Minimizing Systematic Effects in a Solid-State Electron Electric Dipole Moment Measurement

Kyle A. Virgien

Advisor: Larry R. Hunter

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Foreword

I have always enjoyed forewords, the few lines of ex post facto proem where an author can say whatever it is that he wants to say but for some reason avoided writing in the text. I am particularly fond of the ones written by the great Russian authors—Russians more than anybody else seem to have the ability to write for pages and pages about nothing in particular and yet still to be able to convey some point in the end. As I sit here composing my very own forward, I find it only fitting that I include a quotation from one of my favorites: the prologue to Andrei Bely’s masterpiece *Petersburg*.

*Petersburg not only appears to us, but actually does appear—on maps: in the form of two small circles, one set inside the other, with a black dot in the center; and from precisely this mathematical point, which has no dimension, it proclaims forcefully that it exists: from here, from this very point surges and swarms the printed book; from this invisible point speeds the official circular.*

As I sit here adding the final touches to the work that has consumed so much of my life of late, I find this nonsensical geographic description strangely fitting. The electron also appears to us as merely a dimensionless point, but within the electron lies an entire universe of physics, some of which we are able to fathom, and some of whose profound depths we are still unable to pierce. It may be no more than a point, but it contains a vast profusion of mysteries; from it spring forth mind-bogglingly complex and physically interesting phenomena. And it is one particularly interesting, still unquantified, and fundamentally important aspect of this point that will serve as the subject of the pages to follow.

And as for what I want to say here that never made its way into the body of my thesis, I want to remind you, my reader, how conceptually beautiful the subject of this thesis is. The idea of symmetry is simple and at the same time has far-reaching implications for all of physics. At times this thesis becomes bogged down in details and technical specifics, but I urge you not to lose sight of the underlying elegance that motivates it. I know I never did.
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Introduction

In this thesis, I describe my work on an ongoing experiment here at Amherst College to measure the electron’s intrinsic electric dipole moment (or EDM) in a solid-state system. This experiment has been the subject of four previous undergraduate theses, written by Noah Charney, Oliver Elliot, Ben Heidenreich, and Alex Bridges. In the first three chapters of my thesis, I provide an introduction to the problem that our research tackles and to our experimental methods. In chapters four and five, I present work that I have done to better understand and to minimize two sources of systematic error that have the potential to severely limit the precision with which we make our measurement. Chapter six is a conclusion where I present my own preliminary measurement of the electron’s EDM.
Part I

Background
Between two absolutely different spheres, as between subject and object, there is no causality, no correctness, and no expression; there is, at most, an aesthetic relation: I mean, a suggestive transference, a stammering translation into a completely foreign tongue—for which there is required, in any case, a freely inventive intermediate sphere and mediating force.

-Friedrich Nietzsche, *On Truth and Lies in a Nonmoral Sense*

The problem Nietzsche alludes to here lurks in the foundations of physics. Its laws attempt to provide as absolute a version of reality as possible, but at the same time remain a theoretical system fundamentally distinct from the reality that they describe—they exist as abstract concepts while reality consists in concrete objects. We often think of physical theory as though it has some sort of intrinsic connection to the world around us, but it remains nothing but an educated guess, a construct that we have created in order to bridge the gap between reality and the human understanding of it. Over the years, physicists have built up a rigorous and complex science that we can use to calculate how objects behave in a wide variety of circumstances. Experimental results have refined our understanding of the world around us, and laws and equations have been modified, combined and manipulated as physics has evolved into its present form. However, as complete, far-reaching, and unequivocally true as the laws of physics may seem, all their predictions about the behavior of our world are based on a few assumptions that we have to make about the nature of the universe. We justify these assumptions with the fact that they seem to allow us to do a pretty good job of explaining why things occur as they do, but there is nothing intrinsically or necessarily true about them. No matter how elaborate and polished a system physicists develop, at some level we will always have to make guesses as to nature’s intentions, guesses that have no fundamental justification.

One of these underlying assumptions is that of symmetry: the idea that basic principles are true not just for single isolated events but also for other similar events
as well. There are many different specific types of symmetry, but on a basic level all refer to similarities between two events that allow us to predict that they will unfold in the same way. The idea of symmetry is essential to the study of Physics. Without symmetries it would be impossible to use the outcome of one event to predict the outcome of any other and we could never say anything useful about what will happen in the world around us. Additionally, many laws of physics are directly derived from the assumption of the existence of these symmetries—the laws of conservation of energy and momentum, for example, require the assumption of symmetry of time and space translation\(^1\)—but we have no justification in assuming that these symmetries are true besides the simple fact that the system of physics that we derive from this assumption seems to work to describe the world that we observe. Even beyond physics, we frequently use symmetries in our daily lives without thinking twice; we know that one way to correct a cannon whose cannonball always lands 10 feet short of its target is to move the cannon forward 10 feet, and that when a ball is thrown into the air its return trip to its point of origin will simply be the mirror image in time of its ascent, assuming that friction has a negligible effect. These symmetries seem intuitively obvious to us, but they may not be as universally true as we think they are. A few cases have been observed where some of these fundamental symmetries do not hold, and there are certainly more cases remaining to be discovered. The ultimate goal of the work described in this thesis is to observe a breakdown of some of the symmetries that today’s standard model of physics takes to be universally true. This would represent a case where physics as we know it would fail to accurately explain the world around us, and it would force the laws of physics to evolve in order to better

\(^1\)Noether’s theorem tells us that for each symmetry in a given problem there exists a corresponding law of conservation. The law of conservation of energy follows from the assumption that the universe is time translation symmetric, and the law of conservation of momentum is derived from the assumption that it is space translation symmetric. See [13].
agree with nature.
Chapter 1

Symmetry and Asymmetry

1.1 Symmetry

A symmetry is a claim that two events different from each other in a specific way will share the same outcome. It defines the relationship that they must have and tells us that any two events that are related in this way will experience the laws of physics\(^1\) in the same way, that physics is invariant between the two events. Symmetries are defined mathematically by the transformation that describes this relationship. There are different specific types of symmetry, each based on its own transformation.

Time translation symmetry is a familiar example of a type of symmetry. It is the symmetry that allows us to expect the same result when we perform an experiment and then repeat this experiment in exactly the same way at a different time. It is based on a transformation where an event is moved backward or forward in time, and it tells us that two events differing by nothing but a shift in time will have the same outcome. In other words, the laws of physics do not differentiate based on when

\(^1\)Note that I did not claim here that the two events appear the same. I merely claimed that they would obey the same physical laws. A symmetry would claim that Newton’s second law, for example, is equally true in both the transformed and untransformed worlds that it describes.
something takes place and remain constant for all of time. Mathematically speaking, the operator $\mathcal{O}$ this symmetry is based on,

$$\mathcal{O} : t \to t + a$$  \hfil (1.1)

shifts time wherever it appears in an equation by some fixed amount $a$, and time translation symmetry tells us that applying this operator to the set of equations involved in a problem leaves the problem’s result unchanged.

## 1.2 Symmetries of Inversion

Of particular relevance to this thesis are the three major symmetries of inversion: parity inversion, charge conjugation, and time reversal. These symmetries involve reflections rather than translations and thus cannot be performed continuously. One can imagine a spatial translation, for example, occurring gradually as one’s frame of reference is pushed to a new location. Inversions, on the other hand, involve only two states, regular and inverted, with nothing in between to connect these two states continuously. For each of these transformations the only way to return a transformed system to its original state is to perform the transformation operation again, reflecting the system back to the way it started.

Parity inversion symmetry is based on the operator

$$P : x_i \to -x_i$$  \hfil (1.2)

This operator reflects each spatial coordinate $x_i$ about the origin. A transformation closely related to parity that is much more familiar to us is a mirror reflection. Parity transformation is actually equivalent to a mirror reflection followed by a 180° rotation,
but the important part is the same for both: each creates a mirror-image version of
the world that is fundamentally different from the original version. A right hand
viewed in a mirror is not a right hand somehow rotated or moved around but is a new
object, a left hand. In this same way, the version of the universe that results from a
parity inversion is fundamentally distinct from the version we are accustomed to.

Parity symmetry then tells us that the universe is invariant under this transforma-
tion. According to this symmetry, performing a parity transformation on a problem
leaves its result unchanged, and the same laws of physics apply to the world we see
when we look into a mirror as apply to our own.

Rather than inverting spatial coordinates, the time reversal operator inverts time.

\[ T : t \rightarrow -t \]  \hspace{1cm} (1.3)

This operator switches the direction of time’s flow wherever it occurs in a problem.
Performing a time reversal is equivalent to watching a video of the universe played
backwards; events occur in the same way that they would normally, except time has
switched directions. Time reversal symmetry claims that the fundamental laws of
physics will continue to hold under this transformation, that we can use the system
of physics that we have developed to describe our world in order to explain one with
time reversed.

Charge conjugation symmetry is based on the charge conjugation operator

\[ C : \psi \rightarrow \bar{\psi} \] \hspace{1cm} (1.4)

This operator replaces each particle involved in a problem with its antiparticle, mul-
tiplying every additive quantum number associated with each particle by \(-1\). Charge
conjugation symmetry states that if we perform an experiment and then create another experiment exactly the same as the first one except we have substituted every particle with its antiparticle, both experiments will have the same outcome. There are various additive quantum numbers, but the one that we come across the most frequently is probably a particle’s electric charge. In terms of electromagnetism, charge conjugation symmetry tells us that if we take an event and then create another event exactly the same as the first one except we have substituted every positively charged particle with its negative counterpart and vice versa, both events will unfold in the same way. In other words, the laws of physics do not discriminate between positive and negative charge. A negative charge attracts a positive charge just as a positive charge attracts a negative charge, and two positive charges repel exactly as do two negative charges. Charge conjugation symmetry extends to the other additive quantum numbers as well, but I do not make use of those properties of particles in this thesis and for our purposes here it can be considered to be simply a charge reflector that switches the sign on all charges present in a problem.

In our everyday experience we often encounter translated frames of reference, but we have no first-hand experience with inverted systems. This lack of experience may cause symmetries of inversion to seem less obviously true than symmetries of translation, but both types of symmetry have long played important roles in physics. After some consideration these symmetries seem to make just as much intuitive sense to us as symmetries of translation do. We are unable to interact with the world that we see when we look into a mirror, but it would still seem strange to us if we were to look into a mirror and see something that defies the laws of physics that our unreflected world obeys. Likewise, a video played backwards looks different from what we’re used to, but we would not expect to see anything in such a video that violates the fundamental laws of physics.
1.3 CP Symmetry

Violations of charge conjugation, parity inversion, and time reversal symmetries have been observed. In 1957 it was discovered that the weak force does not obey parity or charge conjugation symmetry[7]. However, soon after this discovery, physicists realized that they could construct a new symmetry, called CP symmetry, that it still obeys. CP symmetry is the combination of charge conjugation symmetry and parity symmetry. It states that the universe is symmetric under the product of a charge conjugation transformation and a parity transformation. That is, even though a parity-inverted world and a charge-conjugated world do not behave in the same way as ours, a world that is both parity-inverted and charge-conjugated can be described by our laws of physics. The weak force obeys this symmetry; however, a violation even of this new symmetry was discovered in 1964 in the decay of K mesons [17]. The K meson’s decay also represents a violation of time symmetry. Although the mechanism for the violation of CP and time symmetry is not yet well understood, it is certain that experiments have shown it is possible to violate each of the three symmetries of inversion, as well as CP symmetry. The experiment discussed in this thesis is an attempt to discover a violation of time and parity symmetries, and, by extension, CP symmetry. We hope that the results of our experiment will help us to gain a better understanding of why CP and time asymmetries exist.

1.4 Apparent Asymmetry

This thesis is an attempt to discover an asymmetry of inversion. I am trying to observe a process that looks fundamentally different reflected and unreflected, and when it is viewed forward in time and backward in time. In order for this experiment
to show anything worthwhile, the process and its reflected twin would need to be *fundamentally* different: it would not be enough for the reflected version to look strange—it would need to violate some law of physics. For example, we can recognize the word “*γισμανιφασ*” as belonging to a mirror-image version of reality and not to our own, but its existence in our unreflected reality does not represent a violation of parity symmetry. It looks strange to us, but there is no law of physics that prohibits writing backwards. We need to dig deeper to find a true instance of asymmetry.

Entropy seems a likely candidate. It is grounded in fundamental facts about our world, whereas our idea of which way words should face is not. It distinguishes between forward and backward in time, and we can use this distinction to recognize a time-reversed universe as different from our own. Our universe moves toward a higher-entropy state, but we would see a universe where time runs backwards move in the opposite direction, toward a lower-entropy state. We would, for example, see that rather than burning houses down, fires construct them from piles of ashes. This universe would look strange to us, and we would certainly be able to tell it apart from ours. However, this reversal of the way entropy changes does not represent a violation of symmetry. The time reversal of a house burning down would look different from things that we are used to, but, as with the word “*γισμανιφασ*”, the fact that the system looks strange to us is not enough to constitute an asymmetry. If we were to look very

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2Entropy is a measure of the number of possible states of objects in a system. Entropy ($S$) is defined as $S = k \ln \Omega$, where $k$ is Boltzmann’s constant and $\Omega$ is the number of microstates available in any given macrostate of a system. Each macrostate has its own entropy value. There is an equal probability of a system being in any given microstate, so there is a greater probability that the system will find itself in a macrostate with more possible microstates than in one with fewer. Since the entropy of a macrostate with more microstates is higher than that of a macrostate with fewer microstates, this probability tells us that a system is more likely to be in a state of higher entropy than one of lower entropy. In a system with a large number of particles, this probability becomes extremely high. The second law of thermodynamics, that there is no process that will cause the entropy of the universe to decrease, represents this extremely high probability. This law is not, then, a law of physics that must always be true, but rather a reflection of a very high probability that events will occur in a certain way.
closely at what is happening as the house reconstructs itself, we would see various molecules and photons colliding with each other in just the right way to form larger molecules, which would assemble themselves in exactly the right way to form a house. Each of these collisions would behave according to the laws of physics; in each case the smaller molecules would collide with each other with just the right momentum and would be given just the right amount of energy by the photons in order to form the larger molecule. We don’t normally see processes that decrease the entropy of the universe, but it is not because they are forbidden by some physical law, but rather because they are extraordinarily improbable. The chances that all of the necessary molecules arrive at the same place at the same time with the right amount of energy and build a house are so low that we will never see this occur. Processes that decrease the entropy of the universe, however, are not prohibited by any law of physics, and, while we would be able to use entropy to identify a time-reversed universe as different from our own, we cannot use it as an instance of asymmetry.

1.5 True Asymmetry

It is possible, however, to imagine a true instance of asymmetry, although we will need to develop some more mathematical terminology in order to describe it.

A vector is defined as any element in a vector space that transforms with its vector space. When its vector space rotates it must rotate accordingly, and when its vector space is reflected about any plane it must be reflected about this plane as well. It is obvious from this definition that any system involving only vectors will automatically obey parity symmetry. Any reflection of the system will reflect all of the vectors that are used to describe the behavior of the system as well, and the effect of any relevant physical law will remain unchanged by the transformation. Consequently, one cannot
describe an asymmetric system in terms only of vectors.

However, there is another type of elements of vector spaces. These elements, called pseudovectors, transform like vectors under rotation, but gain an additional multiplication by $-1$ under reflection. A system consisting of only pseudovectors would also obey parity symmetry. A reflection of this system would result in a different system, but every vector in this new system would differ by the same sign change. The same laws of physics would hold true in the reflected system as did in the unreflected system; any physical equation that would produce a vector would produce a vector multiplied by $-1$, and the sign change would cancel out in any equation that would produce a scalar.

In order to break parity symmetry, we need a system that involves a mix of both vectors and pseudovectors. Under a coordinate reflection, the vector would transform normally but the pseudovector would flip relative to the vector. In the reflected system the vector and the pseudovector would point in different directions relative to each other than they did in the unreflected system.

The asymmetry that we search for in our experiment would arise from the combination of a vector and a pseudovector together in the electron. Electrons are exchange antisymmetric, so two electrons cannot simultaneously exist in any single quantum state. We observe that as the orbitals of an atom in its ground state fill up with electrons, they do so with no more than two electrons in each orbital. This tells us that there must be exactly two quantum states available within each orbital, that there are two different kinds of electrons that can be put in each orbital. We already know that we can distinguish between electrons in a given orbital using their spin. Electrons have two possible spin projections along an axis, $\frac{1}{2}$ and $-\frac{1}{2}$; when an orbital is filled by a pair of electrons one will have a positive spin projection and the other a negative one. This distinguishability on the basis of spin allows electrons to form an otherwise
exchange symmetric pair in each orbital. Furthermore, this distinguishability also rules out the existence of any other factor that we could use to differentiate between two electrons with a given spin. If, for each type of spin, there were two different kinds of electrons, all four species would be able to occupy the same orbital at the same time. However, we never see more than two electrons together in any orbital. Because of this observation we know that there is nothing beyond the spin of an electron that we can use to differentiate it from other electrons within a given orbital. Any quantity that can be used to describe an electron must therefore either be the same for both electrons in a given orbital, or must change in the same way that the spin changes. The electron’s magnetic dipole moment, for example, can vary within a single orbital, and we can also use it to differentiate between electrons; however, its changes are related to changes in the spin. It does not, then, create any additional types of electrons and does not allow additional electrons to fit in an orbital.

The laws of physics tell us that we can only describe two different types of electron, one with positive and one with negative spin projection. This limitation presents us with a system where we can find an instance of asymmetry. If we were able to attach both a vector and a pseudovector to the electron, we would be able to identify four, not two, different types of electron. In our unreflected universe there are two different kinds of electrons, those with positive spins and those with negative spins. Both the electron’s vector quantities and its pseudovector quantities would be coupled to its spin, but under a parity inversion, the vector quantities and the pseudovector quantities would flip relative to each other, and we would be able to differentiate between a normal electron and a parity-inverted one. Both the normal kind of electron and the parity-inverted kind would exist in both a positive spin and a negative spin state. Now, if we allow parity inversion symmetry to hold, we could describe four different possible types of electrons, non-inverted positive spin, non-inverted negative
spin, parity-inverted positive spin, and parity-inverted negative spin. But the laws of physics only allow for two different types (if there could be four types we would expect to see all four in a full atomic orbital). This parity-inverted version of the electron could not be explained by the same laws of physics that govern the original version, and it would represent a violation of parity symmetry.

However, the only directional properties of the electron that physicists are currently able to measure are pseudovectors. Without a true vector that we can attach to the electron, we cannot apply this reasoning, and we will not be able to use the electron as a violation of symmetry.

The spin of an electron is a type of angular momentum and is a pseudovector. If one takes a simplistic view of the electron as a spinning sphere of charge whose direction of rotation is defined by the spin, one can see the pseudovector nature of its spin.
CHAPTER 1. SYMMETRY AND ASYMMETRY

Figure 1.1: Reflecting the electron over a plane perpendicular to its spin leaves the spin unchanged, and reflecting it over a plane parallel to its spin causes the spin to switch directions.

If one were to reflect the electron over a plane perpendicular to its spin vector, the reflected version of the electron would still be spinning in the same direction and its spin vector would be unchanged; if one were to reflect it over a plane parallel to its spin, the reflected version would spin in the opposite direction and its spin vector would be multiplied by $-1$. In both cases, when we compare how the electron’s spin transformed with how a true vector would transform, we find that it has gained an additional multiplication by $-1$, confirming its pseudovector nature.

The electron’s magnetic dipole moment is the only other factor we know of that we can use to distinguish between two electrons. Magnetic fields are created by moving charges, and the electron’s magnetic field is created by the motion of the negative charges on our simplistic model’s sphere as it rotates. Its magnetic dipole moment
arises directly from its spin, and we can determine the direction it will point in by looking at the direction of the spin. If one were to reflect the sphere, the magnetic dipole would transform in the same way as the spin, defining it, too, as a pseudovector.

The electron as we know it is symmetric. If we perform a parity transformation on it, every directional property of the electron that we are able to measure will transform in the same way, and we will be left with an electron that appears fundamentally the same as the one we started out with. Similarly, if we were to perform a time reversal on the electron, the direction of its spin would reverse

\[ -1 \]

—imagine looking at a video of a spinning ball of charge played backwards—but its magnetic dipole moment would also reverse accordingly because the moving charge giving rise to it in our simplistic model of the electron would have reversed direction.

What we would need to find in order to observe an asymmetry is something no one has ever been able to measure, a true vector that describes some property of the electron. The vector we attempt to measure in the experiment I describe in this thesis is the electron’s electric dipole moment (EDM).

Figure 1.2: Reversing the direction of the flow of time also serves to multiply the electron’s spin by $-1$. 

—an image showing the direction of spin changing with time reversal.
1.6 Electric Dipole Moments

An electric dipole is a separation of positive and negative charge over some distance. The dipole moment is a way to measure this separation. It is a vector that points from negative charge to positive charge, and its magnitude represents the strength of the dipole.

\[
\vec{d} = (q_2 - q_1) \vec{r}
\]  

Figure 1.3: An electric dipole moment is a vector that points from negative charge to positive charge.

The electric dipole moment \( \vec{d} \) of two point charges \( q_1 \) and \( q_2 \) is related to the difference between the charges and the displacement between them \( (\vec{r}) \).

Bigger differences in charge and greater distances between the charges will result in a stronger dipole and a larger electric dipole moment.

We are looking for the electric dipole moment of the electron. It might seem as though the electron, which is composed entirely of negative charge, could not have a dipole moment, which requires a separation of positive and negative charge. But even charged objects can have dipole moments. In this case, a dipole moment would not mean that the electron has a positive end and a negative end, but that one end is less negative than the other. This is like superimposing a small amount of positive charge on one of its ends and a small amount of negative charge on the other.

Of course, the electron is a dimensionless particle, so it is not entirely correct to think of it as having ends that have different amounts of charge. It may have a dipole
moment, but this dipole moment doesn’t necessarily reflect the separation of charge over any distance—all of the electron’s charge is confined to a single point in space. Equation 1.5 is a useful tool to visualize what an electric dipole is, but we will need to develop a more mathematically rigorous model in order to describe the electron as an electric dipole.

In order to develop this model we will examine the electric potential resulting from a charge distribution $\rho(\vec{r}')$, where $\vec{r}'$ is the displacement from the origin of a point in the charge distribution. We can represent the potential as a function of position from the origin ($\vec{r}$)

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{1}{|\vec{r} - \vec{r}'|} \rho(\vec{r}') \, d\tau$$

(1.6)

where $d\tau$ is an infinitesimal element of volume. $|\vec{r} - \vec{r}'|$ can be rewritten using the law of cosines

$$|\vec{r} - \vec{r}'|^2 = r^2 + r'^2 - 2rr' \cos \theta$$

(1.7)

where $\theta$ is the angle between $\vec{r}$ and $\vec{r}'$. Solving for $|\vec{r} - \vec{r}'|$, 

$$|\vec{r} - \vec{r}'| = r \sqrt{1 + \frac{r'}{r} \left( \frac{r'}{r} - 2 \cos \theta \right)}$$

(1.8)

Inserting this expression back into equation 1.6 we obtain

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{1}{r \sqrt{1 + \frac{r'}{r} \left( \frac{r'}{r} - 2 \cos \theta \right)}} \rho(\vec{r}') \, d\tau$$

(1.9)

If we assume that the charge distribution is close to the coordinate origin and concern ourselves only with the potential at points far away from it—that is, points far enough away that the charge distribution’s dimensions are insignificant relative to the distance between it and the point—we can simplify this equation with an approximation. At
these points $\frac{\rho'}{r}$ is much less than 1, and in turn $\frac{\rho'}{r}(\frac{r'}{r} - 2\cos\theta)$ is also much less than 1. We can use a binomial expansion on the term $\frac{1}{\sqrt{1 + \frac{\rho'}{r}(\frac{r'}{r} - 2\cos\theta)}}$ to obtain

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{1}{r} \left[ 1 - \frac{1}{2} \left( \frac{r'}{r} \right) \left( \frac{r'}{r} - 2\cos\theta \right) + \ldots \right] \rho(\vec{r'}) d\tau$$

(1.10)

or, neglecting all terms higher than second order in $\frac{1}{r}$,

$$V(\vec{r}) \approx \frac{1}{4\pi\varepsilon_0} \int \frac{1}{r} \left[ 1 + \frac{r'}{r} \cos\theta \right] \rho(\vec{r'}) d\tau$$

(1.11)

Now we can write the field produced by this distribution of charge as the field that would be produced by a point charge at the origin, along with another term to correct for any deviations from this field

$$V(\vec{r}) \approx \frac{1}{4\pi\varepsilon_0} \frac{1}{r} \int \rho(\vec{r'}) d\tau + \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} \int r' \cos\theta \rho(\vec{r'}) d\tau$$

(1.12)

The second term in this equation is the dipole potential, and, since the electric dipole moment is defined for a distribution of charge as

$$\vec{d} = \int \vec{r} \rho(\vec{r'}) d\tau$$

(1.13)

we can rewrite the potential in terms of the distribution’s total charge and electric dipole moment

$$V \approx \frac{1}{4\pi\varepsilon_0} \left( \frac{Q}{r} + \frac{\vec{d} \cdot \hat{r}}{r^2} \right)$$

(1.14)

[18, §§3.4.1-2]. In the case of the electron we can express this potential as

$$V \approx \frac{1}{4\pi\varepsilon_0} \left( \frac{e}{r} + \frac{\vec{d}_e \cdot \hat{r}}{r^2} \right)$$

(1.15)
where \( e \) is the electron’s charge and \( \vec{d}_e \) is its EDM. Equation 1.15 makes no reference to the electron’s dimensions,\(^3\) and it will better serve to describe its dipole.

The electron’s charge has been precisely measured and its contribution to the electron’s potential is well-known; however, the contribution of the second part of equation 1.15 has never been measured. Our experiment attempts to characterize the electron’s potential precisely enough to see the effects of the electron’s EDM. If we succeed in measuring these effects, we will have observed a violation of both parity and time reversal symmetry.

![Figure 1.4](image)

Figure 1.4: An electron with an EDM is different from its mirror reflection. In this example, the EDM and the spin point in the same direction in the unreflected electron and in opposite directions in the reflected version. Parity inverting an electron with an EDM is not the same as rotating it, but creates an entirely different kind of object that we do not see in our unreflected universe. This new type of electron is not allowed by the laws of physics that govern our unreflected universe, so an electron with an EDM would represent a violation of parity symmetry.

\(^3\)Although equation 1.15 does not require that the distribution of charge have some dimension, equation 1.13, from which it was derived, does. If a point particle like an electron can have a dipole moment, then my definition of the dipole moment above is incomplete. Rather than thinking of a dipole moment as related to the spatial distribution of charge, we should picture it as nothing more than the property of a charge distribution that causes it to produce the potential described by the second term in equation 1.15.
§igure 1.5: The time reversal of an electron with an EDM would also result in a fundamentally different object. In this case, a time reversal would cause the electron’s spin to switch directions but would leave its EDM unchanged. This pair of electrons, one normal and one time-reversed, could not exist together. The existence of an electron EDM would therefore violate time reversal symmetry as well.

1.7 Why Asymmetry Makes Sense

So far I have explained what the electron EDM is and what its existence would imply, but there is one more question that I have yet to answer—why is it worthwhile to try to measure it? We want to discover an electron EDM because it would violate the standard model of physics, but why would we expect to see something that the laws of physics as they stand claim is impossible?

Several arguments can be made for the existence of a time or CP violation in the electron. One such justification is based on the matter-antimatter asymmetry of our universe. According to the big bang model of the formation of the universe, matter came into existence in the universe’s early stages from a state of extremely high energy density. All matter was created in particle-antiparticle pairs, and consequently it must have been balanced by an equal amount of antimatter. However, all available evidence suggests that our modern universe is composed predominantly of matter. Over the course of its lifetime, some of the antimatter that was so abundant in its early stages
must have been converted into matter. But this conversion of antiparticles to particles would violate the symmetries of charge conjugation and time reversal [10]. This argument isn’t perfect—we cannot be sure that there is no antimatter in some other part of our universe that compensates for the matter where we are—but it provides an elegant and intuitive reason to believe in the existence of symmetry violations in everyday particles. Furthermore, the minor symmetry violations associated with the standard model of physics are too small to explain the degree of the matter-antimatter asymmetry that we observe in our universe. 4

The instances of time and CP symmetry violation that have already been observed provide additional justification for the existence of an electron EDM. The standard model of physics fails to describe these instances, and various theoretical modifications that take these instances of asymmetry into account have been proposed. Each of these explanations also predicts a symmetry violation in the electron.

These modifications to the standard model of physics also place constraints on the magnitude of the electron EDM.5 Each theory predicts a different value, as shown in table 1.1.

<table>
<thead>
<tr>
<th>Standard Model Extension</th>
<th>Electron EDM Prediction (e-cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Model</td>
<td>$&lt; 10^{-40}$</td>
</tr>
<tr>
<td>Supersymmetry</td>
<td>$&lt; 10^{-27}$</td>
</tr>
<tr>
<td>Left-Right Symmetric</td>
<td>$10^{-26}$ to $10^{-28}$</td>
</tr>
<tr>
<td>Higgs Dynamics</td>
<td>$&gt; 10^{-27}$</td>
</tr>
<tr>
<td>Lepton Flavor Changing</td>
<td>$10^{-27}$ to $10^{-29}$</td>
</tr>
</tbody>
</table>

Table 1.1: electron EDM values predicted by various models of physics (from [11, 30])

If we measure an electron EDM, we will determine which of these predictions it lies within, providing justification for the corresponding theories. Even if we fail to

---

4Explanations directly linking an electron EDM with the mechanism for this matter-antimatter symmetry under two different speculative theories can be found in [16, 6].

5For a derivation of some of these constraints, see [2, 27].
measure a non-zero electron EDM, provided that we make our measurement with enough precision to improve on the value’s upper limit, we will be able to rule out any theory that predicts a higher value. Electron EDM experiments are one of the few tests where physicists can achieve enough precision to examine these corrections to the standard model, some of which have far-reaching implications for theoretical physics. Electron EDM limits have already been successful in ruling out some of the theories with larger predictions, and even a modest improvement over the current limit would be able to rule out a few more. We don’t know whether any of these standard model modifications is correct or whether we will discover an electron EDM, but we are certain that improvements in the precision with which we can measure this quantity will help to hone our understanding of physics beyond the standard model.
Chapter 2

The Experiment’s Design

2.1 The Solid-State Approach

Most past experiments attempting to measure the electron’s electric dipole moment have made their measurements on the electrons of atoms in an atomic beam or a gas. This type of experiment examines a relatively small number of electrons spread over a large space. This approach has some advantages in that it is not prone to systematic errors due to interactions between atoms, but it is limited by the simple fact that it involves making measurements on a very small number of electrons. An experiment undertaken at Berkeley refined this method as much as is probably possible and still failed to measure a non-zero electron EDM [1]. This experiment set what is currently the lowest upper bound on the electron EDM, but if we are to further sharpen this limit, we will probably need to use a different method to do so. Various new efforts to make this measurement are currently underway, but thus far no experiment has been successful in achieving a precision greater than that of Berkeley’s atomic beam experiment. The experiment described in this thesis is an attempt to use a solid-state system to improve on this precision. It was one of the first modern solid-state
electron EDM experiment to be constructed,\textsuperscript{1} although there are currently a few more experiments exploring similar approaches.

The basic idea behind the solid state approach to this problem is a simple one. In order to make our measurement of the electron’s EDM, we squeeze as many electrons as possible into a given space and make our measurement on the group of electrons as a whole. This approach allows us to examine a larger number of electrons than we could in an atomic vapor, dramatically improving the experiment’s statistics. In order to achieve the high density of electrons that this method requires, we have to make our measurement on electrons in a solid. This approach has some drawbacks because solids suffer from effects due to interactions between individual atoms that might mask or mimic what we are trying to measure, but the simple advantage of a higher density of electrons may allow this type of experiment to improve upon the precision of measurements made using other methods.

\section{Measuring an Induced Voltage}

If the electron were to have an EDM, we should be able to measure an electric potential drop corresponding to its dipole moment. This potential drop would be minuscule, and it would be impossible to use a conventional voltage detector to measure it. However, if we can line up the EDMs of a large number of electrons, all of the individual EDMs will add to form a much larger potential drop that might be possible to measure. This idea forms the basic concept of our experiment.

\textsuperscript{1}In 1978, a group of Soviet scientists made a solid-state electron EDM measurement, achieving a result of $|d_e| < 2.0 \cdot 10^{-22}$ electron-centimeters [35]. Their experiment was crude by our standards, and we have already made preliminary measurements many times more precise than those obtained in this experiment.
Figure 2.1: In order to create a potential drop that is large enough to measure, we align the EDMs of a large number of electrons. Within the bulk of our group of electrons, the positively charged end of each will meet the negatively charged end of its neighbor and the difference in charge due to each electron’s EDM will cancel out. There will be a net difference in charge, however, across the group of electrons as a whole because the electrons at the group’s surfaces don’t have neighbors to cancel the effective surface charge that their dipole moments create. One surface will be composed of the positive ends of the electrons’ EDMs, and the opposite surface will be contain only the negative ends, causing an effective difference in charge between the two.

But how can we force our electrons to align so that they all share the same orientation? You will recall from my discussion in §1.6 that the electron is defined by the directions of both its electric and magnetic dipole moments. The two must point in the same relative direction, either parallel or antiparallel, for all electrons—a
CHAPTER 2. THE EXPERIMENT’S DESIGN

particle with a different relative orientation of the two would be distinguishable from an electron and would therefore not be an electron. In our experiment we use an external magnetic field to magnetize our solid, aligning the magnetic dipole moments of the electrons whose EDMs we are going to measure. By aligning these electrons’ magnetic dipole moments, we force their EDMs to line up in the same direction, either parallel or antiparallel to the applied magnetic field. With their EDMs all facing in the same direction, the electrons of our solid will generate a potential difference between the solid’s two ends. On a basic level, our experimental method is quite simple: we apply a magnetic field to a solid, causing its electrons to align, and then we measure the resulting potential drop.

To correct for small offsets in our voltage detector we flip the magnetic field back and forth and measure the change in the potential difference across the solid after the magnetic field switches. Any bias voltage in our detector would add in the same way to the signals that we measure for the two magnetic field orientations, but a voltage arising from the electron’s EDM would change in sign from one to the other. We can therefore use the potential drops for the two field positions ($V_+$ and $V_-$) in order to find the potential difference due to the electron EDM ($V_{EDM}$) while screening out any extraneous detector offset voltage ($V_D$)

$$\frac{V_+ - V_-}{2} = \frac{(V_{EDM} + V_D) - (-V_{EDM} + V_D)}{2} = V_{EDM}$$

2.3 Atomic Shielding of the Electron’s EDM

In order to determine the value of the electron’s EDM from our experimental data, we will need to develop a way to interpret the potential drop that we measure across our sample in terms of the electron’s EDM. It might seem as though we could simply add
up each electron’s contribution to the overall potential drop across the solid in order to find a conversion factor that we could use to find the electron’s EDM from the potential drop across the sample. However, this method is not correct. The electrons whose EDMs we measure are contained within atoms, and the other particles that inhabit these atoms modify the potential produced by their electrons.

It appears at first glance that the other charged particles in the atom would shift their positions in order to compensate for the electron’s EDM, acting to cancel it out. Schiff’s theorem tells us that in a non-relativistic atom this canceling is complete—the electron’s EDM is completely shielded by its atom’s other resident charged particles [31]. However, if we are to take relativity into account, there is a modification that we must make to the prediction made by Schiff’s theorem. In most atoms, this correction is relatively minor and their electrons’ EDMs are well shielded. However, large atoms, whose electrons have more energy, experience relativity’s effects more strongly. The correction factor becomes more important, and the effects of the electron’s EDM become much more noticeable in these atoms. I am not going to go into the details of the calculation of the magnitude of the effect an atom has on its electrons’ EDMs, but will instead simply state the result of a calculation in [20, §6.2]. The atomic EDM can be represented in terms of its electrons’ EDMs as

\[ d_{\text{atom}} \sim (Z^3 \alpha^2) d_e \]  

(2.2)

where \( Z \) is the atom’s atomic number and \( \alpha \) is the fine structure constant. We call the constant of proportionality between the atomic EDM and the electron’s EDM the \textit{atomic enhancement factor}.

This atomic enhancement factor scales as \( Z^3 \); heavier atoms amplify their electrons’ EDMs to a much greater extent than lighter atoms do. For this reason, we
want to use an atom with as large an atomic number as possible in our measurement. We choose gadolinium in its Gd$^{3+}$ ionic state because it has both a relatively high atomic number of 64 and seven unpaired $f$ orbital electrons that we can simultaneously align.\footnote{In our data analysis, we make use of a more detailed calculation of this enhancement factor, which can be found in \cite{29, 34}.
}

## 2.4 Gadolinium Iron Garnet

We make our measurement on the Gd$^{3+}$ ions in a polycrystalline sample of gadolinium iron garnet, or GdIG.\footnote{For an introduction to the properties of rare-earth garnets like GdIG, see \cite{21, 441-442} and \cite{9, 212-217}.} This crystal is ferrimagnetic, so we can use an applied magnetic field to give it a permanent magnetization. Its high resistance allows us to be confident that our small signal will not be lost through sample conduction, and it has an additional useful thermal property that I will explain in §2.7.

GdIG is an excellent material to use in a measurement of this type, but it has a drawback in that the other atoms in its crystal structure partially shield the EDMs of its gadolinium ions. Its overall enhancement factor was calculated in \cite{28, 33}, and it was found that the potential difference due to the electron EDM across a completely magnetized sample at room temperature would be $-\frac{1}{\epsilon_0}3\mu n_{Gd}l$, where $\epsilon_0$ is the electric constant, $n_{Gd}$ is the number density of gadolinium ions in the crystal, and $l$ is the sample’s length.\footnote{Alex presents an in-depth discussion of this calculation in his thesis \cite[§2.2.2-3]{3}.} This calculation ignores the effects of the EDMs of the electrons in our GdIG’s iron ions, which is an acceptable approximation because iron has a small atomic enhancement factor relative to that of gadolinium. Scaling this factor to adjust for the fact that the magnetic susceptibility of GdIG is lower at the low temperatures where we take our data,\footnote{I show this dependence of magnetic susceptibility on temperature in figure 2.2.} we achieve a factor relating the electron EDM

\[
-\frac{1}{\epsilon_0}3\mu n_{Gd}l
\]
to the potential drop across our sample of $1.8 \cdot 10^{-17}$ e·cm/V.

### 2.5 Our Experiment’s Geometry

To review, here is the model of our experiment that we have at this point in this thesis: we magnetize a cylinder of GdIG with an applied magnetic field, flip the magnetic field, and measure the potential drop across the sample in both magnetic field configurations. We can use these two potential drops to screen out any bias voltages and to find the potential drop across the sample that is due to the electron’s electric dipole moment. We can then use the atomic enhancement factor of Gadolinium and the correction factor for the structure of GdIG to find the electron’s EDM from this potential drop.

Unfortunately, this model of our experiment as I have described it would not work. In order to magnetize our GdIG sample we would need to apply high magnetic fields to it, and we would need to flip these fields back and forth in order to make our measurement. However, it is impossible to apply a magnetic field to a cylindrical object without creating a stray field in the area around the cylinder. Magnetic fields, unlike electric fields, have no sources or sinks, so any field line traveling out one end of the cylinder will have to loop back to reach the cylinder’s other end. These stray fields will act as demagnetizing fields, making it difficult to magnetize the cylinder, and will also create large induction effects in the sensitive electronics that we need to use to measure the small voltage produced by the electrons’ EDMs.

We avoid these effects with a careful choice of GdIG sample geometry. By choosing a toroidal shape for our sample, and for the fields that we use to magnetize it, we allow our applied magnetic field to close on itself. Its field lines remain entirely within the sample, and it will create no stray field or demagnetizing field. However, while
this sample geometry fixes the problem of stray fields, it creates another in that a toroid has no ends that we could measure a potential drop across. We can align all of the electrons so that they point around the toroid, but this leaves us with no surfaces with only positively charged or only negatively charged ends of the dipole that we can use to make our measurement. We rely on one final useful property of GdIG in order to solve this problem.

2.6 Temperature Dependence of Sample Magnetization

GdIG is a ferrimagnetic crystal. That is, the various magnetic ions within the crystal’s structure point in different directions when it is magnetized. In the case of GdIG the magnetic dipole moments of the crystal’s gadolinium ions, along with those of some of its iron ions, line up in one direction, and the rest of the iron ions’ magnetic dipole moments line up in the opposite direction. The crystal’s net magnetization is determined by the relative strengths of the iron and gadolinium ions’ magnetizations, which in turn depend on the sample’s temperature.

Below a certain temperature, called the compensation temperature, the gadolinium ions’ magnetization is strong enough that the crystal’s total magnetization will point in the direction their magnetic dipole moments are pointing. Above the compensation temperature, the iron ions that point in the opposite direction are able to overcome the gadolinium ions’ magnetization, and the crystal’s total magnetization will point against that of the gadolinium ions. This effect allows us to use the temperature of our sample to choose whether the gadolinium ions’ magnetic dipole moments are pointing with the sample’s magnetization or against it.
CHAPTER 2. THE EXPERIMENT’S DESIGN

2.2. GADOLINIUM IRON GARNET

Figure 2.2: Partial crystal structure of an iron garnet, showing the immediate environment of the rare-earth ion. \( \text{vAfter [24, Figure 5.8b].w} \)

Figure 2.3: Magnetization of the GdIG sub-lattices with temperature. Total magnetization is the dashed curve, and its sign indicates the direction relative to gadolinium. \( \text{vAfter [24, Figure 5.9].w} \)

The magnetizations of the various ions within the GdIG crystal depend differently on the temperature of the crystal. This plot shows the contributions of each type of ion to the crystal’s total magnetization (represented by the dashed line). At the compensations temperature, the three ions’ magnetizations add to zero. Above it, the magnetization of the (d) iron ions dominates; below it, that of the gadolinium and (a) iron ions dominates. The letters a, d, and c refer to the ions’ location within the crystal’s lattice, a consideration with which we do not need to concern ourselves here, except to note that the location of the iron ions determines whether their magnetization points with or against that of the gadolinium ions [9, figure 5.9].

Through this property of GdIG, we can control the direction of the potential drop that the electron’s EDM induces across our sample. Because the direction of the gadolinium ions’ magnetic dipole moments depends on the temperature, the direction in which the electron’s EDM magnetizes our crystal must also depend on temperature in the same way. The potential drop across our GdIG sample due to the electron’s EDM will therefore switch from positive to negative when we pass through the compensation temperature.

We can further control our sample’s ions’ magnetizations by adjusting the compensation temperature. Yttrium iron garnet, or YIG, has many of the same properties
as gadolinium iron garnet, but Y$^{3+}$, the ionic form of yttrium in YIG’s crystal lattice, is nonmagnetic. The crystals that we use in our experiment are not pure GdIG, but instead are composed of a mixture of GdIG and YIG, with yttrium ions and gadolinium ions spread randomly in a given proportion throughout the crystal lattice. In our mixed crystals, there are not as many gadolinium ions, and thus gadolinium’s contribution to the sample’s overall magnetization is less. The iron ions opposing the gadolinium ions’ magnetization will be able to overcome it at lower temperatures than they would in a crystal containing no yttrium. GdIG has a relatively high compensation temperature of 296 K, but the two mixtures of GdIG and YIG that we use in our experiment have compensation temperatures of 104 K and 153 K.

When we construct our toroid, we do so in two halves, one of each GdIG-YIG mixture. We place electrodes between the two halves at the two points where they meet. Each half has a different compensation temperature, so as we increase the sample’s temperature, holding fixed the direction of the sample’s magnetization, we can individually flip the potential drop across each half of the toroid. When the toroid is below 104 K, both halves will be electrically polarized in the direction of the toroid’s magnetization, and their potential drops will cancel out. However, when the toroid is at a temperature between 104 K and 153 K, one half will be polarized with the toroid’s magnetic field, and one will be polarized against its magnetic field. Within this temperature range, the potential drops across the two halves of the toroid will add rather than canceling, and we should observe a potential difference between our two electrodes. Above 153 K, both halves will be polarized against the magnetic field, and the potential drops across the two should once again cancel. By using temperature to tune our signal, we are able to measure a potential drop between our two electrodes without sacrificing the benefits that a toroidal geometry provides our experiment.
CHAPTER 2. THE EXPERIMENT'S DESIGN

Figure 2.3: Below 104 K, the gadolinium ions’ spins in both of the toroid’s halves line up with the applied magnetic field and any signal from the electron’s EDM on one half will cancel with the signal on the other.

Figure 2.4: Between 104 K and 153 K, the gadolinium ions’ spins in one half of the toroid align with the applied field, but the ions’ spins in the other half have flipped direction and now line up against it. The signals on the two halves of the toroid will add and we will be able to measure a potential difference between the two electrodes sandwiched between the toroid’s two halves.
Figure 2.5: Above 153 K, all of the gadolinium ions’ spins align against the applied field, and once again any signal from the electron’s EDM on one half of the toroid will cancel out with its complement on the other half.

Our experimental apparatus’ dependence on temperature also presents us with another advantage. As we change the temperature at which we run our experiment we effectively turn on and off the signal that we will see due to the electron’s EDM. This serves as an excellent check for systematic errors: if we see a signal that does not disappear at temperatures where we would expect not to see one, then we know it is erroneous, and if we see one that we can successfully turn on and off by varying the temperature at which we take data we can be more certain that it is a result of the electron’s EDM and not of some extraneous source of error.
Chapter 3

Experimental Methods

The solid-state electron EDM apparatus that I use in my thesis has been built, refined, and developed over the past seven years through the thesis work of Noah Charney [8], Oliver Elliott [15], Ben Heidenreich [19], and Alex Bridges [3]. It would take much more space than I am willing to devote here to describe every detail of our apparatus, so I will present a brief overview of its elements most pertinent to my thesis. More specific information can be found in these other theses.

3.1 Magnetizing the GdIG Sample

In order to make our measurement of the electron’s EDM we need to magnetize our sample in one direction, measure the potential drop across its two electrodes, flip the direction of its magnetization, and then make another measurement of the potential drop. We magnetize it by running current through a toroidal electromagnetic coil wrapped around the Faraday cage that contains our sample. The simplest scheme we could use to magnetize our sample would be to apply a magnetic field in one direction, make a measurement, switch the direction of the applied field, and then
make the second measurement. We, however, use a slightly more complicated method to magnetize our sample. We apply the magnetic field in short pulses, magnetizing the sample, and then make our measurement of the potential drop across the sample while the applied magnetic field is turned off. Figure 3.1 is a schematic view of how the pulse we apply relates to the sample’s magnetization.

![Figure 3.1: We magnetize our sample with a pulsed applied magnetic field. We apply a field in one direction until the sample magnetizes, and then we shut it off. Our sample retains the magnetization that it gained from this field pulse while we make our measurement. We then apply another field pulse in the opposite direction, reversing the sample’s magnetization. We measure the potential drop across the sample one more time with the direction of the magnetization reversed, and then we repeat this process. The units in this graph are arbitrary.](image)

Although we apply the magnetic field to our sample for a small portion of the cycle, the sample remains magnetized in the same direction as the applied field until we apply another pulse to flip the direction of its magnetization. Because it is a ferrimag-
CHAPTER 3. EXPERIMENTAL METHODS

net, it will retain any magnetization that it gains until it experiences a magnetic field opposing the direction of its magnetization, and since it is in the shape of a toroid, it does not suffer from self-demagnetizing fields that would cause its magnetization to decay.¹

There are two major advantages to applying the magnetic field in short pulses. First, it allows us to apply much larger magnetic field values than would be possible otherwise. As we run current through our coil, it dissipates power as heat. The heat released by the coil when we turn on our magnetic field causes the Faraday cage surrounding our sample to warm, in turn heating the GdIG sample. Our sample’s magnetic permeability depends on its temperature, and allowing it to heat could adversely affect our data. Another potential problem if our sample were allowed to heat would be the formation of thermal gradients across the sample. These gradients could cause extraneous electric effects, hampering our precision measurement.² We can regulate the temperature of the Faraday cage to a certain extent, but our apparatus can absorb only a limited amount of heat from the magnet before it stops being able to maintain the cage’s temperature. If we were to leave our magnetic field on while we make our measurement, we would be unable to apply a magnetic field larger than 53

¹My description here is only valid when the sample is at 127 K and the magnetic susceptibilities of its two halves match. At 88 K and 178 K, one of the two halves will be more susceptible to the applied field and will gain a greater magnetization than the other half. However, this unbalanced magnetization will give rise to a demagnetizing field on the half of the toroid with the greater magnetization. Once we turn off our applied field, this demagnetizing field will cause the more magnetized half to decrease in magnetization until the two halves are equally magnetized. This process takes a very short amount of time, and by the time that we begin to take data the two halves have the same magnetization. We chose 88 K and 178 K as the temperatures where we would take null data because they are the temperatures where the less susceptible half of the toroid has the same magnetic susceptibility as the toroid does at 127 K. Once the toroid’s magnetization has relaxed to that of its less susceptible half, it will have the same magnetization as it did at 127 K. This standardization of the toroid’s magnetization allows us to make better comparisons between data taken at different temperatures.

²There are multiple possible mechanisms through which thermal gradients could effect our signal. One example is the Seebeck effect, where charge carriers would gain different thermal energies and would distribute themselves unevenly across the thermal gradient as a result. For an overview of various galvanomagnetothermoelectric effects, see [5, 190-193].
Oersted to magnetize our sample. However, since our pulsed magnetic field is applied for a small amount of time relative to the period when it is turned off as we make our measurement, it dissipates much less heat for a given value of the applied field. We can therefore reach much higher applied fields, as large as 425 Oersted, before being limited by heating of our sample.

The other benefit of a pulsed applied field is that our electronics will not be disturbed by the field while we are making our measurement. The field that we apply should be mostly confined to the toroidal shape enclosed by the magnetic coils, but it will inevitably produce some stray remnants. The first stage of our electronics has an extremely high input impedance and sits just above the magnet; it is potentially susceptible to the effects of such stray magnetic fields. If we were to keep our magnetic field on while making our measurement, small fluctuations in the field could conceivably affect the data that we take. We expect our signal to reverse with the sample magnetization, so effects of stray fields could mimic the signal that we are trying to observe. We can use mathematical legerdemain to remove effects that cause random noise or bias voltages, but this type of effect has a good chance of making its way into our final result. Because the magnetic field that we apply is turned off when we make our measurement, we avoid this potential problem.

### 3.2 Inductive Effects

Before we can make our high-precision measurement of the potential drop that we create when we magnetize our sample we must take one more source of error into account. Maxwell’s laws tell us that changing magnetic fields give rise to electric effects, and the magnetic field pulses that we apply to our sample involve rapid changes in the magnetic field in the region where we are making our measurement.
Our apparatus contains Faraday shields that prevent these effects from reaching the sensitive parts of the detector, but even so they can make their way into the signal that we measure. During the part of the cycle where we make our measurement, our applied field is turned off and there are no inductive effects to disturb our signal. However, induction effects during the field pulses continue to disrupt our electronics for a long enough time after the pulse that they would continue to mask any signal during the measurement phase of our experiment’s cycle.

We avoid problems due to inductive effects by disabling our electronics during magnetic field pulses. We switch off our detector, apply a pulse, and then turn the detector on again when it is time for us to make our measurement. We call the signal that turns our detector off the blanking signal. Figure 3.2 is a schematic view of how we configure this signal.\(^3\)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{blanking_signal.png}
\caption{Blanking Signal and Magnetic Field Pulse}
\end{figure}

Figure 3.2: In order to minimize inductive effects we turn our detector off during the magnetic field pulses. This figure shows the blanking pulses that disconnect the detector from the electrodes during pulses.

\(^3\)Ben provides a thorough discussion of this issue in his thesis [19, §6.3].
CHAPTER 3. EXPERIMENTAL METHODS

3.3 The Measurement

When we magnetize our sample, the electron’s EDM generates a potential difference between the two electrodes. This potential drop is extremely small, and we need to use a precise detector in order to measure it. Our detector is designed in stages, each amplifying and enhancing the signal before passing it along to the next stage.

We are trying to measure the potential drop across our two electrodes, so in theory we only need to measure that single potential drop. In practice, however, it is to our advantage to independently measure each electrode’s potential, using ground as a reference potential, and then compare the two potentials mathematically. We construct our detector with two identical channels, one for each electrode, and in our analysis we subtract the value of the potential on one electrode from its value on the other.

The first stage of our detector is the preamplifier. Because the signal on our electrodes is so weak, the preamplifier must sit immediately above the sample so that we do not lose the signal as it travels from the electrodes to the detector. The sample must be kept at low temperatures, and our preamplifier must be able to function at temperatures near those of the sample. The two electrodes where we measure our signal are insulated from each other by our sample, so they have an output impedance equal to the sample’s resistance. The sample has an extremely high resistance on the order of $10^{15} \, \Omega$, and if we are to take full advantage of this high resistance, we need our preamplifier to have an input impedance close to this value.

Our preamplifier is based on a JFET cascode pair—two transistors connected to each other so as to create an extremely stable amplifier that takes an input voltage and amplifies it into an output current [26, §3.8].
JFET cascode pairs can be constructed to be very precise, and they serve very well to amplify our small signal. Our JFETs are also reliable at low temperatures and, because they measure the voltage across a reverse-biased junction, have very high input impedances [26, §1]. Since the JFETs themselves have such high input impedances, we can control the input impedance of the preamplifier by placing a resistor of our desired impedance across the cascode pair’s input. We choose a resistor of $10^{13} \, \Omega$.

Of course, one necessary drawback of a preamplifier that has such a high input impedance and that measures such a small signal is that it is highly vulnerable to electromagnetic interference. In order to mitigate this source of noise, we enclose the preamplifier in a Faraday cage.
Figure 3.4: Our preamplifier sits immediately above our sample (the toroidal coil wrapped around our sample’s Faraday cage is visible here). In the picture on the left, the preamplifier circuit is visible. In the picture on the right, it is enclosed by its Faraday cage.

The signal leaving the preamplifier has been amplified and is low impedance. We feed this signal out of the part of our apparatus that we keep at low temperatures, and into the second stage of our detector. This stage consists of a high-precision op-amp configured as a simple linear amplifier, along with a five-pole Bessel filter designed to remove high-frequency noise from our signal. The signal from the main stage of our detector is then read as a function of time by an oscilloscope and transmitted to a computer, where the data are stored for analysis.
CHAPTER 3. EXPERIMENTAL METHODS

Figure 3.5: Our detector contains a JFET cascode pair preamplifier and a second stage with a high-precision operational amplifier and a Bessel filter. The dashed preamplifier box contains the portion of our electronics that sits immediately above our sample and that is maintained at a temperature of 127 K. Visible in this diagram are the two optical isolator switches that our blanking pulse opens during applied magnetic field pulses. When these two switches are open, the second stage of our detector is isolated from the preamplifier and the electrodes, and when they are closed, it is connected.

3.4 Data Analysis

The next step of the process is to convert the raw data that we receive from our apparatus into a result for the electron’s EDM. First, in order to filter random noise from our data, we repeat our experiment many times, flipping the sample’s magnetization back and forth, and then average the data that we take over all of the repetitions.
We use the same timing for each, allowing us to make averages of the voltage data as a function of time. The result of this averaging process, which occurs on the oscilloscope before the data are transmitted to the computer, is a plot equivalent to what we would measure over one cycle of the sample’s magnetization, but with significantly less noise.

In order to calculate the electron’s EDM, we need to convert these averages from a series of data points represented as a function of time into a single value for the potential drop induced by each direction of the sample’s magnetization. To do this we average the voltage over the range of time where the sample is magnetized in a given direction.

Figure 3.6: Pictured here is a plot, averaged over 256 traces, of the potential on one of our two electrodes as a function of time for a typical set of data. The dashed lines are the regions over which we average to determine the value of the potential for each sample magnetization direction.

Even with the precautions that we take against induction, the magnetic field pulse has some effects on our electronics; we wait until these effects have died down before
we begin our average.

Averaging the electric potential on each electrode leaves us with four results, $V_{A+}$ and $V_{A-}$ for the potentials we measure on electrode A for the two magnetization orientations, and $V_{B+}$ and $V_{B-}$ for the two potentials we measure on electrode B. Returning to Equation 2.1, we can now calculate the potential drop induced across our sample by the electron’s EDM.\footnote{The process that we have used to obtain the limits on the electron’s EDM reported in [23, 19, 3] and in my thesis is more complicated than the process I have described here. I will explain the modifications we have made to our method of calculating an EDM, as well as the reasons for those modifications, in Chapter 5 of this thesis.}

$$V_{EDM} = \frac{(V_{A+} - V_{B+}) - (V_{A-} - V_{B-})}{2} \quad (3.1)$$

We call $V_{EDM}$ the difference asymmetry, because it involves both a difference between the two electrodes and an asymmetry between the two sample magnetization directions.

We would expect to measure a non-zero potential drop between the electrodes at 127 K, and no potential drop at 88 K and 178 K when we have effectively turned off signal from the electron’s EDM. In practice, however, we measure a non-zero potential at all three temperatures. In order to determine a single value for the potential drop that an electron EDM would induce across our sample, we compare the value for $V_{EDM}$ at the three temperatures.

$$V_{EDM} = V_{EDM}^{127\text{K}} - \frac{(V_{EDM}^{88\text{K}} + V_{EDM}^{178\text{K}})}{2} \quad (3.2)$$

This comparison corrects for linear temperature dependences in our data. After we have calculated $V_{EDM}$, it is a simple matter of applying the relation that I cited in §2.4 to obtain a value for the electron’s EDM. If we obtain an answer within the range...
of its uncertainty of zero, then we have discovered an upper bound, whose magnitude is given by the uncertainty; if our result is inconsistent with zero, then we will have successfully measured a non-zero electron EDM.

It is worth noting that in equation 3.1, $V_{EDM}$ is essentially a quantification of the amount of asymmetry in our system. Assuming for the moment that the two halves of our toroid contain pure GdIG rather than of two different Gd-YIG compositions, our experimental apparatus will be symmetric on parity inversions about the center of the toroid. A parity inversion of our system would invert the positions of the two electrodes and would leave the direction of the magnetization unchanged (inverting the system along an axis perpendicular to the central axis of the toroid would cause the magnetization to reverse, but inverting the system again along the other axis perpendicular to the toroid’s would flip the magnetization back to its original orientation).

Figure 3.7: a schematic of our system, along with its parity inversion

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5I am assuming here that we are able to construct our apparatus to be perfectly symmetric. In reality, of course, it is certain to have a few flaws. These irregularities explain why we see some small asymmetric sources of error despite the fact that they are theoretically excluded.
Parity symmetry tells us that both the original system and its parity inversion should respond in the same way to the applied field—that when we replace electrode A with electrode B we will see the same value for the potential. This is possible only if electrodes A and B are at the same electric potential. But if the two electrodes are at the same potential for both orientations of the magnetization, then $V_{EDM}$ in equation 3.1 will be zero. The value of $V_{EDM}$ depends on the extent to which our system is asymmetric—that is, to the extent that introducing differing relative concentrations of Gadolinium and Yttrium between the two halves of our toroid unbalances the symmetry between the two electrodes. Our experiment is designed so that this difference in relative ionic concentrations in our garnet will have no effect on the data that we observe unless the electron’s EDM is inconsistent with zero. Any asymmetry in our data implies a non-zero electron EDM, and we can represent the magnitude of the EDM in terms of this asymmetry.

### 3.5 A Recent Modification

Our method of data analysis separates out the asymmetric elements of the signal that we observe from the symmetric ones, and it will also separate asymmetric sources of error from symmetric ones. Symmetric biases cancel out in our analysis, while asymmetric ones will find their way into our final result. For this reason we take great pains to make our experimental setup as symmetric as possible. A symmetric apparatus will produce symmetric sources of error. If our system is perfectly symmetric we can, at least in theory, be sure that any asymmetry that we measure is not an unexpected result of our apparatus, but is related to the effect that we are studying. This section explains a small modification, suggested by Ben Heidenriech, that I made to our method of applying field pulses in order to make it more symmetric.
We regulate our applied field pulses with a current switch, which in turn is driven by two function generators. Our sample sits relatively far from these function generators and is well isolated from them; for this reason we had not concerned ourselves with their contribution to the overall symmetry of our experiment. However, Ben noted that the signal on our electrodes that we measure exhibits a slight coupling to the signals on our function generators. One of our function generators was configured so that the signal that it produced was asymmetric with respect to our sample’s magnetization, creating the potential for an asymmetric source of error in our measurement. I modified the setup of these function generators in order to avoid this problem.

Our current switch has two inputs, each driven by a function generator. One input, called the voltage input, determines when the switch will turn on, allowing current to flow to the coil. When this input receives a voltage pulse from the function generator connected to it, it sends a pulse of current to the coil. The other input, the polarity input, determines the direction in which the current pulse will flow through the coil. When it receives an input voltage from its function generator it causes current to flow in one direction around the coil; when it receives no input voltage it causes the current to flow in the other direction. The voltage input behaves in the same way for both current pulses and is therefore symmetric. The polarity input, on the other hand, must carry different voltages for the magnetic field’s two directions and is potentially asymmetric.
CHAPTER 3. EXPERIMENTAL METHODS

Figure 3.8: This figure shows the two signals that we used to drive our current switch before this modification. The signal on the voltage input (V) is symmetric. The signal on the polarity input (P), however, is asymmetric.

Before this modification to the method that we use to drive our current switch, the signal on the current switch’s polarity input was in fact asymmetric. When the sample is magnetized in one direction there is no voltage on the input, and when it is magnetized in the other direction, there is a positive voltage. The voltage on this input would affect the two sample magnetizations differently and would register as an asymmetry. Our calculations would fail to filter out this asymmetric source of error, and it would find its way into our final result.

The current switch’s polarity input must take on different values for the two pulse directions. However, the value that it takes while the magnetic field pulses are off does not matter. I modified the signal coming from the function generator that drives the polarity input so that it has the same value over the entire course of its cycle, except during one of the field pulses. The function generator now applies the same voltage to the polarity input during the two regions of the cycle where we make our measurement. This modification, while slight, should improve our results by changing an asymmetric source of error to a symmetric one whose effects we can filter out of our data.
3.6 Previous Results

Our experiment as it stands today suffers from sources of error that severely limit its capabilities, and it is these sources of error that will serve as the subject of the remainder of this thesis. However, we have been able to make some preliminary measurements of the electron’s EDM despite the trammels that these effects have cast on our measurement capabilities.

In the summer of 2005 we measured an upper bound on the electron EDM of $|d_e| < 5 \cdot 10^{-24}$ electron-centimeters. We published this result the following fall [23]. After regrinding the sample to correct for a slight asymmetry and making some improvements in the way in which the electrodes are attached to the sample, Alex Bridges and Tim Ripper made a measurement in the summer of 2006, setting an upper bound of $|d_e| < 1.6 \cdot 10^{-24}$ e-cm [3]. This new limit was significantly lower than the one that we published, but it is not quite as convincing because it involves an unjustifiable, although probably proper, correction for a troublesome systematic effect.
Part II

Systematic Error
The final part of this thesis will focus on the work that I have done to understand and to minimize the two primary systematic errors that have limited our ability to make precise, convincing measurements of the electron’s EDM. These two errors enter our measurements in different ways and affect our results differently as well. One, which we call the m-even systematic, is intrinsic to the structure of our apparatus and mimics in many ways the signal that we would expect an electron EDM to generate. We can cancel this source of error to a large extent, but in doing so we blur the precision with which we can make our measurement. The m-even systematic is currently the limiting factor on our results and puts a fundamental constraint on the resolution that we can achieve with our current experimental setup.

The other effect, which manifests itself as a sample orientation dependence in our results, is somewhat less insidious than the m-even systematic. It presents a signature that differs in a few obvious ways from that of the signal that we are trying to measure, and because of this we can, at least in theory, completely correct for it in our results. It does not limit the precision with which we can make our measurement; however, its existence casts some doubt on our experimental methods and will limit the credibility of our results.

If we are to make a measurement of the electron’s EDM that can stand up against the results that have been achieved by other experiments, we must eliminate both of these effects. The second part of my thesis describes my efforts to better understand the mechanisms through which they sneak into our data, as well as my preliminary attempts to remove them from our results.
Chapter 4

The M-Even Systematic

4.1 The M-Even Systematic and the M-Even Effect

The structure of Gd-YIG excludes the generation of potentials odd with respect to the applied magnetic field, assuming no parity violations, [24, 12], but does not prohibit the development of even ones. We prevent even potentials from making their way into our results by carefully constructing our experiment to be as symmetric as possible. Our sample creates even potentials, but we will see remnants of these potentials in our final results only to the extent that our apparatus is asymmetric.

The M-even systematic that we see in our results is such a remnant. It is the asymmetric residual of a larger symmetric effect, which we label the \textit{M-even effect}.\footnote{Here I am defining a new, somewhat confusing, term. The M-even effect is the symmetric effect that is largely filtered out over the course of our data collection and analysis. The M-even systematic is the small asymmetric component of this effect that remains after we filter our data. It is the M-even systematic that limits the precision with which we can make our measurement, although the two are closely related and reductions in the magnitude of the M-even effect will likely reduce the M-even systematic as well.} The M-even Systematic would be zero in a perfectly symmetric apparatus, and the most straightforward way for us to reduce its effects is to improve the symmetry of our
apparatus. In Alex’s thesis research, he attempted to do just this, sending the sample back to the factory to correct for a small asymmetric defect, and carefully reepoxying and reassembling the apparatus so that it would be as symmetric as possible. These upgrades reduced the size of the M-even systematic in his results by a factor of six [3, §3.4]. Some of this reduction was likely due to a reduction of the M-even effect itself due to some modifications to the way the sample was assembled that I will discuss later in this chapter, but a significant part of it can probably be attributed to the apparatus’ enhanced symmetry.

This improvement was encouraging, but it still left us a few orders of magnitude from the precision that we will need to challenge the best limits on the electron EDM, and it is unlikely that we will be able to reduce the asymmetries in our apparatus much more than Alex did. If we are to have any hope of measuring an electron EDM or of refining the limit on it, we must attack the M-even systematic at its source: the M-even effect. If we can reduce the magnitude of the symmetric effect behind the M-even systematic, we will increase the sensitivity of our measurement; if we cannot, then we have no hope of making a meaningful EDM measurement with this incarnation of our experiment.²

4.2 Correction for the Effect

This systematic is large enough to swamp any signal that we would measure from the electron’s EDM. We can, however, minimize its impact on our results with some statistical prestidigitation. We filter the M-even effect to a large extent by designing our experiment and our analysis to select only asymmetric signals. The M-even systematic and the signal we would expect to obtain from an electron EDM are both

²It is worth noting that this effect has, to the best of our knowledge, never been observed before. For us it is an annoyance, but nonetheless it represents a new discovery.
asymmetric and both remain after this filtering, but they differ in their dependence on the magnitude of the magnetic field that we apply to magnetize our sample. While the EDM signal should be the same for all applied fields strong enough to magnetize the sample, the M-even systematic exhibits a clear dependence on the applied field $H$ that can best be modeled in the high-field limit as a $\frac{1}{H}$ decay.\footnote{We have two motivations for modeling our signal with this $\frac{1}{H}$ decay. First, it consistently provides us with closer fits than any other functional form does. Second, a decay that might be relevant to this signal is the sample’s approach to magnetic saturation, which also goes as $\frac{1}{H}$. Our choice of fit seems to coincide with a possible theoretical model that somehow involves this approach to saturation.} We make use of this asymptotic decay to isolate the systematic and to correct for it in our data.\footnote{Some might take issue with this method of correcting for the M-even systematic. We are certain that the electron’s EDM does not depend on the magnitude of the applied field, at least in the approximation that our sample reaches magnetic saturation. For this reason we can confidently discard the component of our signal that exhibits this dependence. However, we have no a priori reason to take what is left after we filter out any field dependence and claim that this result represents the signal from an electron EDM. The same systematic that manifests itself in our data as a decay at high fields may also have a component that does not depend on the field. When we remove our signal’s asymptotic behavior, we may only be partially canceling this systematic. At this point in our experiment, however, it is probably safe to assume that removing the applied field dependence successfully purges this extraneous signal from our data. The result for the electron’s EDM that we obtain from the asymptotic values of our data plots is generally consistent with zero given the range of the statistical uncertainty associated with our fit of the decay. If there is any residual element of the M-even systematic remaining in our results, it is insignificant when compared to the resolution that we can presently achieve. If we improve our precision to the point where we obtain a non-zero value for the electron’s EDM, or even to the point where we begin to approach the current world limit, we may have to concern ourselves with demonstrating that our method of correcting for the M-even systematic is valid. Of course, if we manage to improve our precision to this level, we may have reduced the systematic to the point where this type of filtering is no longer needed.}

We can represent the magnitude of the voltage asymmetry ($V_{Err}$) produced by this effect as a function of the applied field $H$,

$$V_{Err} = \frac{a}{H}$$

where $a$ is some constant. By adding this extraneous voltage to the voltage that we would expect to see on the electrodes due to our signal ($V$), we can represent the
total voltage that we measure \( (V_{Meas}) \) as a function of the applied field.

\[
V_{Meas} = V + \frac{a}{H}
\]  

(4.2)

We take data at multiple values of the applied magnetic field and extract a difference asymmetry\(^5\) for each value of the applied field. We then statistically fit these data to equation 4.2, determining values for the magnitude of the M-even systematic and for the component of our measured voltage that is due to the electron’s EDM. This latter component is now presumably free of the M-even systematic, and we convert it into a value for the electron’s EDM with the factor I cited in §2.4. This method should in theory allow us to completely eliminate the effects of this extraneous voltage from our results; however, it is not perfect. In order to remove this source of error, it introduces an extra parameter, \( a \), that we must statistically fit and consequently adds additional statistical error to our results.

\(^5\)We obtain this difference asymmetry using the method outlined in §3.4.
Figure 4.1: The magnitude of the signal that we observe decays to some asymptotic value as the inverse of the applied field in the high-field limit. We take this asymptotic value as our result for the potential that we would measure in the absence of the M-even systematic. The scales on this plot are arbitrary.

This method allows us to reduce the effects of the M-even systematic on our data by about a factor of 100. We can use it to peer through the veil that it has thrown over the results we are hoping to achieve, but this method alone will not be sufficient to lower our statistical uncertainty to the point where we can challenge the world EDM limit. In order to unshroud the measurement hiding underneath the M-even systematic we must eliminate the systematic itself, and in order to do that, we must tackle the M-even effect that underlies it.
4.3 Theoretical Speculation

The primary goal of my work with the M-even effect was to minimize its impact through the M-even systematic on our results. This goal does not require that I understand the effect, but simply that I determine which parameters mitigate it. However, a certain degree of exploration of the effect’s theoretical underpinnings has proven instrumental in my work, and before I begin my discussion of my efforts to minimize the effect, I will briefly summarize a possible theoretical model for the mechanism through which it might manifest itself.

All magnetic materials undergo a process called magnetostriction when placed in a magnetic field. That is, they experience internal magnetic forces as they magnetize, and their crystal structure deforms slightly as a result of these forces. O. P. Sushkov has examined the effect that magnetostriction would have in the bulk of our sample and has placed an upper bound on it that is sufficiently low that we can rule out bulk magnetostriction as the cause of the effect [32]. However, as I will explain later, we are fairly confident that the effect originates at the surface of our sample, where Sushkov’s calculation does not apply.

Different materials undergo magnetostriction to various degrees, depending on a number of factors. Our electrodes involve a junction of a few different substances, the sample, the electrode, and the metal-filled epoxy, and each of these substances experiences this internal strain to a different extent. Differential magnetostriction at the junctions between materials may underlie the M-even voltage that we observe. It is, unfortunately, an extremely complicated matter to quantify this differential magnetostriction. It is simple to look up plots of magnetostriction for various substances and to calculate the difference between the degrees to which the two materials will

\[W. F. Brown, Jr. provides a particularly useful phenomenologically driven treatment of magnetostriction in [4]. I am basing my discussion here on his book.\]
deform, but our effect occurs at the surfaces of samples, where oxide layers and other chemical considerations make this calculation much more complicated. I will stop short of an attempt to quantitatively explain differential magnetostriction, but I will continue to rely on it as the most likely theoretical vehicle for the appearance of the extraneous potential that I am trying to eliminate.

Exactly how differential magnetostriction gives rise to an electric potential is unclear. The potential could form as a feature of the structure of the Gd-YIG sample near the interface. In an idealized model of the crystal, we would expect the effect to be much smaller than we measure, but there are a few ways in which its structure could depart from the ideal close to its surfaces.

Crystals are known to deform, sometimes dramatically, close to their surfaces [25, § 4.2]. The crystal structure at our sample’s surfaces could depart significantly from what we would find in its bulk. The effect that we observe seems to arise at the sample’s surfaces, and it may be that we cannot assume the same symmetries in our result as one would for an effect in the crystal’s interior.

Another mechanism by which our crystal’s structure could be altered involves pressure gradients within the sample. Solid-state physicists attempting to measure magneto-electric effects in YIG have noted that applying a nominally uniform pressure to a sample can cause linear magnetoelectric effects. These effects should be excluded by YIG’s symmetric crystal structure, but they can be explained by pressure gradients within the sample due to small bumps or ledges in the sample’s surface [22]. Our samples could develop similar pressure gradients either through irregularities in the sample’s surface or epoxy bond, or through differential magnetostriction. These gradients would distort the body-centered cubic crystal structure of our sample. It would no longer be symmetric on inversion about its center and electric effects that we would not normally expect to see could begin to creep into our measurements.
CHAPTER 4. THE M-EVEN SYSTEMATIC

These two mechanisms provide possible ways in which our crystal’s symmetry could break down and potentials intrinsic to the crystal structure could develop. There are probably additional mechanisms that could spoil the crystal’s symmetry and have similar effects.

The potential that we measure could also arise outside of the crystal, as a property of the junction of the crystal and the material that it is in immediate contact with (which in our experiment is generally a metal-filled epoxy). The mechanism by which a potential would arise at this junction would probably be complex in nature, but it could be a result of the triboelectric effect, where a difference in charge builds up when two materials with different affinities for electrons, such as wool socks and a carpet, slide against one another. One could imagine that when a strain develops between the sample and the epoxy, electrons could shift from the material with a weaker hold on its electrons to the other. In this case, the effect would depend on the relative electron affinities and ionization energies of the two substances. Unfortunately, these values vary widely depending on the conditions under which they are measured. It will be impossible for us to determine how our data correlate to these factors unless we modify our experiment in order to control these conditions more carefully.

My thesis is driven by the needs of our experiment rather than by a desire to understand this effect, and what I have presented here is a bare-bones and speculative version of a theoretical model for the effect that we observe. I will devote the remainder of this chapter to a more phenomenological approach, explaining the ways in which I was successful in reducing this effect without concerning myself as much with why they should reduce it. However, I will occasionally refer to this rough theoretical model in order to explain certain discoveries and in order to justify certain decisions that I have made.

\[7\] I owe the idea for this possible mechanism to Professor Hunter.
4.4 The M-even Effect Apparatus

In the spring of 2007 Alex Bridges constructed an apparatus to test modifications that we could make to our experimental setup in order to minimize the M-even effect.\textsuperscript{8} I used this apparatus to try to further characterize and better understand the effect.

In order to measure the M-even effect on a toroidal sample, we must wrap a magnetic coil around the sample. Every modification that we make to the sample would require that we remove and rewrap the coil, a time-intensive and unnecessary step. Instead of working with toroidal samples, we characterized how the M-even effect manifests itself on cylindrical samples. With cylinders, we do not have to wrap a coil for every measurement that we make, allowing us to conduct tests more efficiently. We then generalize the results that we obtain with cylinders to what we would expect on the toroid.

The apparatus consists of a number of solenoidal magnetic coils, arranged so as to provide a relatively uniform field in the region where we make our measurement. Inside of these coils we place a brace that holds our cylindrical sample. This brace squeezes our sample with a variable amount of pressure. The sample consists of two cylinders of YIG with an electrode between them. We linearly ramp the current through the apparatus’ coils, driving the magnetic field on the sample with a triangle wave symmetric about zero applied field, and we measure the potential drop between the electrode and ground. We average the signal that we measure over a number of cycles of the triangle wave. Ignoring the M-even effect, we would expect the magnetic field that we apply not to change the electrode’s potential—we use pure YIG samples that contain no gadolinium and thus should not be sensitive to the electron’s EDM. Any potential that we measure using this system should arise from an extraneous

\textsuperscript{8}Alex explains this apparatus in more depth than I will. See [3, §4.1-2].
Figure 4.2: Our M-even effect apparatus is designed as a series of concentric cylinders. The inner cylinder, pictured on the right of this figure, is the pressure brace that contains our sample. We place that cylinder inside of a solenoidal magnetic coil, pictured in the center. At the top of this coil is our detector, which we enclose in a Faraday cage while we are running our experiment. On the left of this figure is the outer set of coils, which surrounds the two inner cylinders. This set of coils consists of one large solenoidal coil to add to the field produced by the inner coil and two smaller coils to mitigate fringing fields within our apparatus.

In order to study the M-even effect, we must extract the even signal from our data. Before I describe how we isolate this signal, I need to define precisely what we mean when we refer to the M-even signal. We define the even signal as opposed to the odd signal, which consists of the inductive effects that we see as we change the applied magnetic field. However, we cannot use the traditional mathematical definition of even and odd functions to isolate the M-even behavior because our ferrimagnetic crystal magnetizes nonlinearly. Our sample exhibits some magnetic hysteresis and retains its magnetization even when a small field is applied to it against the direction in which it is magnetized. The induction signals are not odd with respect to the
applied field but lag slightly behind the point of zero applied field. We could separate our signal into odd and even functions using the traditional mathematical definition, where

\[ f_{\text{even}}(t) = f_{\text{even}}(-t) \]  

(4.3)

and

\[ f_{\text{odd}}(t) = -f_{\text{odd}}(-t) \]  

(4.4)

Under this definition, however, the even signal would acquire significant artifacts from these induction potentials. In order to account for our sample's hysteretic lag, we compare the signals for the two directions of magnetization not by reflecting them about zero applied field, but by shifting them by half of a period of the driving triangle wave. That is, we redefine even and odd as

\[ f_{\text{even}}(t) = f_{\text{even}} \left( t + \frac{T}{2} \right) \]  

(4.5)

and

\[ f_{\text{odd}}(t) = -f_{\text{odd}} \left( t + \frac{T}{2} \right) \]  

(4.6)

where \( T \) is the triangle wave's period. This reformulation may look strange at first, but for periodic functions like ours, it makes some sense. It completely removes inductive effects from our even signal, and it also creates two subspaces, one of even functions and one of odd functions, whose sum spans the space of real functions periodic on \( T \). Because of this useful property, we can represent an arbitrary signal from our apparatus as a sum of an even component and an odd component just as we could with the traditional definition of even and odd functions.

It is a relatively simple matter to separate the components of our signal odd and even under this definition with respect to the applied field. We measure the current
through the magnet, which we can use to find the value of the applied magnetic field, and the potential on the electrode as a function of time. We then calculate the odd and even components of each of these signals with respect to time.

\[
f_{\text{odd}} = \frac{f(t) - f(t + \frac{T}{2})}{2}
\]

\[
f_{\text{even}} = \frac{f(t) + f(t + \frac{T}{2})}{2}
\]

To represent the component of the potential on our electrode even with respect to the applied magnetic field, we plot the component of the potential even with respect to time against the component of the applied magnetic field odd with respect to time. We correct this result for biases and linear drifts in our detector by adding a constant term and a linear term while constraining the average value of the even voltage over the first and last one percent of data points to zero.

This new definition involves a peculiarity that we must keep in mind when considering the results I present here: it adds potentials measured for one direction of the applied magnetic field to potentials measured for the other direction, and in this addition the labels of positive and negative applied field lose their meaning. The applied field axis of our plots runs from negative fields to positive fields, but these values do not correspond to any particular direction of the field in our experiment. Instead, negative applied field values represent data taken while the applied field was decreasing in magnitude, and positive applied field values correspond to potentials measured as the field was increasing in magnitude.

To ensure that our apparatus was not adding an even effect of its own to our data, we ran a control test without a sample or an electrode, but only a wire running to where we would normally connect our sample. This control test returned a M-even
signal consisting only of random noise, confirming that any signal that we observe beyond this noise must result from some feature of the sample in the apparatus.

Figure 4.3: Pictured above is the control for our tests of the M-even effect. This plot demonstrates that there is no M-even voltage intrinsic to our apparatus beyond random noise.

The experiments that we conducted using this apparatus helped to illuminate the nature of the M-even effect and will hopefully allow us to minimize the M-even systematic in our toroidal system. We must bear in mind, however, that the systematic that we are trying to reduce and the M-even effect that we measure using this apparatus are separate phenomena. The M-even systematic on our toroid displays the same behavior as the M-even effect on the toroid, and reductions in the effect in that system should reduce the systematic by the same factor. However, while we are confident that we are measuring the same effect on toroidal samples and on cylindrical samples, we see different versions of the effect on the two.
Figure 4.4: Shown above is a comparison of the M-even effect on our large toroidal sample and on a cylindrical sample in the M-even apparatus. Both samples use copper electrodes bonded with silver-filled epoxy, and both traces were measured with the same experimental methods. The differences between the two must be due to differences between toroidal and cylindrical sample geometries.

The M-even effect on our toroid will certainly depend on the same parameters as the effect on cylinders, but exactly how it will do so is unclear because of the lack of a one-to-one correspondence between the two signals. There is no specific feature of the M-even plot that will determine the nature of the effect in the high-field limit on our toroids that I can cite in my discussion of the results that we obtained with this apparatus. Instead, I will focus on indications of good magnetostrictive matching at high applied fields, where we take data for our EDM measurement.\(^9\) Good matching in this regime should decrease the M-even systematic in our toroidal system, causing it either to decay more rapidly to zero or to decrease in magnitude.

In this chapter I will present data taken with the M-even apparatus by Ben Hei-

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\(^9\)You might ask why we don’t make our EDM measurement at low applied magnetic fields and try to minimize the M-even effect at these values. We could take this approach, but we have found that the value of the M-even effect at low fields is much less reliable. We have discovered some possible sample configurations that produce a potential of nearly zero for small fields, but we have observed a large degree of variance in the values that we obtain at low fields for those configurations. Additionally, when we magnetize our sample with large fields, we can be more confident in our approximation that the entire sample has come into magnetic alignment with the applied field.
denreich, Jeff Grover, and me during the summer and fall of 2007. I was finishing my semester abroad for the beginning of the summer, and it is with great appreciation that I appropriate the data that Ben and Jeff took in my absence. Most of the tests that I describe in this chapter were conducted multiple times; where more than one set of data is available for a given sample configuration, I have presented an average of all of the available data.

4.5 Electrode Material on Unbonded Samples

In order to better understand the M-even effect and to determine how best to construct our apparatus to avoid it, we characterized the effect on as many different configurations of the junction between the sample and the electrode as possible. I will begin my discussion of the M-even effect with our data on the effect’s dependence on the electrode material. We were able to take a wide variety of data on this subject. We ultimately discovered that changing the electrode material in a bonded sample makes very little difference in the magnitude of the M-even effect at high fields, but our examination of different electrode materials has helped us to better understand the nature of the effect and has guided my approach to the other areas of research that I have done in this thesis.

I will begin by describing the results of tests that we conducted using unbonded electrodes that we fixed to the sample by squeezing the sample with our pressure brace. Figures 4.5-9 show our results for unbonded electrodes.
Figure 4.5: These plots share in common a rounded central peak and relatively calm high-field behavior. Copper and palladium match remarkably well here. Tantalum has a slightly different shape, but it shares this group’s major defining features.
Figure 4.6: Aluminum, indium, and tin each give rise to a central peak opposite in sign to that of any of the other electrodes that we tested. They also share a shape that is asymmetric about zero applied field, with an overshoot and a relatively slow asymptotic fall at positive field values.
Figure 4.7: Silver takes on a shape that is exactly opposite that of Aluminum, tin, and indium. It features the positive peak at zero applied field that most of our electrodes show, but also demonstrates the asymmetric shape and the overshoot characteristic of the electrodes with a negative central peak.
Figure 4.8: Molybdenum, tungsten, niobium, and titanium all have small central peaks, but are slow to decay at high fields.
Chapter 4. The M-Even Systematic

Figure 4.9: Nickel, Cobalt, and Iron all exhibit sharp peaks around zero applied field and small potentials at high-fields. Cobalt’s signal includes two extra dips around the central peak. I chose to include it in this group despite this difference because it shared this group’s two unique defining characteristics.

Our data here exhibit an obvious pattern. Most of the electrode materials that we studied can be classified into four rough categories based on the signature that they give to the M-even effect. There is some variation in the amplitude of the signal within these categories, but each category demonstrates a remarkable similarity in the signal’s shape. There is clearly some effect at play here that is dividing these different metals into categories. Figure 4.10 is a periodic table of the elements where I have labeled each element that we tested with a color based on the M-even signal category to which it belongs.
Figure 4.10: In this figure, I have labeled a periodic table of the elements according to our results for the M-even effect. Each color on this table represents a category of M-even signal from figures 4.5-9.

The categories seem generally to form clusters on the periodic table, indicating that elements with similar chemical characteristics give rise to similar M-even effects. A few qualities that these clusters share are readily apparent from the periodic table. Poor metals, metals in the p orbital block of the periodic table, all exhibit the same characteristic shape. Iron, nickel, and cobalt, which are all ferromagnetic, also share a M-even signal shape. This latter category is especially intriguing, since rare-earth garnets and ferromagnetic elements both experience much greater magnetostriction than paramagnetic or diamagnetic materials [14][9, §3.4]. The fact that the M-even effect decays more sharply for these electrodes hints that differential magnetostriction is important: ferromagnetic electrodes would better magnetostrictively match our sample and would hence experience less differential magnetostriction than electrodes less susceptible to magnetostriction. In the center of the plot, where the applied field is close to zero, the magnetization of the electrode and the sample are not likely to be well matched, but once the magnetization increases to the point where the two materials’ magnetizations becomes more orderly, the effect quickly drops
in magnitude. In fact, the edge of the plot’s central peak coincides very well with the applied field value that is required to flip the sample’s magnetization (this value can be determined by noting a large flux change in the odd part of the signal that is filtered out in our analysis).\textsuperscript{10} The sample’s magnetostrictive behavior evidently starts to match that of the ferromagnetic electrodes at this point.

![Graph](image)

Figure 4.11: Ferromagnetic electrodes produce significant potentials near zero applied field. However, once the sample undergoes the abrupt change in magnetization that signals that it has begun its final approach to saturation, these potentials suddenly disappear. This is a plot of the M-even effect on an unbonded nickel electrode. The dashed lines indicate the applied magnetic field value where the sample's magnetization flips.

On the other hand, molybdenum, titanium, tungsten, and niobium have very small central peaks, with M-even effect values close to zero, but larger potentials at high fields. They exhibit behavior opposite that of the ferromagnetic electrodes, and probably have lower magnetostrictive constants that match well with the crystal’s

\textsuperscript{10}To be precise, when we see this flux change, the sample is completing the phase of its magnetization that occurs through bounday displacements of its magnetic domains and entering its approach to saturation, where its individual magnetic dipoles begin to rotate out of alignment with the crystal’s easy axis and into alignment with the applied field [21, 448-450]. This high-field approach to magnetic saturation occurs in rare-earth garnets as \( \frac{1}{H} \), which is also how the M-even systematic decays. This similarity in functional form suggests some correlation between the two, although we are not sure of the underlying mechanism. It seems likely that improvements in the magnetic field region above the sample magnetization flip will correspond to improvements in the results of our fit.
CHAPTER 4. THE M-EVEN SYSTEMATIC

behavior at low fields before it begins to magnetize, but conflict with its behavior once its magnetization has flipped into alignment. It appears that, at least for cylindrical samples, Gd-YIG experiences magnetostriction to a small degree at low fields and to a much greater degree at high fields. We are trying to minimize the high-field M-even effect, and hence we should use electrodes that experience strong magnetostriction.

Another mechanism that may explain the improved high-field signals that we see on ferromagnetic electrodes is that their magnetization affects that of the sample. These electrodes significantly amplify the magnetic field passing through them. They are very thin, and this amplification would likely only occur very close to the electrode. However, the electrode would probably be able to exert some magnetic influence on the layer of the crystal closest to it. It would increase the magnetic field in this region, causing this first layer of the crystal to come into better magnetic alignment with the applied field. The M-even effect arises at this layer, and we generally observe that it decreases in magnitude as the crystal approaches magnetic saturation. It is therefore possible that electrodes with high magnetic permeability decrease the size of the effect.\textsuperscript{11} If this argument is correct, then it provides another justification for ferromagnetic electrodes as a means to reduce the M-even systematic in our EDM measurement.

While the remaining categories of M-even effect signature seem generally to cluster on the periodic table, I can find no clear similarity that their elements share. The fact that there seems to be a pattern without an obvious ordering principle suggests that the nature of this effect is complex. Although it appears as though factors such as an element’s magnetic characteristics and electronegativity are important in determining the effect’s character, other, subtler factors must also come into play.

Unbonded electrodes lent us a few more important insights. First, they provide

\begin{itemize}
  \item This mechanism was suggested to me by Professor Gordon.
\end{itemize}
evidence to support our hypothesis that the effect is localized at the interface between the sample and whatever is in immediate contact with it. We performed tests comparing the behavior of a tin electrode and copper and nickel electrodes plated with a thin coating of tin. Unplated copper and nickel electrodes both gave rise to an effect whose signature was easily distinguishable from tin’s, but both metals, when plated, showed a response very similar to that of the tin electrode. The effect must depend on the composition of the surface of the electrode, rather than that of its volume. These results provide strong evidence that the effect does not occur in the bulk of the electrode but at the surface where it meets the sample.

Figure 4.12: We measured the M-even potential on three unbonded electrodes with tin surfaces: copper and nickel electrodes plated with tin, and a solid tin electrode. We found that the signals on the three electrodes were similar.
We noted in our tests on unbonded electrodes that electrodes constructed out of softer metals exhibited hysteresis as we cycled the pressure of our brace. This hysteresis showed no clear pattern, but it was readily evident in the signals that we measured for aluminum, indium, tin, and lead. It appeared only in unbonded samples, and would disappear when we made measurements with electrodes bonded to our sample with epoxy. This observation’s implications for the M-even effect are unclear, but it does suggest that we should avoid soft metals when we select the electrode material to use in our EDM measurement. Although bonding these electrodes to the sample seemed to mitigate their hysteretic behavior, it will probably be best for us not to risk problems of this nature cropping up.

Also worth mentioning is the test that we ran on an electrode constructed out of a thin sheet of graphite foil. The M-even effect had a fairly large signature on this electrode, proving that the potential that we measure is not unique to metal electrodes. The graphite foil was flimsy and difficult to work with, so we abandoned it as a potential electrode material once we learned that it suffers from the effect just as our metal electrodes do.

Figure 4.13: the M-even signal on a graphite electrode.
4.6 Electrode Material on Bonded Samples

Our tests on unbonded electrodes proved to be useful in improving our understanding of the M-even effect. However, in bonded samples, the choice of electrode material loses its importance as other factors start to dominate. We tested a number of electrode materials that created a wide variety of effects when unbonded, and each of these electrodes, when bonded with a silver-filled epoxy, created the same type of effect. Even the aluminum electrode, whose central peak was negative when the electrode was unbonded, matches the general shape that we see.

Figure 4.14: Pictured here are the signals on four different electrodes, each belonging to a different category of M-even effect signature when unbonded, and each now attached to the sample with a silver-filled epoxy.
CHAPTER 4. THE M-EVEN SYSTEMATIC

These results further support our hypothesis that the effect occurs at the interface between the sample and the material immediately closest to it: even a thin layer of epoxy is enough to nearly completely mask the contributions of the electrode material. We ultimately decided that the electrode most likely to minimize the M-even effect in toroids when bonded is nickel. The M-even effect on all of these electrodes is similar, but nickel exhibits a few features that may improve our precision in our EDM measurement. It has a sharper peak that falls off abruptly once the applied field becomes large enough that sample magnetostriction starts to become an issue. This sharp peak and falloff is not so pronounced as it is with an unbonded sample, probably because it is mitigated by the silver in the epoxy, but the well-matched sample and electrode magnetostriction that it signals will nonetheless help us to minimize the M-even systematic in our EDM measurement.

4.7 Bonding Method

We know that the M-even effect occurs between the surface of the crystal and the material that touches it, so it is not surprising that it depends strongly on the method that we use to bond the crystal to the electrode. The agent that we use to bond the two together comes into direct contact with the sample and consequently will be important in determining the interaction that gives rise to the M-even effect.

We tested various bonding methods on copper and nickel electrodes. We used commercially available silver-filled and nickel-filled epoxies, and we also created an aluminum-filled epoxy by mixing aluminum powder with a non-conductive epoxy. In order to prevent the aluminum from forming aluminum oxide, I prepared the epoxy and epoxied the sample in an environment of nitrogen gas.

We also, having noted that the signals on poor metals were almost perfectly op-
posite those of the gold and silver, decided to prepare another sample with an epoxy filled with a mixture of aluminum, a poor metal, and silver in equal atomic densities. We hoped that the effects of these two metals would simply cancel out, leaving us with a net M-even potential of zero. In addition to metal-filled epoxies, we experimented with nonconductive epoxy, allowing the sample to couple to the electrode capacitively rather than through the conduction. We also melted indium and tin, both of which have relatively low melting points, and used them to attach the electrode to the sample. We took no data on the effects of indium bonding on a nickel electrode, so I have included a comparison of indium bonding with silver-filled epoxy bonding on a copper electrode.
Figure 4.15: Shown here are the results of tests of various methods to bond the sample to a nickel electrode.
Figure 4.15 (continued)

Figure 4.16: Pictured here are the M-even signals on a copper electrode bonded with silver epoxy and one bonded with indium.

The potential that we observe does not arise from the fact that the bonding agent that we use is conductive—it is larger on non-conductive epoxy than it is on metal-filled epoxy.

Although there is some correspondence between an epoxy filled with a particular type of metal and an unbonded electrode constructed out of the same type of metal, it is not as strong as one might expect. Both an unbonded electrode and the metal in the epoxy connecting an electrode to the sample are in direct contact with the
sample, but it seems that there is an important distinction between one fixed to the sample with an epoxy and one simply placed in contact with it. This dependence on how tightly the metal is attached to the sample fits our model of the effect as depending on the degree of correlation of the magnetostrictive behavior of the sample and the metal. An epoxy bond will force the two to physically couple, effectively improving the magnetostrictive agreement between them. This analysis explains why the epoxied samples give rise to relatively small potentials at high enough fields for the sample to become magnetized and for magnetostriction to begin to play a large role.

Both of our attempts at metal bonding resulted in rather large signals, especially at high fields. If, as is my hypothesis, small high-field signals indicate a strong bond, these results imply that indium and tin are weak bonding agents. I confirmed this interpretation of our results when I was able to pull the tin-bonded sample apart after we finished testing it. We want to avoid using weak bonding agents in our EDM measurement both because they appear to make the M-even effect worse and because we want to avoid risking our sample coming apart during the temperature cycling that we must go through as we collect our data. Additionally, all of the metals that melt at temperatures that we can reasonably reach are ones that give rise to large M-even potentials, so we do not stand to gain very much with this method. We chose to abandon this method in favor of bonding with metal-filled epoxies.

Our mixed aluminum-silver epoxy was also unsuccessful in decreasing the M-even effect. The signal that we observed was no smaller than what we measured on other metal-filled epoxies, and had larger high-field values. The two other epoxies that I prepared with aluminum also gave rise to greater high-field potentials, suggesting that this epoxy’s behavior at high fields was a result of the presence of the aluminum in the mixture. This test demonstrates that the simple linear model that we had
hoped to use to describe the effect is insufficient; when we mix two metals, there is some interaction at play beyond a simple addition of the potentials that each would produce alone. The results that we obtained with this mixed epoxy disagreed with our predictions for the mixture’s behavior and were significantly worse than what we could obtain with epoxy filled with silver or nickel. Moreover, mixtures of metals introduce the possibility that the two metals will somehow interact with each other and will give rise to an unexpected effect. Since we do not fully understand the nature of the M-even effect, it is probably best for us to keep the design of our experiment as simple as possible and to avoid unnecessary complexities such as this one. For these reasons, we decided to focus on epoxies filled with a single type of metal as the best bonding agent for our EDM measurement.

All of the metal-filled epoxies seem to give us relatively good results at high applied magnetic fields, although their low-field behavior varies considerably. Both nickel-filled and silver-filled epoxy produce small high-field potentials; they are probably the best bonding agents to use in our EDM measurement. Of these two options, we chose nickel-filled epoxy as the one most likely to improve the precision of our EDM measurement in accordance with our hypothesis that differential magnetostriction plays a key role in the M-even effect. We chose to use a nickel electrode because it appears as though nickel is the best magnetostrictive match for our Gd-YIG sample. A nickel-filled epoxy should undergo magnetostriction comparable to what our electrode and our sample experience, decreasing the strain at the interfaces between the electrode, the epoxy, and the sample, and lessening the effect.
4.8 Sample Surface Roughness

We do not know why our reground, reepoxied sample experienced the M-even effect much less than our original sample did, but one difference between the two samples is that we ground the ends of the sample before bonding them. Our sample came from the factory with smooth edges, and in the original version of our experiment, we made our epoxy bond onto these smooth surfaces. Unfortunately, when we unbond a sample, we must grind its ends in order to remove excess epoxy, and we are unable to conduct tests with smooth sample surfaces without ordering new samples. We can, however, adjust the roughness to which we grind the sample’s surfaces by changing the grain size of the grit that we use in the grinding process (we used size 120 grit, which results in a finer surface, and size 80 grit, which produces a rougher finish). We studied the effect of the size of the grit that we used to prepare our sample as an attempt to better understand whether the fact that we ground the ends of our sample before bonding them could have caused the improvement in our results.

Figure 4.17: A sample ground with grit with a grain size of 120 experiences a larger M-even effect than a sample ground with an 80 size grit.

Figure 4.17 is a comparison of the effect on cylinders with different surface rough-
ness. These data are limited in scope, but it appears as though rougher sample surfaces experience the effect to a lesser degree. The epoxy should form a stronger bond to an 80 grit surface than it would to a 120 grit one, and perhaps this explains the effect’s decrease in size. Just as a metal-filled epoxy generates a smaller potential than an unbonded electrode because it increases the coupling between the sample and the metal in the epoxy, a stronger epoxy bond will further increase this coupling and will lead to an even smaller effect. Although it is difficult to see the sample roughness’ effect at high applied fields in our results, this improvement in the M-even effect should theoretically become more pronounced at higher applied fields, when the sample approaches magnetic saturation and magnetostriction starts to play a more important role. We see significant improvement between the 120 grit and the 80 grit tests, and there would presumably be even greater improvement between an 80 grit sample and a smooth one. The fact that we ground our sample surfaces when we rebuilt it at least partially explains the decrease that we saw in the M-even effect.

4.9 Electrode Roughness

One would imagine that, just as roughening the sample’s surfaces before bonding improves our results, roughening the electrode should increase the strength of the epoxy bond and lower the M-even potential that we observe. We compared the M-even effect on rough and smooth electrodes to determine whether this would be the case. We experimented with two methods for roughening the electrodes: grinding and acid etching. To grind the electrodes, we attached them to a large piece of metal and then ground them using size 120 grit. The electrodes are very thin, and we had some difficulty using this method to roughen them evenly without tearing or deforming them. We also experimented with etching a nickel electrode in nitric acid
heated to approximately 70°C. This method proved to be more effective than grinding, providing an even finish without damaging the electrodes. Figure 4.18 compares our results from smooth and roughened nickel electrodes that were bonded to the sample with silver epoxy.

![Graphs of smooth, acid etched, and 120 grit nickel electrodes with applied field and even voltage](image)

Figure 4.18: Pictured here are M-even signals on smooth, acid etched, and ground nickel electrodes bonded to the sample with silver epoxy.

Roughening the electrode certainly did not have a large impact on the signal that we observe in the case of bonded nickel electrodes. It seems that rough electrodes exhibit central peaks that are not as sharp as those of smooth electrodes. The electrode roughened by grinding appears to produce larger potentials than the smooth one in the high-field limit. The one that was etched with acid, on the other hand,
featured high-field potentials comparable to those on the smooth electrode. Etching appears to be the superior method of roughening an electrode. Perhaps it is less likely to deform the thin sheet of metal, which could increase either the magnetostrictive strain that the electrode experiences or the magnitude of its response to that strain.

Figure 4.19: Shown here are M-even signals on the smooth and roughened nickel electrodes before they were bonded to the sample.

This approximate equivalence between rough and smooth electrodes when bonded disappears in unbonded tests. Rough electrodes experience much larger M-even potentials at high fields. Maybe when we roughened our electrodes, we simultaneously made their behavior worse and their bond strength greater. These two effects would approximately cancel in bonded tests.
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Figure 4.20 shows the M-even signal on a smooth copper electrode and one roughened with grit of grain size 120.

![Graph showing M-even signals on smooth and rough copper electrodes.](image)

Figure 4.20: Pictured are the M-even signals on smooth and ground copper electrodes bonded to the sample with a silver-filled epoxy.

In the case of copper, roughening the electrode makes a noticeable difference on the signal that we observe. Its peak is altered and, more importantly, it no longer falls off as quickly in the high-field limit.

The best of our trials with rough electrodes were equivalent to trials with smooth electrodes; the worst were inferior. It appears as though any improvement in the M-even effect that we gained from the rough electrodes' increased bond strength was lost through deformations that the electrodes suffered in the roughening process. While grinding the surfaces of the sample before epoxying appears to reduce the magnitude of the M-even effect, roughening our electrodes does not seem to provide us with any clear advantage. In the case of acid-etched nickel electrodes, it did not result in a clear disadvantage, either. Rather than making any hasty judgments about whether or not we should roughen our electrodes, I will suggest that tests be conducted on toroidal samples using both smooth and acid-etched electrodes before a decision is made as to which electrode surface will best minimize the effect.
4.10 Brace Pressure

One additional parameter that we can adjust in the M-even apparatus is the pressure with which we squeeze the sample together. It would be difficult to apply pressure to our toroidal system, so pressure is not a parameter that we would consider varying. I present our results on the system’s pressure here simply because they further support some of my hypotheses about the nature of the effect.

We vary the pressure in the brace by loosening and tightening the long threaded rods that hold it together. In order to standardize the pressure that we apply to the sample, we listen to the resonant frequency of the threaded rod and tune it to musical notes. Ben and I have good enough musical ears to resolve differences in frequency of a few hertz, which gives us more measurement precision than this system requires. If we approximate our threaded rods as vibrating strings, we can relate the frequency $f$ of the rod’s first harmonic to the force $F$ squeezing our sample together.

$$f = \frac{1}{2L} \sqrt{\frac{F}{\mu}}$$  \hspace{1cm} (4.9)

where $L$ is the rod’s length and $\mu$ is its linear mass density. We can group $L$ and $\mu$ into a single constant $C$ and represent the force in terms of the frequency as

$$F = (4L^2 \mu) f^2 = Cf^2$$  \hspace{1cm} (4.10)

In this section I am concerned only with relative forces, so I will keep my results in terms of $C$, using units of $\frac{C \cdot Hz}{10^5}$.

We generally standardize the force on our sample at a value of 7.7 on my arbitrary scale for unbonded samples, and at finger tightness\textsuperscript{12} for bonded ones. Figure 4.21

\textsuperscript{12}We define finger tightness as the highest tension that we can reach using just our fingers.
shows plots for an unbonded nickel electrode at various brace pressures. I chose to present my results from a nickel electrode because nickel is one of the least hysteretic over pressure cycling of the materials we have studied. Unlike some of the softer metals that we have examined, nickel exhibits highly reproducible pressure dependence.

![Diagram of M-even signal for an unbonded nickel electrode at various brace pressures.](image)

Figure 4.21: Pictured above is the M-even signal for an unbonded nickel electrode at various brace pressures. The units of force that I use here are arbitrary.

The M-even effect appears to decrease in magnitude at the high-field limit as the pressure on the brace increases, although it is difficult to tell on these traces whether there is any improvement between forces of 7.7 and 10.4. This improvement at high pressures, which appears as a general trend in most of our pressure tests on unbonded electrodes, further suggests that increased mechanical coupling between the sample
and the electrode, in this case because they are being squeezed together with greater force, decreases the interaction that leads to the high-field M-even effect. The pressure dependence in our results disappears when we bond our samples, probably because any mechanical effects that pressure might have are masked by the strength of the epoxy bond.

4.11 Future Prospects

In the work that I have presented in this chapter, I have helped to characterize the M-even effect and I have made my best guess as to the configuration of our apparatus that will minimize the M-even systematic: a sample roughened to 80 grit, bonded with nickel-filled epoxy to a nickel electrode that is either smooth or etched with acid. The next step is to run tests of these two possibilities for the optimal sample configuration in a toroid to determine whether I was correct in my generalizations of the behavior we see on cylinders.

In order to run these tests, we have reincarnated a miniature model of our apparatus last used in Oliver's thesis. It is scaled down by a factor of approximately 4:1, but otherwise is very similar to the toroidal system that we use in our EDM measurement. This system allows us to conduct tests in the geometry of our large system without the hassle of setting it up or the possibility that we will damage one of its more expensive components.
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Figure 4.22: Pictured above is the miniature model of our system that will be used in the next phase of our experiment in order to test my predictions of how to minimize the M-even effect in toroids. When we take data with this system, we cover the preamplifier, the circuit board sitting immediately above the sample, with a Faraday cage and then enclose the entire system in another Faraday cage in order to block 60 cycle electromagnetic interference.

This system is now in working order. We have calibrated it and have run a baseline test on a toroid featuring copper electrodes bonded with a silver-filled epoxy. We are now preparing a test on a sample with 80 grit surfaces, bonded with a nickel-filled epoxy to a smooth nickel electrode. The results of this test will not be ready in time to be included in this thesis, but we are on the verge of discovering whether we are going to successfully decrease the size of the M-even effect on toroidal samples. Hopefully, the predictions that I make in this chapter will be found to be correct, and they can be used to reconstruct the large toroidal system so that it is less susceptible to the M-even effect.
Chapter 5

Orientation Dependence

5.1 The Effect

In the summer of 2006, Alex and Tim measured the electron’s EDM with more precision than we had previously achieved using this experiment. In making their measurement, however, they discovered that the result that they achieved changed when they rotated the toroid by 180°.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Orientation</th>
<th>Difference Asymmetry (μV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>88 K</td>
<td>Normal</td>
<td>$-0.05 \pm 0.13$</td>
</tr>
<tr>
<td>88 K</td>
<td>Reversed</td>
<td>$-1.01 \pm 0.13$</td>
</tr>
<tr>
<td>127 K</td>
<td>Normal</td>
<td>$0.70 \pm 0.05$</td>
</tr>
<tr>
<td>127 K</td>
<td>Reversed</td>
<td>$-0.19 \pm 0.08$</td>
</tr>
<tr>
<td>178 K</td>
<td>Normal</td>
<td>$0.10 \pm 0.08$</td>
</tr>
<tr>
<td>178 K</td>
<td>Reversed</td>
<td>$-0.76 \pm 0.11$</td>
</tr>
</tbody>
</table>

Table 5.1: Listed above are the results that Alex found for the voltage asymmetry induced by the electron’s EDM at our three operating temperatures for both sample orientations. These values can be found in [3, Table 2.3]. In the absence of any orientation dependence in our data, we would expect the difference asymmetries for the two sample orientations to be equal.

They reversed the Faraday cage that the toroid sits in and the magnetic driving
CHAPTER 5. ORIENTATION DEPENDENCE

coil wrapped around the cage, keeping the relative orientations of the toroid, the cage, and the coil fixed. The only modifications that they made in this rotation were to switch the orientation of the entire apparatus relative to the room around it and to switch the channel on our preamplifier that each of the electrodes was attached to. The orientation dependence that Alex and Tim measured must therefore be related to one of these two changes.

It is possible, but rather unlikely, that the effect originates from switching the electrodes’ preamplifier channels. Symmetry is extremely important in our experiment, and we designed the detector with this in mind. Not only are the two channels electronically identical, but they also are constructed to be physically symmetric for a 180° rotation about the toroid’s central axis.

![Diagram](image)

Figure 5.1: Pictured above is the mask from which we make our preamplifier circuit. This mask is symmetric on a rotation of 180° about its center.

When we rotate the sample in relation to the preamplifier, we would expect this symmetry to cause any effects that depend on the relative orientations of the two to cancel out.

The most likely source of the position dependence in our data is the change in the orientation of the sample itself relative to the rest of the laboratory. Our sample is shielded from stray electric fields, but is not very well shielded from magnetic fields. In fact, we have observed that magnetic fields incident on our sample can affect our
data. When we rotate the sample by $180^\circ$, we would expect to see the effect of any ambient magnetic fields in the horizontal plane to invert, changing the magnitude of the voltage asymmetry that we measure. In order to minimize our results' orientation dependence I explored the ways in which magnetic fields can affect the asymmetries that we observe.

## 5.2 Applied Nulling Fields

Our experiment was designed to account for the possibility that stray magnetic fields may affect our data. We worry in particular about effects that the earth’s magnetic field might have on our results. In order to cancel the earth’s magnetic field and any other ambient magnetic field incident on our apparatus, we generate a field equal in magnitude and opposite in direction to the field that we measure at the center of our apparatus. This field, which we call the *applied nulling field*, should cancel with the incident field, resulting in a net magnetic field of zero incident on our toroid. We generate this field using three sets of Helmholtz coils, aligned so that their axes are mutually orthogonal.
The three pairs of Helmholtz coils centered on the toroidal sample can be seen above. Here the preamplifier and the toroidal magnetic coil are visible because the vacuum can and liquid nitrogen dewar that cover them while we are running our experiment have been removed.

We also used these coils to investigate the effects that stray fields incident on our sample could have. We used them to apply fields of various magnitudes to our sample while we measured the asymmetry between the two sample magnetizations for each electrode. We then plotted these asymmetries as functions of the incident field that we applied in order to represent the effects that incident fields might on our data. In order to avoid the effects of the M-even systematic in these measurements, I measured this asymmetry at a relatively high toroidal pulsed magnetic field value of approximately 360 Oe. Since the magnitude of the M-even systematic decays at high fields, we can expect its effects here to be small. I will present the results of this investigation in the following chapter.
5.3 Models of Magnetic Dependence

Our precision measurement relies on our experiment’s inherent symmetry. Magnetic fields are vectors, and ones in the vicinity of our apparatus can specify a particular direction in space that can break this symmetry. Ambient magnetic fields can create asymmetries in the voltage we measure that corresponds to this asymmetry in our apparatus, and we must carefully consider the effects that they might have. I will examine the effects of magnetic fields in the three orthogonal directions defined by the fields produced by of our three Helmholtz coils. I will call these directions \textit{perpendicular}, for fields that point perpendicular to the plane of our two electrodes, \textit{axial}, for fields that point along the toroid’s central axis, and \textit{parallel}, for fields that point parallel to the electrodes but perpendicular to the toroid’s axis.

![Diagram of the toroidal sample](image)

Figure 5.3: a diagram of the toroidal sample showing the three axes that I use to describe the effects of fields incident on our apparatus

I will first examine the effects that a stray perpendicular field could have on our signal. This field would break the symmetry between the two electrodes, increasing
the magnetic field on one and decreasing it on the other. We have seen that the M-even effect depends on the strength of the magnetic field at each electrode. An incident perpendicular field would raise the M-even potential on one electrode and lower it on the other. Then, when we flip the direction of the sample’s magnetization, it would lower the potential on the first electrode and would raise it on the second, creating an effect antisymmetric between the two electrodes. When we change the sample’s orientation, we will effectively reverse the direction of any incident field in the perpendicular direction. This will in turn invert the field’s effect, generating a sample orientation dependence in our results.

Figure 5.4: Shown above are two schematic diagrams of our toroid showing the contributions that ambient perpendicular fields would make to the total magnetic field in the sample. The field circulating around the toroid is the field that we apply to magnetize our sample, and the two straight arrows represent an ambient field.
Figure 5.5: Pictured above are plots for the three temperatures and the two sample orientations involved in our experiment showing the dependence of the asymmetry between the two electrodes on the perpendicular field. In these plots, Electrode A and Electrode B refer to the two electrodes, not to the two channels of our detector, which switch from one electrode to the other as we rotate our sample. These plots are shown in units convenient for work in our laboratory, voltage uncorrected for the gain of our amplifier and current through the Helmholtz coils that produce the magnetic field. Our amplifier has a gain of 4400, and I include conversion factors from current to applied field for each of our Helmholtz coils in equation A.4.
My experimental results agree with this analysis of the effects perpendicular nulling fields might have. Figure 5.5 shows that incident fields in the perpendicular direction have opposite effects on the two electrodes. The potentials that they generate invert when we switch the relative orientations of the sample and the incident field by rotating the sample. These potentials are on the order of $5\mu V$ on our sample, which is about the magnitude we would expect the M-even effect to take on for small changes in the field at the high-field limit.

A stray field in the parallel direction would increase our toroidal field on one half of the sample and would decrease it on the other, causing the sample to differentially magnetize. When we reverse the direction of the toroidal field this imbalance in magnetization would switch from one side of the toroid to the other and any effect that it might have would be antisymmetric with respect to the toroidal field. Differential magnetization could find its way into our results through the M-even effect, whose magnitude clearly depends on the sample’s magnetization. The changes in the sample’s magnetization should be the same for both electrodes, and the effect should take a “common-mode” form. Also, much as the effect changes in sign when we switch the direction of the applied toroidal field, it should also invert when we reverse the sample orientation.

Figure 5.6: the contributions that an ambient parallel field would make to the total field in the toroid
Figure 5.7: Our data’s dependence on the parallel field.

Our data for 88 K agree with this analysis of the effects of incident parallel fields. They show a common-mode dependence that reverses when we rotate the sample. At 127 K, we see very little dependence on the applied field in this direction (the
plot's vertical scale has changed, and the signal that we observe is actually much smaller than it is at the other temperatures). Perhaps this decrease in magnitude occurs because the sample's two halves have equal magnetic susceptibilities at this temperature, providing an extra source of symmetry that would make it more difficult to unbalance their magnetizations. The data at 178 K seem to indicate the presence of a combination of a common-mode effect and an odd effect. In my analysis above, I imagined an idealized version of our apparatus; some small imperfection in our system could have caused this odd effect to appear.

It is more difficult to imagine how an axial field could affect the difference asymmetry that we measure. If we assume that the toroid is perfectly symmetric, then our experiment should be invariant on reflection about the plane perpendicular to the toroid’s axis and passing through its center. Let us imagine for the moment that a stray axial field changes the difference asymmetry that we measure by an amount $(\Delta V_{\text{even}} + \Delta V_{\text{odd}})$, where $\Delta V_{\text{even}}$ and $\Delta V_{\text{odd}}$ are the components of the change even and odd with respect to the stray field. A field of the same magnitude but opposite in direction would change the difference asymmetry by $(\Delta V_{\text{even}} - \Delta V_{\text{odd}})$. The toroid’s reflection symmetry guarantees that these two fields will have the same effect, that $(\Delta V_{\text{even}} + \Delta V_{\text{odd}}) = (\Delta V_{\text{even}} - \Delta V_{\text{odd}})$. It follows that $\Delta V_{\text{odd}} = 0$. Theory therefore excludes any odd dependence on incident fields in the axial direction, although even dependence is allowed.
Figure 5.8: our data’s dependence on the axial field

Our experimental data on incident fields agrees only partially with this analysis. At 88 K and 127 K, we see, as theory predicts, that our signal’s dependence on incident axial fields is unchanged when we reverse the sample’s orientation. At 178
K, the dependence inverts when we rotate the sample. In both cases, the dependence that we see is almost entirely odd with respect to the incident field. This behavior is puzzling, but it can be explained by some small asymmetry in our apparatus that allows an effect that is by nature even to manifest itself as odd, much as how the M-even effect gives rise to the odd M-even systematic. If this were the case, the effect would appear as odd between the two electrodes, but, since the asymmetry would rotate with the sample, it would be not depend on sample orientation.

This result for axial fields is especially troubling. We can correct for odd effects of imperfectly canceled incident fields by averaging the data that we take for the two opposite sample orientations. We cannot, however, correct for effects even with respect to sample orientation. When we compare the values we obtain for the difference asymmetry at our three operating temperatures, we filter out any remaining magnetic effects to the extent that they are independent of temperature. However, we cannot filter an effect that is both temperature-dependent and even. In our analysis, we assume that there are no significant effects even with respect to ambient fields, but my analysis above would argue otherwise. There appear to be even effects at play here, and, even worse, their behavior in the data that I have taken depends on the sample’s temperature. In EDM measurements that we make in the future, we may need to rethink our method of screening out effects related to fields incident on our sample in order to correct for this type of effect as well.

5.4 Fine-Tuning the Applied Nulling Fields

The position dependence in our results suggests that there is a non-zero magnetic field incident on our apparatus. It appears as though our nulling fields are not calibrated precisely. I recalibrated our applied nulling fields in order to remove the position
In the past, we have calibrated the applied nulling fields by replacing our apparatus with a magnetometer and tuning them until the magnetometer reads zero. This method should give us a net incident field reasonably close to zero when we are running our experiment. However, when we replace the magnetometer with our apparatus again and run our experiment, we change some factors that could lead to changes in the field incident on our toroid. We enclose the sample with a vacuum can, which in turn is immersed in a dewar of liquid nitrogen, neither of which is in place when we calibrate our fields with the magnetometer. We also turn on our electronics, which can generate stray magnetic fields. In order to take these factors into account, we must calibrate our nulling fields while our experiment is running. It would be impossible to use a magnetometer to make this measurement because our experiment itself generates large magnetic fields as we flip the magnetization of the toroid from one direction to the other. Instead, in order to remove any orientation dependence from the asymmetry that we measure, I used the asymmetry measurement itself.\footnote{At first glance this method of removing orientation dependence from our data may seem ill-advised. We observe that our applied nulling fields can affect the asymmetry that we measure, but we have no reason to believe that they are the only factor at play. When we use our measurement itself in our calibration, we sacrifice information on the value of the net field that is incident on our sample. We can successfully remove any orientation dependence from our data, but we cannot be certain that we haven’t simply calibrated our applied nulling fields so that they create an effect that exactly cancels some other systematic effect that we haven’t considered. The existence of this possibility is a necessary drawback of the way our experiment is constructed, because it is very difficult for us to measure the field incident on our experiment while we are taking data. The best justification for our method of calibration that we can cite is that we cannot think of any other likely source of error that would have this orientation dependent signature. Since, at least as far as we know, there is no source of error other than incident fields that we could be picking up in this way, we can be reasonably confident that by tuning the error that we measure to zero, we are tuning the field incident on our sample to zero as well.}

I measured the asymmetry between the two sample magnetizations for each electrode at different values of the applied nulling fields. I varied the field in each of the Helmholtz coils’ three orthogonal directions, keeping the field in the other two
CHAPTER 5. ORIENTATION DEPENDENCE

directions fixed, in order to isolate our signal’s dependence on each individual field. The asymmetry seems to depend linearly on the magnitude of each field direction, so I use a linear fit to model this dependence. I used these linear fits to represent the voltage asymmetry $A$ as a function of the three applied fields, $H_\parallel$, $H_\perp$, and $H_{axial}$.

$$A = C + C_\parallel H_\parallel + C_\perp H_\perp + C_{axial} H_{axial}$$  \hspace{1cm} (5.1)

where $C$, $C_\parallel$, $C_\perp$, and $C_{axial}$ are constants determined from the equations of fit for the asymmetry’s field dependence. I obtained two of these equations, one for each sample orientation, for each of the three temperatures. In order to determine the nulling fields that we need to apply to remove the position dependence from our results, I set the two asymmetry equations for each temperature equal, constraining the orientation dependence to zero at each of the three temperatures. These three constraints allow me to solve the system of equations algebraically for the three variables, $H_\parallel$, $H_\perp$, and $H_{axial}$.

It would be preferable to take data on the applied nulling fields at different temperatures in order to add additional constraints to the system of equations. This would allow me to solve for the applied nulling field values statistically and to get a better idea of whether adjusting the applied nulling field truly fixes a source of systematic error or whether it simply creates an effect tuned precisely to mask some other effect at each of the temperatures where we take data. Unfortunately, we rely on the temperature to tune the toroid’s magnetic susceptibility, and we cannot take reliable data at temperatures other than the three that we use in our EDM measurement.

In Appendix A I go into some detail about my calculation for the nulling field recalibration. Here I will present only the results of that calibration. These results are based on a combination of data that Alex took in the fall of 2006 and data that
I took in the winter of 2008. The values that I obtained for the nulling fields that I should apply are roughly on the scale of the earth’s local magnetic field, although they do not point in the correct direction. However, we should not be troubled by this disagreement in direction since there are probably other sources of ambient fields in our laboratory.

<table>
<thead>
<tr>
<th>Field Component</th>
<th>Applied Nulling Field (Oersted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel</td>
<td>-0.67</td>
</tr>
<tr>
<td>Perpendicular</td>
<td>-0.20</td>
</tr>
<tr>
<td>Axial</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 5.2: Listed above are the results of the calculation that I describe in Appendix A. The signs of these numbers are the result of an arbitrary convention that we use in our laboratory, rather than something fundamental about our experiment. The experiment is theoretically symmetric, so there is no inherent reason to label any particular direction as positive.

I should note that I have ignored the large degree of statistical uncertainty associated with my calculation of the nulling fields. The linear fits on which I based my calculation have two parameters and involve only three data points. These fits introduce a significant amount of uncertainty into my analysis. In this section of my thesis, I am not measuring something of fundamental importance, but rather am making a measurement of a phenomenon specific to our laboratory and our apparatus. For that reason I ignored uncertainty here and instead calculated the value that represents my best guess for how we should recalibrate our applied nulling fields. When we consider this value, however, we must take into account that it represents nothing more than a rough estimate.

I made a preliminary measurement of the electron’s EDM using the applied nulling field recalibration that I calculated above. I will discuss my calculation of the electron’s EDM from this measurement in §6.1, but in this section I will reference the sample orientation dependence in my results. Table 5.3 shows the values for this
orientation dependence.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Orientation</th>
<th>Difference Asymmetry (μV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>88 K</td>
<td>Normal</td>
<td>0.63 ± 0.14</td>
</tr>
<tr>
<td>88 K</td>
<td>Reversed</td>
<td>−0.39 ± 0.09</td>
</tr>
<tr>
<td>127 K</td>
<td>Normal</td>
<td>0.36 ± 0.07</td>
</tr>
<tr>
<td>127 K</td>
<td>Reversed</td>
<td>−0.39 ± 0.05</td>
</tr>
<tr>
<td>178 K</td>
<td>Normal</td>
<td>−2.8 ± 0.3</td>
</tr>
<tr>
<td>178 K</td>
<td>Reversed</td>
<td>−2.0 ± 0.1</td>
</tr>
</tbody>
</table>

Table 5.3: Shown above are the difference asymmetry values that I calculated in my preliminary EDM measurement.

These results are not as precise as Alex’s were, in particular at 178 K, but they do not show as clear or as systematic a dependence on the sample’s orientation as his did.

After recalibrating the applied nulling fields according to the calculation above, I completed another iteration of this nulling field calibration analysis. I hoped that this second iteration would confirm the results of the first, proving my method for calibrating the fields to be legitimate. Unfortunately, the results that I achieved were not very encouraging.

<table>
<thead>
<tr>
<th>Field Component</th>
<th>Applied Nulling Field (Oersted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel</td>
<td>7.1</td>
</tr>
<tr>
<td>Perpendicular</td>
<td>0.84</td>
</tr>
<tr>
<td>Axial</td>
<td>33</td>
</tr>
</tbody>
</table>

Table 5.4: Listed above are the results of my second-iteration calculation, which is described in Appendix A.

The predictions from my second-iteration analysis, with the possible exception of the prediction for the perpendicular field, are absurdly large. We may not be able to exactly measure the field in our experimental apparatus while it is running, but we have never measured an incident field of more than an Oersted. It seems unreasonable
that we would need to apply a field of 33 Oersteds in order to cancel the magnetic field incident on our sample. We should take these aberrant predictions not as a new calibration for our applied nulling field but as an indication that we should not place much confidence in results that we obtain with this process.

It is obvious that my method for recalibrating the applied nulling fields isn’t as precise as we had hoped it would be. However, in its first iteration, it seems to have steered us toward a value for the applied nulling fields that was at least somewhat successful in removing the position dependence from our results. The first iteration may have been superior to the second in some way. It was based on a set of data that Alex took and a set that I took, which combined to form more data than the single set of data that I based the second iteration on. Averaging this greater number of data points may have decreased the effects of random noise on our results. My data for my second iteration deviate more from their linear fit than the data from my first iteration do, especially at 178 K. These irregularities could also point to inaccuracies in our detector or in some other part of our apparatus that were present for the second iteration but not for the first. It is possible that for one of these reasons my initial recalibration was more accurate than my second would lead us to believe; however, the poor results of the second analysis make it difficult for me to claim that the values I obtained in my first one are correct.

Although my applied nulling field recalibration seems to have been somewhat successful in reducing the sample orientation dependence in the results of my preliminary electron EDM measurement, we will need to improve our calibration method in the future. An obvious improvement would be to take more than three data points for each applied field direction, reducing the statistical uncertainty associated with the linear fits. Another area where we could refine our method is our means of compensating for the M-even systematic. I tried to minimize its effects by taking data at
high values of the toroidal field. However, merely taking data at high fields does not
totally cancel the systematic. In order to completely rid our calibration of its effects,
we would need to take data at multiple values of the toroidal pulsed magnetic field
and statistically remove the systematic as we do in our measurement of the electron’s
EDM. These modifications would dramatically increase the amount of data that we
would need to take to complete this calibration, but they may be necessary if we are
to achieve an accurate and reliable result.
Chapter 6

Conclusion

6.1 A New Limit for the Electron EDM

In order to test my calibrations for the applied nulling fields, I conducted a preliminary measurement of the electron’s intrinsic EDM. I didn’t take as much data for this measurement as we have taken in our previous EDM measurements, and as a result I didn’t achieve the levels of precision that we have seen before. Figure 6.1 shows the difference asymmetry in the potential drop across the sample as a function of the applied toroidal pulsed magnetic field for each temperature and for each sample orientation. Also included in this figure are the fits to equation 4.2 that we use to isolate our results from the M-even systematic. The asymptotic values of these fits are shown in table 6.1. I took half of the data points for each temperature with the toroidal driving magnetic coil reversed with respect to our pulsed current source, in order to correct for any bias in the current source.
Figure 6.1: the data from my EDM measurement, along with their asymptotic fits
Table 6.1: This table presents my results for the portion of the potential drop across the sample due to the electron’s EDM for each sample orientation and for each temperature.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Orientation</th>
<th>Difference Asymmetry ((\mu V))</th>
</tr>
</thead>
<tbody>
<tr>
<td>88 K</td>
<td>Normal</td>
<td>0.63 ± 0.14</td>
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<td>88 K</td>
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<td>−0.39 ± 0.09</td>
</tr>
<tr>
<td>127 K</td>
<td>Normal</td>
<td>0.36 ± 0.07</td>
</tr>
<tr>
<td>127 K</td>
<td>Reversed</td>
<td>−0.39 ± 0.05</td>
</tr>
<tr>
<td>178 K</td>
<td>Normal</td>
<td>−2.8 ± 0.3</td>
</tr>
<tr>
<td>178 K</td>
<td>Reversed</td>
<td>−2.0 ± 0.1</td>
</tr>
</tbody>
</table>

These data do not appear to depend on the sample’s orientation in any systematic way, but they may carry some orientation dependence. To cancel this possible dependence, I averaged the values from the two sample orientations at each temperature. My average value for \(V_{EDM}^T\) for each temperature is shown in table 6.2.

Table 6.2: the values for \(V_{EDM}^T\) that I obtained for each of our three temperatures

<table>
<thead>
<tr>
<th>Temperature</th>
<th>(V_{EDM} (\mu V))</th>
</tr>
</thead>
<tbody>
<tr>
<td>88 K</td>
<td>0.13 ± 0.16</td>
</tr>
<tr>
<td>127 K</td>
<td>−0.01 ± 0.08</td>
</tr>
<tr>
<td>178 K</td>
<td>−2.4 ± 0.3</td>
</tr>
</tbody>
</table>

Plugging these values into equation 3.2, I obtain a final value for \(V_{EDM}\) of 1.1 ± 0.3 \(\mu V\). This value for \(V_{EDM}\), along with the scaling factor that I discuss in §2.4, gives an EDM value of \((2.0 ± 0.6) \cdot 10^{-23}\) e-cm. This value is inconsistent with zero within the range of uncertainty; however, it is not large enough relative to its uncertainty that it should be considered very significant, and it is much larger than the current experimental limit on the electron’s EDM from [1]. Although it seems to indicate a non-zero electron EDM, I will take its maximum value as a limit.

\[|d_e| < 2.6 \cdot 10^{-23} \text{ e-cm}\]
CHAPTER 6. CONCLUSION

The precision of this limit is severely hampered by the large uncertainty involved in the fits to our data for 178 K. We could probably decrease our limit by improving the precision with which we take our data at this problematic temperature. We keep the preamplifier at the temperature where it was designed to operate when we take data at 88 K and 127 K. When we make our measurement at 178 K, we have to increase its temperature by 18 K, which slows the detector’s response to changes in the potential on the electrodes. The signal from the detector takes a larger portion of the measurement cycle to decay to its final value, and we are left with less of the cycle to average over when we measure the potential on each electrode. This decrease in averaging time in turn increases the statistical spread of the data points that we obtain at this temperature. If we eliminate this problem with our measurement at 178 K, we will probably be able to improve on the EDM limit that I quote here.

6.2 The Experiment’s Future

I started working on this experiment three years ago during a time of anticipation of improvement. We had recently discovered a new systematic effect, which would come to be known as the M-even systematic, but we were hopeful that we could overcome it with a few simple modifications. Over the intervening years, we have discovered that the systematic is more insidious than we had imagined and that there was a very real possibility that it could ruin our experiment. At times it has seemed that we would never be able to get past it, but I hope that I have illuminated its nature enough that we can loosen its stranglehold on our results. We may be able reduce it to the point where it no longer is the primary limiting factor on our precision. We also may reduce it as much as possible, only to discover that as much as possible isn’t quite enough. I am confident, however, that we now have achieved an understanding
of the effect that will allow us to start refining our precision. I hope to have left in a
time of optimism once again for this experiment. I was unsuccessful in convincingly
correcting for the effects of ambient fields, but now we know what must be done to
compensate for them. I haven’t proven my conjectures about the nature of the M-
even effect on toroidal samples, but I am hopeful that they will provide us with the
opportunity to reduce the M-even systematic significantly.

Precision measurements can at times be tedious and frustrating, but when the
quantity to be measured is of as much fundamental importance as the electron EDM,
it is almost always worthwhile to keep trying. It would be shortsighted for us not to
acknowledge that the M-even systematic represents a potentially fatal pitfall for our
experiment. However, it would be irresponsible for us not to continue with our work.
Our experiment uses an innovative and unique method, and we have put enough effort
into it that we should push this method as far as it will go. It is entirely possible that
we will be unable to reduce the M-even systematic enough to improve our precision
to the point where we can make a competitive measurement, but the small possibility
that we will be able to make an impact on an area of physics with such fundamental
importance makes our efforts more than worthwhile.
Acknowledgements

Financial support for experiment on which my thesis is based comes from the National Science Foundation (under Grant No. 0555715), Los Alamos National Laboratory, The Howard Hughes Medical Institute, Amherst College, and Pacific Ceramics.

I am deeply indebted to all of the professors I have had at Amherst for the roles they have played in the extraordinary education that I have received here. Particularly important in the writing of this thesis were the professors of the Physics department. My work here draws on so much that each of them has taught me. Beyond the Physics department, Professor Gentzler has devoted an enormous amount of time to helping me to hone my writing abilities; her efforts have certainly improved the quality of this thesis.

I’d like to thank my family for their support and encouragement over the years. I’d also like to thank all of my friends at Amherst, and especially Carl Wang, Morgan Holland, Brendan Milliner, Andrew Whelan, Cong Geng, Ernesto Acosta, Nicole Kinsley, and Kerimcan Oral, for providing me with the distractions that I needed to survive the creation of this thesis, as well as for making my experience here as exciting and rewarding as it could possibly have been.

I’m grateful to Noah, Oliver, Ben, and Alex for all of the amazing work that they have done on this experiment. And of course, we owe the fact that our experiment works at all to Dan Krause and Bob Cann’s technical wizardry, Norm Page’s electrical expertise, and Bob Bartos’ logistical prowess. I also want to thank the students I’ve worked with in this laboratory over the years, Ben Heidenreich, Jeff Grover, and Tom Langin, for putting up with me for all those hours. I owe Professor Gordon for both his experimental know-how and his theoretical perspicacity. And finally, I would like to thank Professor Hunter for sharing his enthusiasm and expertise with me, and for making this thesis such a valuable, fascinating, and enjoyable experience.
Appendix A

Recalibrating the Applied Nulling Fields

The regressions that I performed on the data shown in figures 5.5, 5.7, and 5.8 provided me with the following equations to represent the voltage asymmetry in terms of the applied fields. All constants in these equations have units of millivolts or millivolts per ampere.

\[ A^{Normal}_{88K} = -6.5 - 2.1I_{\parallel} + 19.9I_{\perp} + 25.4I_{axial} \]
\[ A^{Reversed}_{88K} = -12.0 - 3.3I_{\parallel} - 20.2I_{\perp} + 21.3I_{axial} \]
\[ A^{Normal}_{127K} = 2.2 + 7.8I_{\parallel} - 36.5I_{\perp} - 14.7I_{axial} \]
\[ A^{Reversed}_{127K} = 8.5 - 2.9I_{\parallel} + 34.7I_{\perp} - 11.8I_{axial} \]
\[ A^{Normal}_{178K} = -9.7 - 36.6I_{\parallel} + 25.4I_{\perp} + 11.9I_{axial} \]
\[ A^{Reversed}_{178K} = 9.3 + 37.6I_{\parallel} - 26.6I_{\perp} - 12.6I_{axial} \]
APPENDIX A. RECALIBRATING THE APPLIED NULLING FIELDS

Setting the equations of constraint,

\[
\begin{align*}
A_{88K}^{\text{Normal}} &= A_{88K}^{\text{Reversed}} \\
A_{127K}^{\text{Normal}} &= A_{127K}^{\text{Reversed}} \\
A_{178K}^{\text{Normal}} &= A_{178K}^{\text{Reversed}}
\end{align*}
\]  

(A.2)

I can solve these equations to find that

\[
\begin{align*}
I_\parallel &= -0.31 A \\
I_\perp &= -0.14 A \\
I_{axial} &= 0.12 A
\end{align*}
\]

(A.3)

And, using the following conversions,

\[
\begin{align*}
H_\parallel &= \left(2.12 \frac{G}{A}\right) I_\parallel \\
H_\perp &= \left(1.4 \frac{G}{A}\right) I_\perp \\
H_{axial} &= \left(2.6 \frac{G}{A}\right) I_{axial}
\end{align*}
\]

(A.4)

I can represent these results in terms of the fields that we must apply to eliminate the orientation dependence of our data.

<table>
<thead>
<tr>
<th>Field Component</th>
<th>Applied Nulling Field (Oersted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel</td>
<td>-0.67</td>
</tr>
<tr>
<td>Perpendicular</td>
<td>-0.20</td>
</tr>
<tr>
<td>Axial</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table A.1: the results of my recalibration calculation

Figures A.1-3 show the data from the second iteration of my calibration analysis.
Figure A.1: the second-iteration plots for the perpendicular field
Appendix A. Recalibrating the Applied Nulling Fields

Figure A.2: the second-iteration plots for the parallel field
Figure A.3: the second-iteration plots for the axial field
APPENDIX A. RECALIBRATING THE APPLIED NULLING FIELDS

My second iteration of this analysis yielded the following equations of fit.

\[
\begin{align*}
A^{\text{Normal}}_{88\text{K}} &= 5.8 + 6.6I_\parallel - 15.5I_\perp - 19.0I_{\text{axial}} \\
A^{\text{Reversed}}_{88\text{K}} &= 9.2 + 2.6I_\parallel + 16.3I_\perp - 19.7I_{\text{axial}} \\
A^{\text{Normal}}_{127\text{K}} &= -7.9 - 4.0I_\parallel + 33.2I_\perp + 13.3I_{\text{axial}} \\
A^{\text{Reversed}}_{127\text{K}} &= -3.4 + 6.3I_\parallel - 32.2I_\perp + 13.3I_{\text{axial}} \\
A^{\text{Normal}}_{178\text{K}} &= -90.9 + 37.5I_\parallel - 25.5I_\perp - 21.9I_{\text{axial}} \\
A^{\text{Reversed}}_{178\text{K}} &= 24.6 - 42.0I_\parallel + 30.9I_\perp - 12.7I_{\text{axial}}
\end{align*}
\]

(A.5)

Solving these equations, I obtain

\[
\begin{align*}
I_\parallel &= 3.36\, A \\
I_\perp &= 0.60\, A \\
I_{\text{axial}} &= 12.82\, A
\end{align*}
\]

(A.6)

Table A.2 shows the results of this calculation in units of Oersted.

<table>
<thead>
<tr>
<th>Field Component</th>
<th>Applied Nulling Field (Oersted)</th>
</tr>
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<td>Perpendicular</td>
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<td>Axial</td>
<td>33</td>
</tr>
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Table A.2: the results of my second-iteration calculation
Bibliography


