Maxwell’s Equations – The Fundamental Laws of Electromagnetism

• Gauss’ Law
  – The total electric flux through a closed surface is proportional to the charge enclosed:

\[ \oint \mathbf{E} \cdot \hat{n} dA = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

• Gauss’ Law for Magnetism
  – The total magnetic flux through a closed surface is zero.
  – There are no magnetic charges.

\[ \oint \mathbf{B} \cdot \hat{n} dA = 0 \]
Maxwell’s Equations – The Fundamental Laws of Electromagnetism

• Ampere’s Law (valid for constant currents)
  – The integral of $\mathbf{B} \cdot d\mathbf{l}$ is proportional to the current piercing a surface bounded by the curve.
  – This Law will need generalization to the case of non-constant currents (in a few weeks).

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{encl}} \]
Figure 28.15b

Solenoid

$\vec{B}$

$B \sim \text{small}$
Figure 28.16

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \oint_a^b \mathbf{B} \cdot d\mathbf{l} + \oint_c^d \mathbf{B} \cdot d\mathbf{l} + \oint_c^d \mathbf{B} \cdot d\mathbf{l} + \oint_c^d \mathbf{B} \cdot d\mathbf{l} \]

\[ = \int_c^d B \, dl = B \int_c^d dl = BL = \mu_0 I_{enc} = \mu_0 N I \]

\[ B = \mu_0 (N/l) I = \mu_0 n I \]
Maxwell’s Equations – The Fundamental Laws of Electromagnetism

• Faraday’s Law
  – When the magnetic flux through some loop (C) changes, it induces an emf around the loop proportional to the rate at which the flux changes.
  – If loop has $N$ turns, the emf is $N$ times larger.
  – Lenz’ Law: the sign of the emf is such that an induced current opposes the change in flux.

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = -N \frac{d}{dt} \int \hat{\mathbf{B}} \cdot \hat{\mathbf{n}} dA$$
Ways in which magnetic flux can change:

• Magnitude of Field ($|B|$)
• Direction of Field ($\hat{B}$)
• Orientation of Loop ($\hat{n}$)
• Area of loop ($A$)
• Any combination of the above.
Faraday’s Law and Motional emf

- Motional emf:

\[ E = \Delta V = -\int \mathbf{E} \cdot d\mathbf{l} \]

\[ E = \mathbf{E} \cdot d\mathbf{l} \]

\[ E = vB \ell \]

\[ \mathbf{F}_E = \mathbf{F}_B \]

\[ A \mathbf{E} = A \mathbf{v} B \]
Faraday’s Law and Motional emf

- Faraday’s Law:

\[ \varepsilon = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = B \frac{dA}{dt} \]

\[ = B l \frac{dx}{dt} = B l v \]

\[ dA = l dx \]