H is increased by δ. Consequently, if M is the formateur and θ_M is close enough to $\overline{\theta_M}$, M might give up an alliance with H for an alliance with L. This results in a switch from a regressive tax schedule imposed by the MH coalition to a progressive tax schedule imposed by the ML coalition.

The case where w_M decreases by $\delta > 0$ is similar, with the additional condition that $(1-\tau_J^{ML})^{\varepsilon+1}(1+\theta_M\alpha_M(1+\varepsilon)\frac{\tau_J^{ML}}{1-\tau_J^{ML}}) \leq 1$, which ensures that, as w_M decreases, the utility of M will not decrease faster under the ML coalition than under the MH coalition. Therefore, an increase in income inequality between the middle and high income groups can lead either to a more progressive tax schedule or it can have no effect on the existing tax schedule, which proves that the equilibrium tax schedule becomes weakly more progressive. ■

Proof of Remark 7. We are in the situation where the coalition is formed by M and L and $\tau_J^{ML} = \frac{1-\Omega_{ML}}{1+\varepsilon_J-\Omega_{ML}}$ $(J = L, M)$, $\tau_H = \frac{1}{1+\varepsilon_H}$. The coalition tax rate depends on the shares of the population occupied by M and L. If α_M decreases while α_H stays constant (i.e. we have downward population mobility from M to L), then $\frac{\partial\tau_J^{ML}}{\partial\alpha_M} = \frac{\partial\tau_J^{ML}}{\partial\Omega_{ML}}\frac{\partial\Omega_{ML}}{\partial\alpha_M} = \frac{\varepsilon_J\Omega_{ML}^2(-\theta_L+\theta_M)}{(1+\varepsilon_J-\Omega_{ML})^2} < 0$, since $\theta_L > \theta_M$. Thus, τ_J^{ML} increases, there are more tax revenues, thus higher public good provision. Tax progressivity, measured as $\tau_H - \tau_L^{ML} = \frac{1}{1+\varepsilon_H} - \tau_L^{ML}$, decreases as α_M decreases. If α_M increases while α_L stays constant (i.e. we have downward population mobility from the high group to the middle group), then $\frac{\partial\tau_J^{ML}}{\partial\alpha_M} = \frac{\partial\tau_J^{ML}}{\partial\Omega_{ML}}\frac{\partial\Omega_{ML}}{\partial\alpha_M} = \frac{\varepsilon_J\Omega_{ML}^2\theta_M}{(1+\varepsilon_J-\Omega_{ML})^2} > 0$. Thus, τ_L^{ML} increases, public good provision increases and tax progressivity decreases, since τ_L^{ML} is now closer to $\frac{1}{1+\varepsilon_H}$. Finally, if α_L increases while α_M stays constant, then the result is qualitatively analogous to the downward mobility from H to M: $\frac{\partial\tau_J^{ML}}{\partial\alpha_L} = \frac{\partial\tau_J^{ML}}{\partial\Omega_{ML}}\frac{\partial\Omega_{ML}}{\partial\alpha_L} = \frac{\varepsilon_J\Omega_{ML}^2\theta_M}{(1+\varepsilon_J-\Omega_{ML})^2} > 0$. As α_L increases, τ_J^{ML} goes up, there is higher public good provision and progressivity decreases. ■

Proof of Remark 8. Under the ML coalition, assume θ_L increases. Then $\frac{\partial\Omega_{ML}}{\partial\theta_L} = \frac{-\alpha_L}{(\theta_L\alpha_L+\theta_M\alpha_M)} < 0$, and $\frac{\partial\tau_L^{ML}}{\partial\theta_L} = \frac{\partial\tau_L^{ML}}{\partial\Omega_{ML}}\frac{\partial\Omega_{ML}}{\partial\theta_L} > 0$. Thus, τ_L^{ML} increases, there are more tax revenues and higher public good provision. The tax rate on group H remains the same,

$\tau_H - \tau_L^{ML}$ decreases, meaning progressivity decreases. Under the LH coalition, both τ_L^{LH} and τ_H^{LH} increase since $\frac{\partial \tau_J^{LH}}{\partial \theta_L} = \frac{\partial \tau_J^{LH}}{\partial \Omega_{HL}} \frac{\partial \Omega_{HL}}{\partial \theta_L} > 0$, so there is more public good provision. Moreover, $\frac{\partial(\tau_H^{LH} - \tau_L^{LH})}{\partial \theta_L} = \frac{\alpha_L(\varepsilon_L - \varepsilon_H)(\varepsilon_L \varepsilon_H - (1-\Omega_{LH})^2)}{\Omega_{LH}^2 (1+\varepsilon_H - \Omega_{LH})^2 (1+\varepsilon_L - \Omega_{LH})^2} > 0$ for $\varepsilon_L \varepsilon_H > (1 - \Omega_{LH})^2$. Thus, progressivity increases. If $\varepsilon_L \varepsilon_H < (1 - \Omega_{LH})^2$, then $\frac{\partial(\tau_H^{LH} - \tau_L^{LH})}{\partial \theta_L} < 0$ and progressivity decreases. Finally, if the ruling coalition is MH, a change in θ_L has no effect on the equilibrium tax rates, so the progressivity does not change.

In terms of the requirement for $\overline{\theta_M}$, we find that $\frac{\partial V_M}{\partial \theta_L} < 0$ whenever $\theta_L > \theta_M$ and w_M is sufficiently larger than w_L.[33] Thus, M's utility under the ML coalition decreases as L's preference for the public good increases. M's utility under the MH coalition is, however, unaffected by changes in θ_L. Thus, H becomes a more attractive coalition partner and the threshold $\overline{\theta_M}$ from Proposition 4 (under which MH is preferred to ML) increases. If, as a result of the increase in θ_L, M switches from a coalition with L to a coalition with H, then the tax schedule becomes regressive. Thus, in either case progressivity decreases. ■

Proof of Remark 9. If θ_M or θ_H increase under the MH coalition, then τ_H^{MH} and τ_M^{MH} both increase,[34] while τ_L remains unchanged, so more public good is provided and progressivity increases. If θ_M increases under the ML coalition, then τ_L^{ML} and τ_M^{ML} both increase while τ_H remains unchanged, more public good is provided and progressivity decreases. If θ_H increases under the HL coalition, then τ_L^{LH} and τ_H^{LH} both increase, more public good is provided, and, from the proof to Remark 8, it follows that progressivity increases if $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} > 1$, and it decreases if $\frac{\varepsilon_L \varepsilon_H}{(1-\Omega_{LH})^2} < 1$. Finally, if θ_H or θ_M increase when H or M, respectively, are outside the ruling coalition, then this has no effect on the equilibrium tax rates. ■

[33] $\frac{\partial V_M}{\partial \theta_L} = -\alpha_L^2 \Omega_{ML}^3 \left(\frac{\varepsilon_M^{2\varepsilon_M+1} \theta_L w_M^{\varepsilon_M+1}}{(1+\varepsilon_M - \Omega_{ML})^{\varepsilon_M+2}} - \frac{\varepsilon_L^{2\varepsilon_L+1} \theta_M w_L^{\varepsilon_L+1}}{(1+\varepsilon_L - \Omega_{ML})^{\varepsilon_L+2}} \right) < 0$ when $\theta_L > \theta_M$ and w_M is sufficiently larger than w_L. If labor supply elasticities are equal, this reduces to $-\frac{\alpha_L^2 \Omega_{ML}^3 \varepsilon^{2\varepsilon+1}(\theta_L w_M^{\varepsilon+1} - \theta_M w_L^{\varepsilon+1})}{(1+\varepsilon-\Omega_{ML})^{\varepsilon+2}} < 0$ whenever $\theta_L > \theta_M$.

[34] The tax rates remain 0, in the trivial case when the coalition tax rate is 0 and progressivity does not change. However, we are not focusing on the unintersting case.

References

Austen-Smith, D. 2000. Redistributing income under proportional representation. Journal of Political Economy 108, 1235-1269.

Baron, D., and J. Ferejohn. 1989. Bargaining in Legislatures. American Political Science Review 83(4), 1181–1206.

Bassett, W.F., J. P. Burkett and L. Putterman. 1999. Income distribution, government transfers, and the problem of unequal influence. European Journal of Political Economy 15(2), 207-228.

Battaglini,M., and S. Coate. 2007. Inefficiency in Legislative Policymaking: A Dynamic Analysis. American Economic Review 97(1), 118–49.

Berliant, M. and M. Gouveia. 1994. On the political economy of income taxation. University of Rochester, Mimeo.

Besley, T. and S. Coate. 1997. An economic model of representative democracy. Quarterly Journal of Economics 112, 85-114.

Bohn, H. and C. Stuart. 2005. Voting and nonlinear taxes in a stylized representative democracy. Working paper, UCSB.

Carbonell-Nicolau, O. and E. F. Klor. 2003. Representative democracy and marginal rate progressive income taxation. Journal of Public Economics 87(9-10), 2339-2366.

Carbonell-Nicolau, O. and E. A. Ok. 2007. Voting over income taxation. Journal of Economic Theory 134(1), 249–86.

Casamatta, G. , H. Cremer and P. De Donder. 2008. Repeated Electoral Competition over Non-Linear Income Tax Schedules. CEPR Discussion Paper No. DP7054.

Chen, Y. 2000. Electoral Systems, Legislative Process and Income Taxation. Journal of Public Economic Theory 2(1), 71-100.

Coughlin, Peter J. and Shmuel Nitzan. 1981. Electoral Outcomes with Probabilistic Voting and Nash Social Welfare Maxima. Journal of Public Economics 15, 113-22.

Cox, G. 1997. Making Votes Count. Cambridge, UK: Cambridge University Press.

Cukierman, A. and A.H. Meltzer. 1991. A political theory of progressive taxation. In: Meltzer, A.,Cukierman, A., Richard, S.F. (Eds.), Political Economy. Oxford: Oxford University Press,.76–108.

De Donder, P. and J. Hindriks. 2003. The politics of progressive income taxation with incentive effects. Journal of Public Economics 87, 2491–505.

Galli,E. 2002. Tax progressivity and economic and political determinants: an empirical analysis for the OECD countries. Universita di Roma "La Sapienza", Mimeo.

Hansen JM. 1998. Individuals, institutions, and public preferences over public finance. Am. Polit. Sci. Rev. 92, 513–31

Hettich, Walter and Stanley L. Winer. 1988. Economic and Political Foundations of Tax Structure. American Economic Review 78, 701-712.

_ _ _ _. 1999. Democratic Choice and Taxation: A Theoretical and Empirical Analysis. New York: Cambridge University Press.

Hinich, M., J. Ledyard and P. Ordeshook. 1972. Nonvoting and the existence of equilibrium under majority rule. Journal of Economic Theory 4, 144–153.

Jacobs, D. and D. Waldman. 1983. Toward a Fiscal sociology: Determinants of Tax regressivity in the American states. Social science quarterly 64, 550-565.

Kanbur, S.M.R. and M. Tuomala. 1994. Inherent inequality and the optimal graduation of marginal tax rates. Scandinavian Journal of Economics 96, 275–282.

Laver, M. and N. Schofield. 1990. Multiparty Government: The Politics of Coalition in Europe. paperback ed. Oxford University Press.

Levy, G. 2004. A model of political parties. J.Econ.Theory 115, 250-277.

Lijphart, A. 1994. Electoral Systems and Party Systems. Oxford: Oxford University Press.

Lizzeri, A., and N. Persico. 2001. The Provision of Public Goods under Alternative Electoral Incentives. American Economic Review 91: 225-239.

Lowery, D. 1987. The Distribution of Tax Burdens in the American States: The Determinants of Fiscal Incidence. Western Political Quarterly 40, 137-158.

Meltzer, A. and S. Richard. 1981. A Rational Theory of the Size of Government. Journal of Political Economy 89 (5), 914-927.

Milesi-Ferretti Gian-Maria, R. Perotti and M. Rostagno. 2002. Electoral Systems and the Composition of Public Spending. Quarterly Journal of Economics 117, 609-657.

Mueller, D.C. 2003. Public Choice III. Cambridge University Press.

Osborne, M. and A. Slivinski. 1996. A model of political competition with citizen-candidates. Quarterly J. Economics 111, 65-96.

Perotti, R. 1996. Growth, income distribution, and democracy: what the data say. Journal of Economic Growth 1, 149–187.

Persson, T., G. Roland and G. Tabellini. 2000. Comparative Politics and Public Finance. Journal of Political Economy 108, 1121-1161.

Persson, T., G. Roland and G. Tabellini. 2007. Electoral Rules and Government Spending in Parliamentary Democracies. Quartely Journal of Political Science 2(20, 155-188.

Persson, T., Tabellini, G. 1994. Is inequality harmful for growth?. American Economic Review 84, 600–621.

Persson, T. and G. Tabellini. 2000. Political Economics: Explaining Economic Policy. Cambridge, MA: MIT Press.

Persson, T. and G. Tabellini. 2003. The Economic Effects of Constitutions. Cambridge, MA: MIT Press.

Plott, C. R. 1967. A notion of equilibrium and its possibility under majority rule.American Economic Review 57, 787–806.

Roberts, K. 1977. Voting over income tax schedules. Journal of Public Economics 8, 329–40.

Röell, A., 1997. Voting over non-linear income tax schedules. Tilburg University, mimeo.

Roemer, J.E. 1999. The democratic political economy of progressive income taxation. Econometrica 67, 1-19.

____. 2001. Political competition: Theory and applications. Cambridge: Harvard University Press.

Roemer, J.E. 2008. A positive theory of income taxation where politicians focus upon swing and core voters. Cowles Foundation Discussion Paper No. 1637.

Romer, T. 1975. Individual welfare, majority voting and the properties of a linear income tax. Journal of Public Economics 7, 163–68.

Rubinstein, A. 1982. Perfect Equilibrium in a Bargaining Model. Econometrica 50(1), 97-109.

Snyder, J. and G. Kramer. 1988. Fairness, self-interest, and the politics of the progressive income tax. Journal of Public Economics 36, 197– 230.

Iversen, T. and D. Soskice. 2006. Electoral Institutions and the Politics of Coalitions: Why Some Democracies Redistribute More Than Others. American Political Science Review 100, 165-181.

Weymark, J.A. 1987. Comparative statics properties of optimal nonlinear income taxes. Econometrica 55, 1165–1185.