A. Preliminary Notes:

1. Under conditions of low pressure, high temperature real gases obey the equation of an ideal gas:

   \[ PV = nRT \]

Here:

\( P \) = pressure \([N/m^2]\)

\( V \) = volume \([m^3]\)

\( T \) = temperature \([K]\)

\( n \) = number of moles

\( R \) = universal gas constant \([J/(mole*K)]\)
2. At fixed volume and fixed moles the pressure of an ideal gas is linearly proportional to the temperature of the gas. The slope of the line is proportional to $R$.

3. Pressure is proportional to combined mass of piston, support stand and the atmospheric pressure

$$P = \frac{Mg}{A} + P_{\text{atm}}$$

Here:

- $M =$ total mass of piston, support stand and addition masses [kg]
- $A =$ cross-sectional area of piston [$m^2$]
- $g =$ gravitational acceleration ($9.80$ m/sec$^2$)
- $P_{\text{atm}} =$ atmosphere pressure [$N/m^2$]

4. Volume of gas is the cross-sectional area of piston times the change in the height of the piston.

$$V = A(h_2 - h_1) = A\Delta h \text{ [cm}^3\text{]}$$
B. Equipment and Setup:
double-walled cylinder ACPD#01045
Piston and support stand
Thermometer
Caliper
Ruler
Support block

List of Equipment

Sketch of Setup with relevant parts labeled
C. Procedure: We will make a series of measurements of the pressure vs temperature. As the temperature of the gas is increased the gas will expand and the piston will rise. Masses are added to the support stand on top of the piston to compress the gas back to the original volume. The pressure exerted on the gas is proportional to the mass on the support stand. A plot of the mass vs. temperature expected to be a straight line if the air is behaving like an ideal gas.

Outline of Procedure

1. Initial measurements:
   - Diameter of piston measured with centimeter caliper = 10.00 ± 0.01 cm
   - Area of piston \( A = \pi \left( \frac{10.00 \pm 0.01 \text{ cm}}{2} \right)^2 = 78.54 \pm 0.15 \text{ cm}^2 \)
   - Height of support stand with piston inserted into cylinder \( h_1 = 21.1 \pm 0.1 \text{ cm} \)
   - Height of support stand with piston draw up to exp. Volume \( h_2 = 40.2 \pm 0.1 \text{ cm} \)
   - Fixed experimental volume \( V = A(h_2 - h_1) = (1.48 \pm 0.02) \times 10^3 \text{ cm}^3 \)
   - Combine mass of piston and support stand \( M_{\text{piston}} = 5.000 \pm 0.001 \text{ kg} \)

2. Initial setup:
   - Air valve in piston opened and piston drawn up to height of 40.2 ± 0.1 cm
   - Support stand held in place by support block Ice and water added to water bath between cylinder walls
   - Apparatus allowed equilibrate for 20 mins. Ice-water bath temperature 0 C.
   - Air valve in piston is closed to seal air inside and support block removed.
3. Calculation of number of moles n:
At standard temperature (0°C or 273 K) and pressure (1 atmosphere)
1 mole of gas occupies a volume of 22.4 liters
N = (1.48 ± 0.02 liters) × (1 mole/22.4 liters) = (6.62 ± 0.10) × 10^{-2} moles

4. Measurements of Mass vs. Temperature:
Set cylinder on heating coil and insert thermometer into water bath.
Adjust temperature of heating coil and allow water bath to equilibrate for 15 min.
As water bath warms, the gas inside cylinder begins to expand and piston rises.
After 15 mins no observable change in temperature and height of piston remaining constant. Add masses to support stand to compress gas back to original height of 40.2 cm.
Table of measurements of Mass and Temperature:

<table>
<thead>
<tr>
<th>Time</th>
<th>Mass (kg)</th>
<th>Temperature (°C)</th>
<th>Temperature (K)</th>
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</thead>
<tbody>
<tr>
<td>14:30</td>
<td>7.201</td>
<td>27.0</td>
<td>300.0</td>
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<tr>
<td>14:46</td>
<td>10.340</td>
<td>36.8</td>
<td>309.8</td>
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<tr>
<td>14:55</td>
<td>13.560</td>
<td>43.2</td>
<td>316.2</td>
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<tr>
<td>15:01</td>
<td>14.550</td>
<td>46.7</td>
<td>319.9</td>
</tr>
<tr>
<td>15:17</td>
<td>16.256</td>
<td>58.1</td>
<td>331.1</td>
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<td>15:30</td>
<td>20.109</td>
<td>68.3</td>
<td>341.3</td>
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<td>15:45</td>
<td>23.034</td>
<td>76.9</td>
<td>349.9</td>
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<tr>
<td>16:00</td>
<td>26.322</td>
<td>87.0</td>
<td>360.0</td>
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<tr>
<td>16:16</td>
<td>28.178</td>
<td>97.1</td>
<td>370.1</td>
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</table>

Sequential Measurements in neat table. Column headings labeled with quantity and units.

Incorrect measurement with single line strike. Comment on incorrect measurement.

Questionable measurement: water bath may not have reached equilibrium.
1. Preliminary graph of Mass vs. Temperature exhibits a linear relationship. This is expected for an ideal gas.

Outline of Analysis:

Title: Y (vertical axis) vs. X (horizontal axis)

Axes labeled with units

Axes scale divided into intervals of 1, 2, 5 or 10
2. Perform regression analysis of the Mass vs. Temperature to find best slope and intercept of trend line.

Print and paste Regression Analysis into notebook as permanent record.

### SUMMARY OUTPUT

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<td>Observations</td>
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<table>
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<th>t Stat</th>
<th>P-Value</th>
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<th>Upper 95%</th>
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<td>0.32093044</td>
<td>0.27499816</td>
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</tbody>
</table>
3. Results of Excel regression analysis:
Slope $a = (0.298 \pm 0.009)$ kg/K
Intercept $b = (-80.6 \pm 3.2)$ kg

4. Relationship of slope to universal gas constant $R$:
Substitute expressions for volume and pressure into ideal gas equation
$$(P_{\text{atm}} + Mg/A)(A h) = nRT$$
Solve for mass $M$:
$$M = \frac{(nR/g h)T - (A/g)P_{\text{atm}}}{\Delta \text{atm}}$$
Relate terms of equation to slope and intercept
Slope $a = nR/g h$
Intercept $b = (A/g)P_{\text{atm}}$

5. Solve for the universal gas constant $R$:
$$R = \frac{ag\Delta h}{n} = \frac{(0.298 \text{ kg/K})(9.8 \text{ m/sec}^2)(0.189 \text{ m})}{(6.62 \times 10^{-2} \text{ moles})}$$
$$= 8.34 \text{ J/(mole*K)}$$

6. Find the relative uncertainty of $R$:
$$\frac{dR}{R} = \left(\frac{(da/a)^2 + (dh/h)^2 + (dn/n)^2}{2}\right)^{1/2}$$
$$= \left(\frac{(0.012/0.304)^2 + (0.002/0.189)^2 + (0.10/6.62)^2}{2}\right)^{1/2}$$
$$= 0.0354$$

7. Our measurement of the universal gas constant $R$:
$$R = (8.34 \pm 0.30) \text{ J/(mole*K)}$$
We obtained an experimental value of the gas constant $R = (8.51 \pm 0.55) \text{ J/(mole*K)}$. This is consistent with the accepted value of $R = 8.31 \text{ J/(mole*K)}$. The largest contribution of uncertainty is the statistical uncertainty in the slope (3.9%) of the best fit line from the regression analysis. The uncertainties from the measurement of the height and number of moles are 1.1% and 1.5% respectively. From our plot of mass vs. temperature it is evident that the air inside the cylinder is behaving like an ideal gas under our experimental conditions. The mass vs. temperature plot is a straight line, consistent with the equation of an ideal gas at fixed volume and fixed moles.