Math 211, Fall 2011: Midterm I Study Guide

The first midterm will take place on Friday 30 September at 1:00pm in our usual classroom, the Paino Lecture Hall. It will be 50 minutes long. You will not be allowed to use notes, books or calculators of any kind. The exam will cover sections 12.1 through 13.3, except for 12.6. (This is everything we covered in class up until Thursday 22 September.)

Format of the exam

The exam will have five questions, each will be worth 10 points. You should do as many of the questions as fully as possible. You will get partial credit if you have the right idea for a question, even if you do not get it completely correct. You should also explain your answers fully, to an amount of detail appropriate to the question. The rule of thumb is that you should explain the key steps needed to solve the problem, but do not have to explain minor steps that you use along the way. In the solutions to the practice exam problems, I'll try to indicate what is expected.

Background

Here is a summary of *some* of the things you need to know as background to this course. Obviously this is not a comprehensive list but hopefully clarifies some of what is expected. Please ask if you have any questions.

- polynomial and power functions
  - be able to sketch the graphs of linear and quadratic functions, and of x^r for $r = -1, -2, \frac{1}{2}, \frac{1}{3}$
- trig functions
  - know the definitions of the sine, cosine and tangent functions
  - be able to draw the graphs of the sine, cosine and tangent functions
  - know, or be able to calculate, the values of the sine, cosine and tangent functions at multiples of $\pi/6$ and $\pi/4$
  - know that $\sin^2(u) + \cos^2(u) = 1$ for any real number u
  - (you do not need to know by heart any other trig identities for this test)
- exponential and logarithmic functions
  - know the domains and ranges, and be able to sketch the graphs, of the functions e^x and $\ln(x)$.
- differentiation

  - be able to differentiate polynomials, power functions x^r, trig functions (sine, cosine and tangent), exponential and logarithmic functions
  - be able to use the product, quotient and chain rules to differentiate combinations of the above types of functions
  - know the meaning of the derivative as the rate of change, or graphically as the slope of the tangent line

- integration
  - be able to integrate polynomials, power functions (including x^{-1}), sine and cosine functions and exponential functions
  - be able to use integration by substitution (the chain rule in reverse) to integrate simple variants of the above, such as $\sin(2t)$ or e^{3t+1}
  - (more advanced integration techniques such as integration by parts, or partial fractions, and harder types of integrals, such as those requiring a trig substitution, will not be required for this test)
  - be able to calculate definite integrals involving the above functions

- the Fundamental Theorem of Calculus
  - know that integration is inverse to differentiation so that integration amounts to finding an antiderivative
  - know that for a differentiable function f

$$f(b) - f(a) = \int_{t=a}^{t=b} f'(t)\, dt$$

  - know that if a function $G(t)$ is defined by

$$G(t) = \int_{u=a}^{u=t} g(u)\, du$$

    then $G'(t) = g(t)$

Syllabus

Here is a summary of what you need to know and be able to do. Unless stated otherwise, you may use any fact or result from class or from the sections of the textbook we have covered. You may *not* use any results from later in the textbook. You should:

- coordinate systems in three-dimensions (12.1)
  - understand the ordinary (rectangular or Cartesian) coordinate system in three-dimensional space $\mathbb{R}^3$, where a point is specified by giving three real numbers (x, y, z) representing the distances from a chosen origin in the directions of the x-, y- and z-axes.

- be able to find the distance between two points in $\mathbb{R}^3$ from their coordinates
- be able to find the equation of a sphere with given center and radius
- from the equation of a sphere be able to work out the sphere's center and radius

- vectors (12.2)
  - understand the three different ways that vectors (in $\mathbb{R}^2$ or $\mathbb{R}^3$) can be thought of: (i) as a pair or triple of real numbers $\langle a_1, a_2 \rangle$ or $\langle a_1, a_2, a_3 \rangle$ (the numbers a_1, a_2, a_3 are the *components* of the vector); (ii) as an arrow with a particular direction and a particular length, but no fixed start or end points; or (iii) as representing a point in $\mathbb{R}^3$ (that is, as a *position vector*)
  - know that the position vector of the point $P = (x, y, z)$ is the vector $\langle x, y, z \rangle$, and that the arrow representing this position vector is that arrows starting at the origin and ending at P
  - be able to add and subtract vectors either in terms of their components, or geometrically using their representation as arrows
  - know that $\vec{AB}$ represents the vector starting at point A and ending at point B, and be able to use this notation to express vectors obtained by adding or subtracting
  - be able to multiply a vector by a scalar (i.e. a real number) and know what this means geometrically using the representation of the vector as an arrow
  - be able to find the length or magnitude of a vector from its components, and understand that this equals the length of the corresponding arrow
  - know and be able to use the various properties satisfied by addition, subtraction and scalar multiplication of vectors (page 819)
  - know the definitions of the standard basis vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$, and be able to write any vector in the form
    $$a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$
  - be able to find a unit vector that points in the same direction as a given nonzero vector

- the dot product (12.3)
  - know the definition of the dot product $\mathbf{a} \cdot \mathbf{b}$ of two vectors in terms of the components of the vectors
  - know, and be able to use, the properties of the dot product relating it to length, addition and scalar multiplication (page 825)
  - know the geometric interpretation of the dot product, and be able to use this to calculate the angle between two given vectors
  - know that two non-zero vectors are perpendicular (or *orthogonal* or *normal*) if and only if their dot product is equal to zero
  - be able to find the projection of one vector in the direction of another (called the *vector projection* in the book)

- the cross product (12.4)
  - know the definition of the cross product $\mathbf{a} \times \mathbf{b}$ in terms of the components of the vectors
  - know, and be able to use, the properties of the cross product relating it to addition and scalar multiplication (properties 1-4 on page 836)
  - know the cross products of the standard basis vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$
  - know the geometric interpretation of the cross product $\mathbf{a}\times\mathbf{b}$: that it is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, of a certain length, and that its direction is determined by some rule such as the right-hand rule
  - know that two non-zero vectors are parallel if and only if their cross product is zero
- equations of planes (12.5)
  - be able to find the equation of the plane containing a given point (x_0, y_0, z_0) and perpendicular to the vector $\mathbf{n} = \langle a, b, c\rangle$
  - be able to find a vector perpendicular to a plane with given equation
  - be able to find equations of planes from other given information by working out a point on the plane and a vector perpendicular to it
- vector functions and curves in space (13.1)
  - know what is meant by a vector function $\mathbf{r}$ and understand vector functions described in terms of their components, i.e. in the form

$$\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$$

  - know what is meant by a curve in $\mathbb{R}^2$ or $\mathbb{R}^3$, and that the same curve can be described by different vector functions, or parametrizations
  - be able to draw the curve in $\mathbb{R}^2$ given by a particular vector function $\langle f(t), g(t)\rangle$
  - know what is meant by the limit of a vector function in terms of limits of the component functions f, g, h
  - know what it means for a vector function to be continuous at a particular point
  - be able to find where two curves intersect, where a curve meets a plane or sphere
  - be able to tell if the particles following curves given by vector functions would collide
- vector equations of lines in space (12.5, but in the spirit of 13.1)
  - be able to find a vector equation, or parametric equation, for a line in $\mathbb{R}^2$ or $\mathbb{R}^3$ containing a given point and parallel to a given vector
  - from the vector or parametric equation of a line be able to find a vector parallel to the line
  - be able to find an equation for a line from other given information by working out a point on the line and a vector parallel to it

- derivatives and integrals of vector functions (13.2)
  - know what it means for a vector function to be differentiable at an input value and be able to find its derivative in terms of the component functions
  - know that the derivative $\mathbf{r}'(a)$ is, if it is not zero, a tangent vector to the curve given by the function $\mathbf{r}$ at the point $\mathbf{r}(a)$
  - know what is meant by the unit tangent vector $\mathbf{T}(t)$ for a vector function and be able to find it
  - be able to find the equation of the tangent line to a curve at a given point from a parametrization of the curve
  - be able to find the angle between two intersecting curves as the angle between their tangent vectors at the point of intersection
  - know and be able to use the rules for differentiating vector functions (page 874)
  - know what it means to integrate a vector function in terms of the component functions
  - know the Fundamental Theorem of Calculus as it applies to vector functions
- arc length (13.3)
  - be able to find the length of a piece of a curve from a given parametrization
  - be able to find the arc length function for a curve with given parametrization and given starting point
  - know what is meant by an arc length or unit speed parametrization for a curve, and be able to find such a parametrization
- curvature (13.3)
  - know what is meant by the (scalar) curvature of a curve at a particular point, and how it is related to the derivative of the unit tangent vector
  - be able to find the curvature of a curve given by vector function $\mathbf{r}(t)$ (by either formula 9 or 10 on page 880)
  - be able to find the curvature of the graph of a function $y = f(x)$ at a particular point
  - be able to find the unit normal vector $\mathbf{N}(t)$ of a curve at a particular point, and understand how the unit normal vector is related to the direction the curve is turning

This is not a complete list of what you might have to do on the test but it covers most of the ideas involved. In particular, you may have to combine several of these ideas or techniques, and you may to think to decide what to use to solve a problem.

Preparing for the test

The best way to prepare for the test is to do practice tests. This means you should sit down, without a textbook or your notes, and try to do as much as you can of the practice test in 50 minutes, as though it were the real thing. This will give you an idea of how well prepared you are, what topics you might need to review, and how you react under test conditions. This is especially important if you don't have much experience taking timed tests, or if you have had anxiety problems with tests in the past. The more practice you do, the better prepared you will be.

You should also go back over past homework problems, especially those for which the grader has written a comment or deducted points and make sure you understand the comment or why you lost points. If you can't work this out or have any other questions about the grading, please come and ask me about it. I'll have extra office hours on Wednesday and Thursday next week to help you prepare.

You should also just work through more practice problems. If you didn't do the practice problems assigned for the homeworks, now would be a good time to do those. If you did, you can make up some more problems on your own (which is also a good exercise to see if you understand the material) and try to solve them. You can always ask me if you are unsure of something.

Beyond that, please let me know how else I can help you prepare, and good luck!