Math 211, Fall 2011: Midterm II Study Guide

The second midterm will take place on Friday 28 October at 1:00pm in our usual classroom, the Paino Lecture Hall. It will be 50 minutes long. You will not be allowed to use notes, books or calculators of any kind. The exam will cover section 13.4, and 14.1 through 14.6.

Format of the exam

The exam will have five questions, each will be worth 10 points. You should do as many of the questions as fully as possible. You will get partial credit if you have the right idea for a question, even if you do not get it completely correct. You should also explain your answers fully, to an amount of detail appropriate to the question. The rule of thumb is that you should explain the key steps needed to solve the problem, but do not have to explain minor steps that you use along the way. In the solutions to the practice exam problems, I'll try to indicate what is expected.

Syllabus

Here is a summary of what you need to know and be able to do. Unless stated otherwise, you may use any fact or result from class or from the sections of the textbook we have covered. You may *not* use any results from later in the textbook. You should:

- velocity and acceleration (13.4)
  - be able to find the velocity, speed and acceleration of an object whose position at time t is given by a function $\mathbf{r}(t)$
  - be able to find the velocity and position of an object whose acceleration at time t is given by a function $\mathbf{a}(t)$ and whose velocity and position at some particular time t_0 are specified
  - be able to find the velocity and position of an object moving under the influence of gravity (given by a constant acceleration $g = 9.8ms^{-2}$
  - be able to solve word problems involving position, velocity and acceleration
  - know what is meant by the tangential and normal components to the acceleration of an object and be able to find these for an object whose position vector at time t is a given function $\mathbf{r}(t)$
- functions of two variables (14.1)
  - know what is meant by a function of two variables and be able to find and sketch the domain of a given function

- know what is meant by the graph of a function of two variables and be able to identify the graph of a function from a formula defining that function (you will not be asked to sketch any graphs in three dimensions)
- know what is meant by the level curves of a function of two variables and be able to sketch a selection of level curves (that is, the contour map) for a given function (you will only be expected to be able to sketch circles, straight lines and parabolas)
- understand the relationship between the graph of a function and the level curves of the function

- limits and continuity for functions of two variables (14.2)
  - know the definition of what it means for a limit of a function of two variables to be equal to some value
  - be able to show that a limit does not exist by showing that the function approaches different values (or does not approach any value) when the limit point is approached from different directions
  - be able to show that a limit is not equal to some value by showing the that the function approaches some other value (or no value) when approached from some particular direction
  - be able to use the ϵ/δ-definition of limit to show that a limit is equal to a particular value
  - know what it means for a function of two variables to be continuous and be able to find a limit involving a continuous function
  - know that any function built via a single formula from constants, powers, sums, differences, products, quotients, trig functions, exponentials, logarithms and composition of these, is continuous at all points in its domain
  - be able to find the limit of a given function by showing that it is equal to a continuous function except at the limit point

- partial derivatives (14.3)
  - know the limit definition of partial derivatives and be able to calculate the partial derivatives of a function at a particular point using the limit definition
  - be able to find the partial derivatives of a function by taking one variable to be a constant and differentiating with respect to the other variable
  - understand that the partial derivatives of a function of two variables are themselves functions of two variables
  - know the meaning of the partial derivative of f as the slope of the cross-section through the graph of f given by keeping one of the inputs constant
  - be able to estimate the values of partial derivatives from a contour map (i.e. diagram of level curves) of a function
  - be able to use both f_x and $\frac{\partial f}{\partial x}$ as notation for the partial derivative with respect to x

  - be able to find second-order partial derivatives and understand the various notations for them
  - know conditions under which the mixed second-order partial derivatives at a point are equal

- tangent planes and linear approximation (14.4)
  - know how to find the equation of the tangent plane to the graph of a function f at a particular point
  - know what is meant by a linear function of two variables
  - know how to find the linear approximation to a function of two variables at a given point, and be able to use the linear approximation to estimate the value of the function at a nearby point

- differentiability for functions of two variables (extra emailed notes)
  - know what it means to say that a function of two variables is differentiable at a given point
  - be able to show a function is differentiable by showing that the function and its partial derivatives are continuous at and around the point in question
  - be able to show a function is not differentiable at a particular point by showing that one of its partial derivatives does not exist at that point
  - be able to show a function is not differentiable by showing that the relevant limit either does not exist or is not equal to zero
  - be able to show that a function is differentiable at a particular point by showing that the relevant limit is equal to zero (with an ϵ/δ-argument if necessary)
  - understand that being differentiable means that the function has a particularly good linear approximation at the point in question

- the chain rule (14.5)
  - be able to apply the chain rule (either case 1 or case 2 as described in the book) to find derivatives or partial derivatives of functions defined by composing functions of two variables
  - be able to apply the chain rule to find expressions for the partial derivatives of an arbitrary function with respect to new variables (such as finding the partial derivatives of $f(x, y)$ with respect to u and v when x and y are functions of u and v (see Example 7)

- directional derivatives and the gradient vector field (14.6)
  - know and be able to use the limit definition to find the directional derivative of a function at a particular point in a given direction
  - in the case of a differentiable function, be able to find the directional derivatives using the partial derivatives or gradient vector

- know what is meant by the gradient vector field of a function of two or three variables and be able to find the gradient vector for a particular function at a given point
- understand that the gradient vector represents the direction and magnitude of the maximum directional derivative of a function at a particular point and be able to prove this using the dot product formulas
- know that the gradient vector is always perpendicular to the level curves of a function (or is the zero vector)
- know that the gradient vector of a function of three variables is perpendicular to the level surfaces of the function and hence be able to find the equation of the tangent plane to a surface given by an equation of the form $f(x, y, z) = c$
- understand the relationship of the gradient vector to the graph of a function of two variables as representing the direction and slope of the steepest uphill

This is not a complete list of what you might have to do on the test but it covers most of the ideas involved. In particular, you may have to combine several of these ideas or techniques, and you may to think to decide what to use to solve a problem.

Preparing for the test

The best way to prepare for the test is to do practice tests. This means you should sit down, without a textbook or your notes, and try to do as much as you can of the practice test in 50 minutes, as though it were the real thing. This will give you an idea of how well prepared you are, what topics you might need to review, and how you react under test conditions. This is especially important if you don't have much experience taking timed tests, or if you have had anxiety problems with tests in the past. The more practice you do, the better prepared you will be.

You should also go back over past homework problems, especially those for which the grader has written a comment or deducted points and make sure you understand the comment or why you lost points. If you can't work this out or have any other questions about the grading, please come and ask me about it. I'll have extra office hours on Wednesday and Thursday next week to help you prepare.

You should also just work through more practice problems. If you didn't do the practice problems assigned for the homeworks, now would be a good time to do those. If you did, you can make up some more problems on your own (which is also a good exercise to see if you understand the material) and try to solve them. You can always ask me if you are unsure of something.

Beyond that, please let me know how else I can help you prepare, and good luck!