1. (a) State the product rule for finding the derivative of a product \( f(x)g(x) \) of two functions \( f \) and \( g \).

(b) Use the product rule to show that the derivative of 
\[ f(x) = x^{\frac{3}{2}} \]

is given by 
\[ f'(x) = \frac{3\sqrt{x}}{2}. \]

(In your answer, you may only use the power rule to differentiate \( x^n \) where \( n \) is a positive integer.)

(a) \[ \frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \]

(b) We have 
\[ f(x)f(x) = x^3. \]

Differentiating both sides we get 
\[ f'(x)f(x) + f(x)f'(x) = 3x^2 \]

and so 
\[ 2f'(x)f(x) = 3x^2. \]

Therefore, 
\[ f'(x) = \frac{3x^2}{2f(x)} = \frac{3x^2}{2x^{\frac{3}{2}}} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}. \]

2. Find the derivative of the following function 
\[ P(t) = t^2 \cos \left( \frac{2t + 1}{2t - 1} \right). \]

By the product rule, we have 
\[ P'(t) = 2t \cos \left( \frac{2t + 1}{2t - 1} \right) + t^2 \frac{d}{dt} \left( \cos \left( \frac{2t + 1}{2t - 1} \right) \right). \]

By the chain rule 
\[ \frac{d}{dt} \left( \cos \left( \frac{2t + 1}{2t - 1} \right) \right) = -\sin \left( \frac{2t + 1}{2t - 1} \right) \frac{d}{dt} \left( \frac{2t + 1}{2t - 1} \right) \]
and by the quotient rule
\[
\frac{d}{dt} \left( \frac{2t + 1}{2t - 1} \right) = \frac{(2t - 1)(2) - (2t + 1)(2)}{(2t - 1)^2} = \frac{-4}{(2t - 1)^2}.
\]

Putting all of this together we have
\[
P'(t) = 2t \cos \left( \frac{2t + 1}{2t - 1} \right) - t^2 \sin \left( \frac{2t + 1}{2t - 1} \right) - \frac{4}{(2t - 1)^2} \sin \left( \frac{2t + 1}{2t - 1} \right)
\]
\[
= 2t \cos \left( \frac{2t + 1}{2t - 1} \right) + \frac{4t^2}{(2t - 1)^2} \sin \left( \frac{2t + 1}{2t - 1} \right)
\]

3. The height \( h \) (measured in meters) of a toy rocket at a time \( t \) seconds after taking off satisfies the equation
\[
\frac{h^2}{t^2} + 5h = 100 - 50t.
\]

How fast is the rocket moving 1 second after take off?

Differentiating the given equation with respect to \( t \), we get
\[
\frac{t^2(2hh') - h^2(2t)}{t^4} + 5h' = -50.
\]

We want to find the value of \( h'(1) \) so we evaluate this at \( t = 1 \) to get
\[
\frac{2h(1)h'(1) - 2h(1)^2}{1} + 5h'(1) = -50.
\]

To figure this out we need to find \( h(1) \). From the original equation we have
\[
\frac{h(1)^2}{1} + 5h(1) = 100 - 50
\]
and so
\[
h(1)^2 + 5h(1) - 50 = 0
\]
and so
\[
(h(1) + 10)(h(1) - 5) = 0
\]
so either \( h(1) = -10 \) or \( h(1) = 5 \). Since the height cannot be negative we must have \( h(1) = 5 \). Therefore the previous equation becomes
\[
10h'(1) - 50 + 5h'(1) = -50
\]
and so
\[
h'(1) = 0.
\]

Therefore, the speed of the rocket 1 second after take off is zero.
4. Find the equation of the tangent line to the function

\[ H(r) = \sin(2r + \pi) \]

at the point \( r = 0 \).

The slope of the tangent line is given by \( H'(0) \). We have

\[ H'(r) = 2 \cos(2r + \pi) \]

so

\[ H'(0) = 2 \cos(\pi) = -2. \]

Since \( H(0) = \sin(\pi) = 0 \), the line passes through the point \( (0, 0) \). Therefore, the equation of the line is

\[ y = -2r. \]

(Since \( r \) was the input variable of the original function, it makes sense to give the equation of the line in terms of \( r \) instead of \( x \).)

5. A television camera watching the launch of the Space Shuttle is 500 meters from the launch site. If the take-off speed of the shuttle is 100 meters per second, how fast does the camera initially need to turn (in radians per second) to keep the Shuttle in the center of the picture?

It’s a very good idea to draw a diagram here to show what is going on. Let \( h(t) \) be the height of the Shuttle (in meters) above the launch pad after \( t \) seconds and let \( \theta(t) \) be the angle between the camera direction and the horizontal after \( t \) seconds. Then we have

\[ \tan(\theta) = \frac{h}{500}. \]

We are interested in the value of \( \theta'(0) \).

Differentiating the equation we get

\[ \frac{1}{\cos^2(\theta)} \theta' = \frac{h'}{500}. \]

At the point of launch, \( t = 0 \), we have \( \theta = 0 \) and \( h' = 100 \). Therefore, this equation gives

\[ \frac{1}{\cos^2(0)} \theta'(0) = \frac{100}{500}. \]

Therefore

\[ \theta'(0) = \frac{1}{5}. \]

So the initial rate at which the camera needs to turn is 0.2 radians per second.