The third midterm will take place on Friday 2 December at 9:00am in Seeley Mudd 206. It will be 50 minutes long. You will not be allowed to use notes, books or calculators of any kind. The exam will cover sections 3.1 through 4.4 (with the exception of 3.2, 3.6 and 3.8).

Format of the exam

The exam will have five questions, each will be worth 10 points. You should do as many of the questions as fully as possible. You will get partial credit if you have the right idea for a question, even if you do not get it completely correct. You should also explain your answers fully, to an amount of detail appropriate to the question. The rule of thumb is that you should explain the key steps needed to solve the problem, but do not have to explain minor steps that you use along the way. In the solutions to the practice exam problems, I’ll try to indicate what is expected.

Syllabus

Here is a summary of what you need to know and be able to do. Unless stated otherwise, you may use any fact or result from class or from the sections of the textbook we have covered. You may *not* use any results from later in the textbook. You should:

• maximum and minimum values of a function (3.1)
  – know what is meant by an absolute or local maximum and minimum for a function;
  – be able to identify absolute and local maximum and minimum values from a graph of a function
  – know that a local maximum or minimum must occur at one of three types of points:
    * a point where the derivative of the function is zero (a critical point)
    * a point where the function is not differentiable
    * an endpoint of an interval the function is defined on
  – be able to use the Extreme Value Theorem to justify the existence of an absolute maximum or minimum
  – be able to find the absolute maximum and minimum values of a function defined on a closed interval $[a, b]$ by comparing the values of the function at the three types of points mentioned above

• increasing and decreasing functions (3.3)
  – know what it means for a function to be increasing or decreasing on an interval
  – know that the sign of the derivative tells you the intervals on which a function is increasing or decreasing
be able to show that a critical point is a local maximum or minimum by understanding whether the first derivative goes from positive to negative, or negative to positive

be able to give an example to illustrate how a critical point could be neither a local maximum nor a local minimum

be able to sketch a graph of a function from knowledge of its first derivative to show the intervals on which the function is increasing or decreasing

- concave up and concave down functions (3.3)
  - know what it means for a function to be concave up or concave down on an interval
  - know what is meant by an inflection point for a function
  - be able to use the second derivative of a function to work out the inflection points and intervals on which it is concave up or concave down
  - be able to use the second derivative to decide if a critical point is local maximum or a local minimum

- limits at infinity (3.4)
  - know what is meant by the limit of a function as the input approaches \( \infty \) or \(-\infty\)
  - know the limits of the functions \(x^r\) as \(x \to \infty\) and \(x \to -\infty\), when \(r\) is a rational number
  - know the limit laws that apply to limits at infinity
  - be able to find the limit of a function \(f(x)\) as \(x \to \infty\) or \(x \to -\infty\) by rewriting the function in a different form, and using limit laws
  - know what is meant by a horizontal asymptote for the graph of a function

- curve sketching (3.5)
  - be able to sketch a graph of a given function including the following information:
    * the domain of the function
    * where the graph crosses the \(x\)– and \(y\)–axes
    * vertical and horizontal asymptotes
    * intervals on which the function is increasing or decreasing
    * local maximum and minimum values
    * inflection points and intervals on which the function is concave up or concave down

- optimization problems (3.7)
  - be able to solve word problems that ask for a certain quantity to be maximized or minimized by constructing an appropriate function and finding its absolute maximum or minimum value

- estimating areas and distances (4.1)
- for an object travelling with a given velocity, be able to find an estimate for the distance travelled by the object over some time interval by breaking up the interval into shorter time intervals and using a constant velocity to estimate the distance travelled in each short interval
- be able to estimate the area under a graph using a Riemann sum, that is, by dividing the relevant interval into small pieces and using rectangles to estimate the area under the graph for each piece

- definite integrals and area (4.2)
  - know what is meant by the definite integral of a continuous function $f(x)$ from $x = a$ to $x = b$ in terms of area, including what happens when the function is negative
  - be able to calculate certain integrals from knowledge of the area of a rectangle, triangle or circle
  - be able to calculate integrals that involve adding or subtracting areas of different regions
  - know the basic properties of integration involving: (a) the integral of a sum or difference of functions; (b) the integral of a constant times a function; (c) the sum of integrals of the same function over different intervals; (d) how the integrals of $f(x)$ and $g(x)$ are related when $f(x) \geq g(x)$

- the Fundamental Theorem of Calculus and antiderivatives (4.3, 3.9)
  - know the statement of the Fundamental Theorem of Calculus (FTC) as stated in class (or what is called Part 2 in the textbook)
  - be able to use the FTC to evaluate integrals by finding an antiderivative for the function to be integrated
  - know how to find the derivative of a function defined by an integral with a variable limit, including the case where the limit is itself a function of the variable (this is Part 1 of the FTC in the textbook)
  - be able to find antiderivatives of power functions (except $x^{-1}$), polynomials, sine, cosine functions
  - be able to rewrite a function to express it as a sum of terms for which the antiderivative can be found (such as by multiplying out parentheses)
  - know that any two antiderivatives of the same function differ by a constant function
  - know that an indefinite integral is notation for the general antiderivative

- integrals and rates of change (4.4)
  - be able to use integration to solve a word problem involving finding the net change in a quantity from its rate of change

This is not a complete list of what you might have to do on the test but it covers most of the ideas involved. In particular, you may have to combine several of these ideas or techniques, and you may need to think to decide what to use to solve a problem.
Preparing for the test

The best way to prepare for the test is to do practice tests. This means you should sit down, without a textbook or your notes, and try to do as much as you can of the practice test in 50 minutes, as though it were the real thing. This will give you an idea of how well prepared you are, what topics you might need to review, and how you react under test conditions. This is especially important if you don’t have much experience taking timed tests, or if you have had anxiety problems with tests in the past. The more practice you do, the better prepared you will be.

You should also go back over past homework problems, especially those for which the grader has written a comment or deducted points and make sure you understand the comment or why you lost points. If you can’t work this out or have any other questions about the grading, please come and ask me about it. I’ll have extra office hours on Wednesday and Thursday next week to help you prepare.

You should also just work through more practice problems. If you didn’t do the practice problems assigned for the homeworks, now would be a good time to do those. If you did, you can make up some more problems on your own (which is also a good exercise to see if you understand the material) and try to solve them. You can always ask me if you are unsure of something.

Beyond that, please let me know how else I can help you prepare, and good luck!