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Preface

Local Lorentz invariance is the idea that the laws of physics do not change under boosts and rotations. Because it is so engrained in physics it may sound absurd that anyone is trying to test it. However, some modern theories beyond the Standard Model and General Relativity predict such a violation, and there are many ongoing experiments that are trying to look for its effect.

My initial attempt in trying to understand the reasoning behind predictions of Lorentz violation has failed due to the high level of background physics the papers on such theories require. None of the previous theses have looked into this subject matter in detail, so there wasn’t any obvious starting point. After much trial and error, however, I have found some accessible books on the subject, and have made some progress. As a reference for future thesis students, I have sketched out the basic theoretical motivation behind looking for Local Lorentz Invariance in a comprehensible manner and have provided a reasonable starting point for pursuing the subject further. Also, as a reminder, in the theory section, I have set $\hbar$ and $c$ to 1 for the most part.

With that said, this is an experimental thesis, so a thorough understanding of the apparatus, the experimental techniques, and the mathematics behind making the measurement is crucial. Fortunately, the apparatus and the mathematics behind making the measurements has been discussed in previous theses. Therefore, a detailed treatment has been given only when I found a different way to think about these subjects.

The Experimental Apparatus section includes few subsections on basic experimental techniques for using the equipment. Only a cursory treatment has been given of actual data collection protocols because Stein’s thesis [20]
explains it in detail.

To analyze the data more efficiently, I have written a Mathematica notebook code for rejecting points that lie outside the $3\sigma$ range for a given parameter, and another code for sine fitting the data points. I hope these come in handy for future thesis students.
Chapter 1

Experimental Motivation

1.1 Lorentz Transformation

A Lorentz Transformation is defined as a linear transformation $x^\mu \rightarrow \Lambda_\nu^\mu x^\nu$ that leaves the following quantity invariant.

$$\eta_{\mu\nu}x^\mu x^\nu = -t^2 + x^2 + y^2 + z^2$$  \hspace{1cm} (1.1)

where $\eta_{\mu\nu}$ is a diagonal matrix with elements \{-1,1,1,1\}. In general, the combination of boosts and rotations create Lorentz transformations, and the set of all possible Lorentz transformations forms a group.$^1$

A generic group element $g(\theta)$ that is parametrized by a continuous parameter $\theta$ can be represented by a linear operator $D_R(g(\theta))$. The linear operator, in turn, can always be represented as \(^2\)

$$D_R(g(\theta)) = e^{i\theta T_R}$$  \hspace{1cm} (1.2)

where $T_R$ is the generator of the group. The variable $\theta$ can be multi dimensional, in which case the exponent becomes $i\theta_{a_1,a_2,a_3...}T_R^{a_1,a_2,a_3...}$

By Noether’s theorem, when a Lagrangian has a symmetry group, the generator of the group is related to the conserved charge of the system described by the Lagrangian.$^3$ This can be best understood by looking at some simple
examples from introductory quantum textbooks.4

\[ f(x + x_0) = e^{i\hat{p}x_0}f(x) \]

\[ \Psi(x, t + t_0) = e^{-i\hat{H}t_0}\Psi(x, t) \]

These equations indicate that \( e^{i\hat{p}x_0} \) is a space translation operator and \( e^{-i\hat{H}t_0} \) a time translation operator. The continuous variables here are \( x_0 \) and \( t_0 \). The momentum \( \hat{p} \) is a conserved “charge” in space translation, and it generates the space translation operator. Likewise, the Hamiltonian without explicit time dependence is a conserved quantity in time translation, and it generates the time translation operator. Therefore, \( \hat{p} \) is multiplied by the continuous variable \( x_0 \) to generate the space translation operator as indicated in equation 1.2, and \( \hat{H} \) is multiplied by \( t_0 \) to generate the time translation operator.

Since \( D_R(g(\theta))'s \) are required to be linear transformations of the group elements, \( g(\theta)'s \), the \( D_R(g(\theta))'s \) need to preserve the group structure.5

\[ D_R(g(\theta_1)g(\theta_2)) = D_R(g(\theta_1))D_R(g(\theta_2)) \quad (1.3) \]

Substituting equation 1.2 into the above equation, taking the log of both sides and performing a Taylor expansion, we obtain at first order,6

\[ [T^a, T^b] = i f_c^{ab}T^c \quad (1.4) \]

Because the above equation is the defining relationship of the Lie group, it is called the Lie algebra. A set of transformations exhibit Lie group structure if and only if this equation is satisfied. The coefficients \( f_c^{ab} \) for Lorentz Transformations can be identified by calculating the commutation relation for a specific Lorentz charge. The easiest Lorentz charge to calculate is that of a relativistic point particle because the Lagrangian of a relativistic point particle is proportional to the line element. Starting from the infinitesimal Lorentz transformation \( x^\mu \rightarrow x^\mu + \epsilon^{\mu\nu}x_\nu \) and the Lagrangian, \( L = -mc^2\sqrt{1 - v^2/c^2} \), the Lie algebra for Lorentz Transformation is derived. First, using the invariance of the line element under Lorentz Transformation, \( \epsilon^{\mu\nu} \) is characterized as an
antisymmetric matrix.

\[
\delta(\eta_{\mu\nu}x^\mu x^\nu) = 0
\]

\[
= 2\eta_{\mu\nu}x^\mu(\delta x^\nu) = 2\eta_{\mu\nu}x^\mu(\epsilon^{\nu\rho}x_\rho) = 2\epsilon^{\nu\rho}x_\nu x_\rho = x_\nu x_\rho(\epsilon^{\nu\rho} + \epsilon^{\rho\nu})
\]

Therefore, \(\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}\). This simplifies the calculation of the Lorentz charge, \(M_{\mu\nu}\).

\[
L = -mc\sqrt{-\eta_{\mu\nu}\frac{\partial x^\mu}{\partial \tau}\frac{\partial x^\nu}{\partial \tau}} = -mc\sqrt{-\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} = -mc \cdot ds
\]

\[
\epsilon^{\mu\nu}M_{\mu\nu} = \frac{\partial L}{\partial x} \delta x = \frac{\partial(-mc\sqrt{-\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu})}{\partial \dot{x}} \epsilon^{\mu\nu}x_\nu
\]

\[
M_{\mu\nu} = \frac{mc\eta_{\mu\rho}\dot{x}^\rho}{\sqrt{-\eta_{\mu\rho}\dot{x}^\mu\dot{x}^\rho}} x_\nu = \frac{mc \cdot dx_\mu/d\tau}{\sqrt{-\eta_{\mu\rho}\dot{x}^\mu\dot{x}^\rho/d\tau}} x_\nu = mc \frac{dx_\mu}{ds} x_\nu = p_\mu x_\nu
\]

where \(\tau\) is the proper time of the particle. Since \(\epsilon^{\mu\nu}\) is antisymmetric, the symmetric part of \(M_{\mu\nu}\) has no contribution in \(\epsilon^{\mu\nu}M_{\mu\nu}\). Therefore, \(M_{\mu\nu}\) can be defined as an antisymmetric tensor.

\[
\epsilon^{\mu\nu}M_{\mu\nu} = \frac{1}{2}(2\epsilon^{\mu\nu}p_\mu x_\nu) = \frac{1}{2}(\epsilon^{\mu\nu}p_\mu x_\nu + \epsilon^{\nu\mu}p_\nu x_\mu) = \frac{1}{2}(\epsilon^{\mu\nu}p_\mu x_\nu - \epsilon^{\nu\mu}p_\nu x_\mu) = (-\frac{1}{2}\epsilon^{\mu\nu})(x_\mu p_\nu - p_\mu x_\nu) \quad (1.5)
\]

\[
M_{\mu\nu} \text{ differs from } x_\mu p_\nu - p_\mu x_\nu \text{ only by a constant factor. Since the } D_R(g(\theta)) \text{ only depends on the product of the Lorentz charge and the coordinate, the constant can be absorbed into the coordinate by re-parametrization and the Lorentz charge can be redefined as } x_\mu p_\nu - p_\mu x_\nu \quad (1.6)
\]

\[
M^{\mu\nu} = x^\mu p^\nu - p^\mu x^\nu \quad (1.7)
\]

The final step in obtaining Lorentz algebra is calculating the commutation relation between the Lorentz charges. Using equation 1.7 and the Heisenburg uncertainty relation \([x^\mu, p^\nu] = i\eta^{\mu\nu}\), it is a trivial exercise to obtain the Lorentz
Algebra.\(^9\)

\[
[M^{\mu\nu}, M^{\rho\sigma}] = i\eta^{\mu\rho} M^{\nu\sigma} - i\eta^{\nu\rho} M^{\mu\sigma} + i\eta^{\mu\sigma} M^{\nu\rho} - i\eta^{\nu\sigma} M^{\mu\rho}
\]  

(1.8)

The Lorentz Algebra ensures that the Lorentz transformations form a Lie group. If there is no set of operators that satisfy the above equation in a particular theory, then there is no group of transformations that conserves the line element, \(\eta_{\mu\nu} x^\mu x^\nu\). Therefore the theory must violate Lorentz symmetry. In other words, the Lorentz Algebra completely characterizes Lorentz symmetry.

### 1.2 Sources of Lorentz Violation

There are two general ways a theory can exhibit a violation of Lorentz symmetry. The first one is by lacking a set of transformation operators that satisfy Lorentz algebra. The second is by having a background field that points to a specific direction and permeates through the entire spacetime.

There is an important distinction to be made here. In the first case, the Lorentz violation is an intrinsic property of the theory, and the symmetry might not be recovered. However, in the second case, the Lorentz violation arises from the assumptions in the established physical theories that there is no stray background field. This assumption is implicit in the flat spacetime interpretation of free space in General Relativity. Likewise, the Standard Model does not account for any energy coupling between the particle and a background field of any form in a free particle solution. If such a background field exists, a complete understanding of the nature of this background field and its interaction laws with particles may allow the established theories to incorporate the background field. If this is the case, a successful integration of the background field in General Relativity and the Standard Model would recover the Lorentz Symmetry.\(^{10}\)

The nature of background field can be understood more intuitively by an analogy with an actual physical system. A crystal has a frozen-in electric and magnetic field because the atoms are fixed in a specific arrangement. Due to
these fields, an electron moving in different directions in the crystal feels different forces. If our theory does not account for the background fields and treat the electron as a free particle, then an apparent violation of Lorentz Symmetry would be observed. However, by properly accounting for the direction of background fields and their interaction with the electron, the Lorentz Symmetry is recovered. Similarly, if there is a preferred direction in the universe, a Lorentz violation would be observed in the current theoretical framework. However, by incorporating its direction and its interaction laws with the fundamental particles to the theory, Lorentz Symmetry can be reestablished.

The next two sections give examples of the two different types of Lorentz Violation.

1.2.1 Non-Commutative Geometry

Non-Commutative Geometry is defined by the following relation.\(^{11}\)

\[
[x^i, x^j] = i\theta^{ij} \quad (1.9)
\]

By the definition of a commutator, \(\theta^{ij}\) is bound to be an antisymmetric tensor. There are two natural ways to understand this equation. The first way is to see its connection with equation 1.4, and to identify the above equation as a Lie algebra with \(x^i\)’s as the generators. A more intuitive way to understand the equation is by assigning position operators, \(x\) and \(y\) to \(x^i\) and \(x^j\), and seeing that the two different position operators do not commute. This is analogous to what happens to the position and the momentum operator in the phase space in quantum mechanics. This interpretation naturally leads to the conclusion that two different spatial coordinates of an object cannot be measured simultaneously to an arbitrary precision. Therefore, in a non-commutative geometry, the spatial coordinates of a sphere become indeterminate, necessitating the usage of the term, “fuzzy sphere.”\(^{12}\)

The treatment of position operators as non-commutative entities may seem bizarre since quantum mechanics, the fundamental framework underlying the Standard Model, explicitly assumes the position operators to commute. How-
ever, in a high energy environment, the coordinate symmetry may break down.  
This phenomenon can be seen in the following example of an electron confined  
in two dimension under a strong magnetic field.  

The Lagrangian of an electron in a magnetic field is given by\(^{13}\)

\[
L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} + e\vec{v} \cdot \vec{A} - e\Phi 
\]  

(1.10)

If the field is uniform, then \(\vec{A}(\vec{r})\) can be written as \(-\frac{1}{2}(\vec{r} \times \vec{B})\),\(^{14}\) and if \(\vec{B}\) is repre-  

sented as a antisymmetric tensor, \(B_{ij} = \begin{pmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{pmatrix}\) then \(A_i\) can  

be represented as \(-\frac{1}{2}B_{ij}x_j\) and \(\vec{v} \cdot \vec{A}\) simplifies to \(-\frac{1}{2}B_{ij}\dot{x}^i x^j\). Moreover, since  

the electron motion is confined to two dimension, \(B_{ij}\) simplifies to \(\begin{pmatrix} 0 & B \\ -B & 0 \end{pmatrix}\)  

If the magnetic field is strong enough, then \(e\vec{v} \cdot \vec{A}\) term dominates and the  

corresponding Lagrangian and the canonical momentum becomes,\(^{15}\)

\[
L \approx -\frac{e}{2}B_{ij}\dot{x}^i x^j 
\]

(1.11)

\[
p_j = \frac{dL}{d\dot{x}^j} \approx -eB_{jk}x^k 
\]

(1.12)

Using the commutation relation between the momentum and the position op-  
erator, \([x^i, p^j] = i\delta^j_i\), the following non-commutative relationship between the  
coordinates can be obtained.

\[
eB_{jk} [x^k, x^j] = i\delta^j_i \]

(1.13)

By explicitly summing over \(j, k \in 1, 2\) for \(i = 1\) and \(i = 2\), the commutation  
relation \([x^2, x^1] = i\frac{B}{B}\) and \([x^1, x^2] = -i\frac{B}{B}\) is obtained. Therefore, in a strong, yet  
finite magnetic field, the coordinates of an electron confined in two dimension  
do not commute.  

It is easy to see how Lorentz Symmetry is broken in this “fuzzy world.”  
Imagine a free particle. Using the Lorentz charge calculated in equation 1.7,
the commutation relation between $M^{\mu\nu}$, and $M^{\rho\sigma}$ can be obtained using the values for $[x^i, p^j]$ and $[x^i, x^j]$. Because of the nonzero position operator commutators, $[M^{\mu\nu}, M^{\rho\sigma}]$ gets extra terms compared to equation 1.8, and the Lorentz Algebra is not satisfied.

Non-commutative geometry arises naturally in quantum gravity because the flat spacetime structure breaks down at Plank scale.\textsuperscript{16} For example, String Theory predicts the coordinates to become noncommutative when two D-particle separation reaches less than the string length $l_s$, the only constant in the theory.\textsuperscript{17}

1.2.2 String Theory and its Background Fields

String Theory is explicitly Lorentz Invariant in every step in its formulation: the Nambu-Goto action for a relativistic string is a Lorentz scalar, and the spacetime dimension is fixed to twenty-six in Bosonic String Theory and ten in the Superstring Theory in order to satisfy the Lorentz Algebra. However, an apparent violation of Lorentz Symmetry is expected even at the length scale greater than the string length because the theory predicts the existence of different background fields that interact with the strings. Examples of such fields are the Kalb-Ramond field.

The existence of the Kalb-Ramond field is a unique prediction of the String Theory. In String Theory, the particles that mediate the field interaction arise from quantization of excitation levels of strings.\textsuperscript{18} The massless state levels obtained from quantization of closed strings contain a square matrix, which can be split into a scalar, an antisymmetric matrix, and a symmetric matrix. The massless states with a symmetric tensor can be identified as graviton states, the scalar field is given the name ‘dilation’, and the states with an antisymmetric tensor term is referred to as the Kalb-Ramond states. Because of its antisymmetric nature, the Kalb-Ramond field is expected to behave like an electromagnetic field that couples to an intrinsic string charge. Since the Standard Model and General Relativity do not account for this field, any nonzero net interaction with this field will cause an apparent violation of
Lorentz Symmetry.

1.3 Standard Model Extension and our Experiment

Following the development of physical theories that predict violation of Lorentz symmetry, the number of experiments designed to look for Lorentz violation has steadily increased. In 1997, Alan Kostelecky proposed the Standard Model Extension. By parametrizing the various Local Lorentz violating terms for different particles, it provided a systematic theoretical framework under which the various efforts to look for Local Lorentz violation can be classified. Our experiment is sensitive to the Lorentz violating spin energy coupling parameter represented by the following equation.

$$\mathcal{L}_b' \equiv b_{\mu} \bar{\psi} \gamma_5 \gamma^\mu \psi$$

(1.14)

By the argument in pg. 225 of [9], $\bar{\psi} \gamma_5 \gamma^\mu \psi$ is the only pseudovector associated to a 1/2 spin system. Since angular momentum is a pseudovector, it must correspond to $\bar{\psi} \gamma_5 \gamma^\mu \psi$. Therefore, the above Lagrangian can be thought of as a Lorentz violating energy term that couples to the angular momentum. Since $\bar{\psi} \gamma_5 \gamma^\mu \psi$ is unitless, $b_{\mu}$ has the units of energy and it measures the strength of the energy coupling to a preferred direction of the universe.

For completeness, it should be noted that our experiment measures additional terms associated with the angular momentum vector. The complete energy parameter that our experiment measures is

$$\tilde{b}_j^\omega = b_j^\omega - m \epsilon\omega_{jk}^\alpha - \frac{1}{2} \epsilon_{ijk} H_{KL}^\alpha$$

(1.15)

The above equation and its symbols are explained in detail in Foss’ thesis.

The goal of our experiment is to place an upper bounds on $\tilde{b}_j^\omega$ for the neutron. The current limit on neutron $\tilde{b}_j^\omega$ is $1.1 \times 10^{-31}GeV$, and the present experiment is designed to improve this limit by a factor of two.
Chapter 2

Theoretical Background

The goal of our experiment is to measure the Lorentz violating energy term in the Hamiltonian of the neutron using atomic magnetometers. Because these devices measure the frequency of Larmor precession of atoms around a magnetic field (~10mG), they have a natural sense of direction defined by the direction of the applied magnetic field. The variation of the angle between the experimental direction and the preferred direction of the universe creates the sensitivity to the Lorentz violating energy coupling. In principle, the measurement can be done using only a Hg magnetometer because Hg has an unpaired neutron spin and completely paired electron and proton spins. However, much higher precision can be obtained using another magnetometer sensitive to electron spin to reject magnetic field noise, and using a rotating table to reject long-term drift in the signal. This chapter gives a quantitative treatment on how the Lorentz violating term can be measured using atomic magnetometers, the fundamentals of atomic magnetometers, the mathematics behind rejecting magnetic noise using the Cs magnetometer, and the reasoning behind using the rotating table. Since these materials have been covered extensively in previous theses[20, 7], lengthy derivations on some of the results are omitted.
2.1 Lorentz violating energy coupling

Any generic potential energy term, \( V \), that represents a spin coupling to a preferred direction in the universe can be written as
\[
V = K (\vec{s} \cdot \hat{r})
\]
where \( \hat{r} \) represents the preferred direction of the universe, \( \vec{s} \) stands for the spin of the particle, and \( K \) is the coupling strength. For a particle in a magnetic field, the total Hamiltonian, \( H \) is
\[
H = H_{\text{magnetic}} + V = -\vec{\mu} \cdot \vec{B} + K (\vec{s} \cdot \hat{r}) = -\vec{\mu} \cdot \left( \vec{B} - \frac{K}{\gamma} \hat{r} \right)
\]
(2.1)
\[
= -\vec{\mu} \cdot B_{\text{eff}} = -\vec{s} \cdot \left( \gamma \vec{B} - K \hat{r} \right)
\]
(2.2)

The effective magnetic field is the vector sum of the the applied magnetic field and \( -\frac{K}{\gamma} \hat{r} \). Since \( |\vec{s}| = \hbar/2 \) for a spin 1/2 system, \( K \) is related to \( \tilde{b}_x \) by a factor of \( 2/\hbar \). This effective magnetic field can be accurately measured by measuring the Larmor precession frequency of the atoms, \( \omega = \gamma |B_{\text{eff}}| \).

In the “sensitive” experimental position, the magnetic field of our magnetometers points along the equatorial plane. As the earth rotates, the direction of the magnetic field sweeps different directions in space. By requiring the direction of our experiment to repeat every sidereal day with a period \( T \), the \( |B_{\text{eff}}| \) can be calculated explicitly.

\[
\omega = \gamma |B_{\text{eff}}| = \gamma \sqrt{\left( \vec{B} + \frac{K}{\gamma} \hat{r} \right) \cdot \left( \vec{B} + \frac{K}{\gamma} \hat{r} \right)}
\]
(2.3)
\[
= \gamma \sqrt{B^2 + \left( \frac{K}{\gamma} \right)^2 + 2 \frac{K}{\gamma} |\vec{B}| |\hat{r}| \cos \theta}
\]
(2.4)
\[
\approx \gamma B + KB \cos \left( \frac{2\pi t}{T} \right)
\]
(2.5)

In the last step, \( B \) is assumed to be much greater than \( \frac{K}{\gamma} \). The equation implies that the Lorentz violating term in the Hamiltonian causes a sinusoidal variation in the precession frequency, \( \omega \), with a period, \( T \), which corresponds to a sidereal day.
2.2 Atomic Magnetometer

The present experiment uses two different atomic magnetometers to look for Local Lorentz violation. As mentioned in the previous section, atomic magnetometers use Larmor precession to precisely measure the magnetic field using the relation, $\omega = \gamma B$. The measurement of the precession frequency is done via optical pumping. The two main ideas in using optical pumping to measure atomic precession are: 1) the x and y component of the angular momentum of an atom changes sinusoidally with the frequency of the precession 2) the absorption probability of a resonant photon to an atom depends on their relative angular momentum.

To see how these two ideas come together, imagine a spin $\frac{1}{2}$ atom with its angular momentum precessing around a magnetic field pointing in the z direction, and photons with angular momentum +1 moving in x direction. Because the atoms with spin $\frac{1}{2}$ can’t absorb a photon with $l = 1$, the absorption probability of the photon is maximum when the atom has minimum x component of the angular momentum and minimum when the x component of the angular momentum is at maximum. In other words, the photon absorption probability represents the x component of the atomic angular momentum.

![Figure 2.1: Angular Momentum and Photon Absorption Probability](image)

(a) minimum absorption probability  
(b) maximum absorption probability

In the energy level diagram language, the photon forces an atomic transition from spin $-\frac{1}{2}$ to $\frac{1}{2}$ and the Larmor Precession takes the angular momentum state of the atom back to $-\frac{1}{2}$.

However, this picture becomes unrealistic when a sample of atoms is considered because the initial phase of different atoms are random. Instead of
evolving together, the spins of the atoms are randomly distributed in a cone around the applied magnetic field, so the spin distribution in the total sample remains essentially constant in time.

To force the spins of the atoms to precess coherently, an oscillatory transverse magnetic field is applied. When the magnetic field oscillation is near the Larmor Precession frequency of the atoms, it coherently drives the precession, similar to a driven mechanical oscillator.\(^{24}\) As in the case of a mechanical oscillator, the phase of atomic procession shifts when there is a difference between the driving frequency \(\omega\) and the natural procession frequency, \(\omega_0\). Since the change in the phase is linear with the difference in the frequencies for a small \(\omega_0 - \omega\), the experiment measures the phase shift and extrapolates the change in frequency.

To summarize, an atomic magnetometer consists of four components: a sample of atoms to start, strong axial magnetic field to cause Larmor Precession, an oscillatory transverse magnetic field to force the atoms to precess together, and a laser beam to polarize and measure the frequency of the precession.

### 2.3 Double Magnetometer Setup

In theory, using a Hg magnetometer to measure the effective magnetic field would suffice to measure the neutron spin coupling to the preferred direction in the universe. However, random magnetic fluctuations and stray magnetic fields contribute an extra \(-\vec{\mu} \cdot \vec{B}_{\text{err}}\) term in the Hamiltonian. Although the
effect of the stray magnetic fields is minimized by the four magnetic shields
that surrounds the magnetometer in our apparatus, to further reduce their
effect, a Cs magnetometer is used to keep the magnetic field constant.

Unlike Hg which has an unpaired neutron spin, Cs has an unpaired electron
spin and an unpaired proton spin. Therefore, Cs is insensitive to neutron
Lorentz violating energy coupling, $K_n$, but sensitive to that of the electron,
$K_e$. By locking the magnetic field using the Cs signal, the magnetic field is
fixed at $\omega_{Cs}$ and the equation 2.5 gives: \[ \omega_{Hg} \approx \frac{\gamma_{Hg}}{\gamma_{Cs}} \omega_{Cs} + \gamma_{Hg} \left( \hat{B} \cdot \hat{r} \right) \left( \frac{K_{Cs}}{\gamma_{Cs}} - \frac{K_{Hg}}{\gamma_{Hg}} \right) \] (2.6)
\[ = \frac{\gamma_{Hg}}{\gamma_{Cs}} \omega_{Cs} + \gamma_{Hg} \cos \left( \frac{2\pi t}{T} \right) \left( \frac{K_{Cs}}{\gamma_{Cs}} - \frac{K_{Hg}}{\gamma_{Hg}} \right) \] (2.7)

A Torsion pendulum experiment by Heckel et al. has put an upper limit on
$\tilde{b}_x$ of the electron at $(0.1 \pm 2.4) \times 10^{-31} \text{GeV}$. Since $\gamma_{Cs} = \gamma_e/4 = 350 \text{kHz/G}$,
\[ \frac{K_{Cs}}{\gamma_{Cs}} \sim \frac{2 \tilde{b}_x \omega_{electron}}{\hbar} < 2 \times 10^{-12} \text{G} \] (2.8)

On the other hand, our target limit on $\tilde{b}_x$ of $5 \times 10^{-32} \text{GeV}$ and $\gamma_{Hg} = \gamma_n = 759 \text{Hz/G}$ gives $\frac{K_{Hg}}{\gamma_{Hg}} < 1.80 \times 10^{-10} \text{G}$. Since $\frac{K_{Cs}}{\gamma_{Cs}} \ll \frac{K_{Hg}}{\gamma_{Hg}}$, our experiment
has much higher sensitivity to $\frac{K_{Hg}}{\gamma_{Hg}}$. Therefore, the contribution of $\frac{K_{Cs}}{\gamma_{Cs}}$ is
negligible and the following equation is obtained.
\[ \omega_{Hg} \approx \frac{\gamma_{Hg}}{\gamma_{Cs}} \omega_{Cs} + \gamma_{Hg} K_{Hg} \cos \left( \frac{2\pi t}{T} \right) \] (2.9)

Therefore, by forcing $\omega_{Cs}$ to be a constant, the effect of stray magnetic fields
is eliminated to first order. Our experiment eliminate the fluctuations in $\omega_{Cs}$
by locking the axial magnetic field to the Cs resonance.
2.4 Rotating Table

The purpose of the rotating table is to reject any long-term drift by adding a control position. In an atomic magnetometer, the axial magnetic field gives a vectorial nature to the experiment, and the coupling of this direction to the a preferred direction of the universe that gives the sinusoidal variation in our measurement (see equation 2.5). However, a direction that does not change with the earth’s rotation is not sensitive to the Lorentz violating energy coupling. Therefore, the direction defined by the earth’s axis of rotation is used as a control direction, and our magnetometers are designed to rotate between the control position and the experimental position.

\[ \vec{B}(t+\Delta t)_{\text{exp}} = \vec{B}(t)_{\text{ctr}} \]

Figure 2.3: Different Magnetometer Configurations

Since the entire magnetometer setup is rotated, the magnetic field vector of magnetometer in the experimental position, \( \vec{B}(t)_{\text{exp}} \), and the field in the control position, \( \vec{B}(t)_{\text{ctr}} \), must be related by \( B(t+\Delta t)_{\text{exp}} = B(t)_{\text{ctr}} \), where \( \Delta t \) represents the time taken to rotate the apparatus from the control position to the experimental position. Ideally, there should be no drift, and \( \vec{B}(t+\Delta t)_{\text{exp}} = \vec{B}(t)_{\text{exp}} \). However, various electronic drifts, such as drifting in the reference voltage used in the feed back loop of the magnetic coil, may cause a long term drift in the axial magnetic field. The effect of this drift can be eliminated to
first order by taking four measurement and using $\Delta \omega$ as a data point, where 
$$
\Delta \omega = \omega_{1,\text{exp}} - \omega_{1,\text{ctr}} - \omega_{2,\text{ctr}} + \omega_{2,\text{exp}}.
$$
Here, $\omega_{1,\text{exp}}$ is measured first, then the apparatus is rotated to take the $\omega_{1,\text{ctr}}$ and $\omega_{2,\text{ctr}}$ consecutively, and the table is then rotated back to measure $\omega_{1,\text{exp}}$.

$$
\begin{align*}
\Delta \omega &= \omega_{1,\text{exp}} - \omega_{1,\text{ctr}} - \omega_{2,\text{ctr}} + \omega_{2,\text{exp}} \\
&= (\gamma B(t)_{1,\text{exp}} - \gamma B(t)_{1,\text{ctr}}) - (\gamma B(t)_{2,\text{ctr}} - \gamma B(t)_{2,\text{exp}}) \\
&= (\gamma B(t)_{1,\text{exp}} - \gamma B(t_1 + \Delta t)_{\text{exp}}) - (\gamma B(t_2)_{\text{ctr}} - \gamma B(t_2 + \Delta t)_{\text{ctr}}) \\
&\approx \gamma \left[ \left(- \frac{dB}{dt} \Delta t \right) - \left(- \frac{dB}{dt} \Delta t \right) \right] = 0
\end{align*}
$$

The Local Lorentz violating term in the $\omega_{exp}$ has been omitted for ease of calculation. Repeating the above calculation with the neglected LLI violating terms gives back the sinusoidal variation in the $\Delta \omega$. Therefore, by measuring $\Delta \omega$ instead of $\omega_{exp}$ the effect of linear magnetic field drift is eliminated. The above argument is based on the magnetic field drifting, but similar calculations can be done for various noise sources ranging from electronics to laser intensity, as long as it has a linear relationship with $\omega$. Therefore, the rotating table rejects any linear drifts in all parameters in our apparatus.


\section*{2.4.1 AC Light Shift}

A major source of systematic error in our experimental setup is the AC light shift. AC light shift refers to a shift in atomic energy levels in the presence of an off-resonant light. Lightwaves affect the state of an atom by shifting its energy levels, and by changing the atomic state via the absorption process. The shifts in the energy level can be understood semi-classically by treating the lightwave as an electromagnetic field and considering its effect on the atom. In view of quantum mechanics, this interaction can be characterized by an extra term in the hamiltonian, called the light-shift operator $\varepsilon$. As for the absorption process, its effect on the depopulation of the ground state is completely characterized by the absorption rate, $\Gamma$, which, in the quantum picture, becomes an operator whose expectation value represents the rate of disappearance of the atoms from a specific state.\footnote{27}

Because the experiment only measures the precession frequency of the ground state atoms, investigating only the time evolution of the ground state suffices. The Hamiltonian for the ground state can be written as the sum $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'$, where $\mathcal{H}_0$ denotes the ground state hamiltonian in the absence of light, and $\mathcal{H}'$ represents the effect of the light. Because the energy level shifts and the absorption process are two distinct effects of a lightwave on the atom, the perturbed Hamiltonian can be decomposed into the light-shift operator, $\varepsilon$ and a term containing $\Gamma$.\footnote{21}

$$\mathcal{H}' = \varepsilon - i(\hbar/2)\Gamma$$  \hspace{1cm} (2.10)

The second term has units of energy and is proportional to $\Gamma$. It accounts for a change in energy due to atoms disappearing from the ground state via the absorption process. The first term represents all other effects due to the electromagnetic field. The matrix can be decomposed into a scalar term, a vectorial interaction term and a higher order tensorial interaction term.

$$\varepsilon = \varepsilon_0 + \vec{\alpha}_i \cdot \vec{J}_i + [T]$$  \hspace{1cm} (2.11)
The second term is summed over, and it accounts for all possible vectorial interactions between the electromagnetic field and the atom. The absence of a permanent electric dipole in electrons implies no electric field coupling in the second term. However, the magnetic dipole moment of an atom can contribute by coupling with an “effective” magnetic field, $\vec{F}$. Since the field transforms as a vector, this effective magnetic field must be related to a vectorial property of the light, the direction of propagation in this case. The effective magnetic field is parallel or antiparallel to the propagation direction depending on the helicity of the light, and it disappears when the light is linearly polarized. $\vec{\sigma}$, the effective magnetic field must be proportional to the polarization vector. Therefore, the $\vec{F}$ must be parallel or antiparallel to $\vec{\sigma}$ depending on the helicity of the light.

This effective magnetic field contributes yet another term for the hamiltonian in equation 2.2.

$$H = -\vec{\mu} \cdot \left( \vec{B} - \frac{K}{\gamma} \hat{r} \right) - \vec{\mu} \cdot \vec{F} = -\vec{\mu} \cdot \vec{B}_{\text{eff}} = -\vec{s} \cdot \left( \gamma (\vec{B} - \vec{F}) - K \hat{r} \right)$$

(2.12)

This change in the effective magnetic field changes the Larmor precession frequency measurement. However, since the Lorentz violating term is measured from the change in the Larmor precession frequency, this has no effect as long as $\vec{F}$ stays constant.

Unfortunately, keeping the AC Light Shift term constant is not trivial because the Light Shift energy depends on the intensity and the frequency detuning of the light. This has been shown in various experiments.\(^{28}\) (see Figure 2.4) The frequency and intensity dependence of AC light shift implies that if the light is detuned from the resonance frequency, a constant laser intensity is required to keep $\vec{F}$ constant. Although both the Hg and Cs laser frequencies are locked using the reference vapor cells in our experiment, these cells are not completely identical to the experimental cells. Since any change in the vapor pressure within a cell and the composition of the buffer gas within affects the resonance frequency of the atoms, the atoms in the reference cell and the atoms in the experimental cell have slightly different resonance frequency.
As a result, the laser light locked to the atoms in the reference cell is not completely on resonance with the atoms in the experimental cells. Therefore, any change in the laser intensity affects the Larmor Frequency measurement, as was shown in detail in [20].
Chapter 3

Apparatus

The experiment uses Cs and Hg magnetometers mounted on a rotating table. An atomic magnetometer consists of a frequency stabilized laser system, an experimental vapor cell, magnetic coils, and the signal detection mechanism. The two magnetometers share the magnetic coils and shields, but they require separate detection mechanisms, laser systems, and vapor cells. The vapor cells, magnetic coils, and the detection mechanisms are mounted on a tower which is encased in four layers of magnetic shields. Therefore, the four main parts of the apparatus are: Cs magnetometer, Hg magnetometer, the rotating table, and the magnetically shielded tower.

3.1 Cesium Laser System

The photons that excite the Cesium transition is produced by Mitsubishi ML2701, a AlGaAs laser diode with its peak wavelength at 894nm. The diode is mounted in a Littrow external cavity configuration which directs the first order diffraction back to the laser diode to allow over a GHz of frequency tuning range. The basics of Littrow Configuration is explained in detail in [20] and won’t be reproduced here. The laser frequency is locked by the dither-locking method using a saturation absorption signal. The temperature of the diode is stabilized by two-stages of temperature control.
3.1.1 Dither-Lock Mechanism / saturated absorption

Because the current experiment is interested in producing the frequency of the light that excites a specific Cs atomic transition, a reference Cs vapor cell is used to lock the frequency of the laser. However, because the Cs vapor cell is at the room temperature, the regular absorption spectrum is Doppler broadened. Saturated absorption is used to eliminate such broadening.\(^{30}\) The current Cs optics configuration is shown below.

![Figure 3.1: Cs Table Schematics](image)

The saturated signal is added to an inverted regular absorption signal with some gain to produce a signal that is concave up for the entire width of the signal (see Figure3.2).\(^{31}\)

The desired frequency is at the minimum of the transmission spectrum. However, the feedback signal required for the system is the one that becomes positive when the laser frequency is lower than the peak frequency, negative when the frequency is greater, and zero when the light frequency is at the peak frequency. In other words, the spectrum is quadratic in detuning, but a signal that is odd in detuning is required for the locking circuit.

A dither-Locking mechanism converts the even saturated absorption signal into an odd signal by taking its derivative. The key idea is to modulate the frequency of the light. Since the photodiode voltage signal in the saturated absorption spectroscopy is a function of the laser frequency, \(V = V(\omega)\), the sinusoidal variation transforms the voltage as a function of time: \(V(t) = \)
Figure 3.2: Saturated Absorption Spectroscopy Signal

\[ V(\omega_{\text{center}} + \Delta \omega \cos(\Omega t)) \]

As long as the frequency of variation is much smaller than the detuning, \( \Omega \ll \Delta \omega \), the oscillatory term can be treated as a non-oscillating variable, and \( V(\omega) \) can be Taylor expanded to powers of \( \Delta \omega \cos(\Omega t) \).

\[
V(\omega_{\text{center}} + \Delta \omega \cos(\Omega t)) = V(\omega_{\text{center}}) + \frac{dV(\omega_{\text{center}})}{d\omega} \Delta \omega \cos(\Omega t) \\
+ \frac{d^2V(\omega_{\text{center}})}{d^2\omega}(\Delta \omega \cos(\Omega t))^2 + \ldots
\]

The term linear to \( \Delta \omega \) is isolated by demodulating the signal with the input signal used to generate the modulation using a lock in detector. The output signal has the desired property of being proportional to the slope of the saturated absorption spectrum near \( \omega_{\text{center}} \), and it can be used to lock the laser frequency by controlling the external cavity length using a piezoelectric device.

### 3.1.2 Temperature/Thermal Expansion Control

The lasing frequency of an external cavity laser depends on the temperature because the thermal expansion of the laser diode causes the wavelength of the light to increase. Also, the expansion and contraction of the external cavity has the effect of steering the beam. Therefore, the temperature control system is critical to the stability of the laser. The temperature control is achieved in
two stages. The first one controls the temperature of the bulk laser mount and cavity, and the second one controls the temperature of a 1” × 1” laser diode block. The schematics for the laser system is shown below.

Figure 3.3: Bulk Cs Laser

A thermistor is inserted in the lower 1/2” thick aluminum plate so that its temperature information can be used to lock the temperature using a thermoelectric device (TED). Since the entire upper portion of the laser system constitutes the heat load on the TED, the large heat capacity of the heat load causes the temperature control to react slowly to the change in temperature setting. Therefore, the time constant for the Stage I is ∼1day. In addition to the temperature control, the Aluminum diode mount and the Adjustable optics mount is attached on top of Invar, which is known for its low thermal expansion coefficient. The linear thermal coefficient of Invar is generally less than 1.3 × 10⁻⁶ K⁻¹, whereas that of aluminum is at 23 × 10⁻⁶ K⁻¹. The low thermal expansion of the Invar minimizes the steering effect from changes in the length of the external cavity. Although it is not shown in the figure, the entire upper thermal mass is shielded from the air currents by an aluminum cover that rests on the horizontal and vertical alu-
uminum plates. For additional thermal and acoustic insulation, a styrofoam box is constructed to encase the aluminum cover.

The Stage II temperature control TED is mounted on the aluminum diode mount shown in figure 3.3. A detailed schematics of the Diode Mount is shown below. The main purpose of the diode mount is to insulate the diode block that contains the laser diode and the collimating lens from the rest of the thermal mass. Besides the Stage II TED, the only point of contact for the diode block is the G-10. Since G-10 plastic is an insulating material, the above configuration prevents any uncontrolled heat flow between the bulk Stage I thermal mass and the diode block. Also, because the entire upper thermal mass is isolated from the air currents by 1/2” thick aluminum cover, the diode block is isolated from convection.

The Stage II TED controls the heat flow between the diode block and the heat conducting wire in figure 3.3 using the temperature feedback information from a thermistor inserted in the diode block. Because the diode block is insulated, it constitutes the only thermal load of the Stage II TED. The small heat-load and fast re-thermalization time allows the time constant in Stage II to be small, on the order of a minute. The extra heat is carried through the Heat conducting wire, and dumped to the vertical aluminum plate, which is part of the heat load for the Stage I.

The benefit of having two-stage temperature control scheme is temperature
isolation from rest of the thermal load. It prevents the thermal mass near the diode block from heating up by dumping the extra heat away from the block. This ensures that when the diode block heats up from turning on the laser, the heat is not transmitted back into the laser cavity. It also gives an extra level of insulation from the ambient temperature fluctuation. Because the rate of heat flow is proportional to the temperature difference, the change in the heat sink temperature affects the voltage required for the TED to create constant heat flow between the load and the sink. However, due to the finite reactivity of the feedback circuit, this TED current adjustment causes a small oscillation in the temperature. By controlling the Stage II heat sink temperature by loading it to Stage I, the two stage system provides the Stage II TED with a thermally stabilized heat sink, and increases the stability of the diode block temperature.

3.1.3 Improvements

Dither Locking frequency

In an external cavity laser, there are two ways to generate a frequency oscillation: by varying the PZT voltage, and by using current modulation. Before the summer, the modulation had been applied via the PZT voltage. However, because the PZT physically moves the grating, the frequency of modulation had been limited to 2kHz.

Unlike the modulation based on PZT vibrations, the current modulation relies on the expansion of the laser diode itself. When the current increases, the laser diode heats up and expands, producing a longer wavelength photons. Because the current modulation doesn’t cause any large large movement it allows a much higher frequency range. During the summer, a ten fold increase in modulation (28kHz) was achieved using the current modulation. Because it is well above the vocal range, the entire locking system became essentially impervious to minor acoustic disturbances.
Temperature/Vibrational Isolation

The temperature and vibrational isolation of the Cs laser system was increased by improving the cover that isolates the Stage I heat load from the air currents (see figure 3.3). The original cover was a thin sheet of copper that had almost no thermal load, and gave no insulation against vibration. The new cover is made out of 1/4” thick aluminum, and it is padded with Sorbothane sheets on the outside. The thick aluminum cover provides an additional heat reservoir to the Stage II, so the transfer of heat between the Stage II load and the vertical aluminum plate has less effect on the temperature of the Stage I load. Moreover, the increased thickness provides more acoustic damping, and the Sorbothane lining improves it further.

Although the new cover increased the thermal stability considerably, an incident breeze still could cause the laser frequency to oscillate by affecting the temperature of Stage I heat sink. Therefore, a cardboard box was made to cover the entire bulk laser system, including the stage I heat sink. Now, the laser frequency is stable under any moderate air currents and acoustic vibrations.

Collimation

Our January 25-28 data run indicates a unidirectional PZT voltage drift. Because PZT voltage is used to correct for the frequency drift in the laser, this implies the tendency of frequency of the laser light to drift in one direction. Although the laser frequency is expected to drift, the drift should not continue in one direction for days. Therefore, besides the effect of steering the beam, the PZT drift indicates a more serious fundamental problem in the laser, the collimation drift.

The slow drift in the collimation not only causes PZT voltage drift, but also decreases the tuning range of the Cs laser. Although the collimation and the diffraction grating angle was adjusted in January to give 1GHz free tuning range, the range dropped to less than 0.5GHz on the 28th. Up to this point, they needed to be adjusted once every few months. Because these adjustments
Figure 3.5: PZT Drifting during Jan 25-28 ’08 Data Run - the jumps correspond to re-locking of the laser

usually take over a week, this indicated a serious problem that needed to be addressed.

The original collimation scheme involved two copper blocks and a rubber o-ring. The front copper block contained the collimating lens and the laser diode was inserted into rear copper block.

Figure 3.6: Original Diode Mount

The collimation adjustment of the laser was achieved by tightening and un-tightening the four hex screws against the o-ring. This configuration made the optimizing the collimation especially difficult because the four hex screws allowed the angle between the collimation lens and the output beam to change. Also, as the o-ring loses elasticity, the distance between the two blocks changes, which results in a drift in the collimation. Since adjustment in the collimation
was needed increasingly frequently, we suspected that this was indeed the case.

The collimation mechanism has been entirely revamped so that there is no more need for an o-ring. The new diode mount makes use of a commercial collimation tube. The diode is fixed at one end of the tube and the collimating lens is screwed into the other end of the tube. The tube keeps the laser beam on axis with the lens, and the distance between the lens and the diode can be easily adjusted using a special wrench. In order to maximize the thermal conductivity between the collimation tube and the aluminum block, small amount of thermal grease is applied on the surface and the open end of the aluminum block is clamped with a screw.

3.1.4 Experimental Techniques

Cs laser locking is quite simple compared to that of Hg. Once the Stage II temperature stabilizes after $\sim$1hr of lasing, the laser current and the PZT offset is tuned. Since both change the frequency of the light, this affects the Fabry-Perot signal and the saturated absorption signal. The peak in the saturated absorption signal is found by looking for the parameter where the saturated absorption signal shows an abrupt change while the Fabry-Perot indicates a single lasing mode. After checking that the frequency can be tuned 1GHz by adjusting only the PZT offset, the tuning range is centered at the center of the saturated absorption peak and locked. After a successful locking, the various sensitivity and phase settings in the lock-in detector are used to optimize the
locking.

When a mode hop occurs before the frequency can be tuned 0.5GHz using only PZT, the external cavity needs to be adjusted. After aligning the beam to the wave meter and removing the 1/2 inch aluminum cover, the grating angle is adjusted using hex screws on the optics mount (see figure 3.3) to obtain the frequency 335.107THz (894nm) with as much tuning range as possible. Usually, 0.5GHz PZT tuning range with the cover off indicates a successful realignment because the tuning range generally increases to 1GHz when the cover is put back on.
3.2 Hg Laser System

The Hg Laser system is designed to produce 253.7nm light to run the $F = 1/2 \rightarrow F = 1/2$ Hg transition. It is a commercial laser system built by Toptica. Because commercial laser diodes generally produce infrared light, the system produces light of the desired frequency by doubling twice the 1016nm light output from a diode laser.

3.2.1 General Setup

The system is composed of four main components: IR stage, 1st frequency doubling cavity, 2nd frequency doubling cavity, and the Noise Eater/ Frequency Lock system. Because the Laser system consist of various large components, only the 2nd frequency doubling cavity and the noise eater system could be placed on the rotating optics table. The IR stage, 1st frequency doubling cavity and all of the associated electronics are placed on a separate, stationary table.

IR Stage

![IR Stage Schematics](image)

Figure 3.8: IR Stage Schematics

The IR Laser box contains an external cavity and the temperature control system. The laser produces light of $\sim$1016nm and the frequency can be fine tuned by adjusting the PZT voltage. The light produced from the IR laser diode is sent to the tapered amplifier which amplifies the 50mW input light to over 1W. The two optical isolators are added to prevent the light from
reflecting back to the IR laser and the tapered amplifier. Part of the output beam is sent to the wavemeter.

1st Frequency Doubling Cavity

The cavity uses KNbO$_3$ crystal to double the frequency of the light. Since the first frequency doubling is a sharply peaked function of temperature, the temperature is stabilized via a thermistor/TED combination. Because the frequency doubling is a nonlinear process that requires extremely high light intensity, the crystal is placed within a bow-tie cavity that is designed to increase the light intensity by $\sim 40x$ inside the crystal. The ring cavity is locked into resonance via Pound-Drever-Hall (PDH) method. The resulting 508nm light is coupled to an optical fiber which brings the light to the 2nd doubling cavity situated on the rotating table.

Because PDH is such a powerful and useful technique, it is explained in full glory here.$^{36}$ It is a frequency locking method that uses the different reflectivity of the carrier signal and the sidebands in a cavity. The sidebands are created by giving a radio-frequency phase modulation in the electric field ($\sim 2$Mhz in our experiment) with a frequency $\Omega$. The resulting electric field of the light is $E = E_0 \exp(i (\omega t + \beta \sin(\Omega t)))$. Since the $\omega$ represents the photon frequency of few hundred THz, as long as $\beta \Omega \ll \omega$, $\omega t \gg \beta \sin(\Omega t)$ for all nonzero $t$. By
treated the oscillating term as a perturbation of $\omega t$, the electric field can be expanded via a Taylor Expansion.

$$E = E_0 \exp(i(\omega t + \beta \sin(\Omega t))) = E_0 \exp(x) \quad (3.1)$$

$$\approx E_0 \left[ \exp(x)|_{x=i\omega t} + \left. \frac{d \exp(x)}{dx} \right|_{x=i\omega t} (x - (i\omega t)) \right] \quad (3.2)$$

$$= E_0 \exp(i\omega t)(1 + i\beta \sin(\Omega t)) \quad (3.3)$$

$$\rightarrow E_0 \exp(i\omega t)(J_0(\beta) + iJ_1(\beta) \sin(\Omega t)) \quad (3.4)$$

In the last step, 1 is replaced by $J_0(\beta)$ and $\beta$ is replaced by $J_1(\beta)$ in order to force $|J_0(\beta) + iJ_1(\beta) \sin(\Omega t)| = 1$ to preserve the magnitude of the electric field. Both $J_0$ and $J_1$ are assumed to be real functions. Substituting $\exp(i\Omega t) - \exp(-i\Omega t)$ for $i \sin(\Omega t)$, the following equation for electric field is obtained.

$$E = E_0 [J_0(\beta) \exp(i\omega t) + J_1(\beta) \exp(i(\omega + \Omega)t) - J_1(\beta) \exp(i(\omega - \Omega)t)] \quad (3.5)$$

The equation shows that the RF modulation with frequency $\Omega$ effectively creates sidebands with frequency $\omega + \Omega$ and $\omega - \Omega$.

When the beam with the three distinct frequency component arrives at the ring cavity, some gets reflected, and some gets transmitted. In an optical cavity, the reflection coefficient, $F$ depends on the frequency of the light, and when the round trip distance of the optical path becomes an integer multiple of the wavelength, the reflectivity vanishes. Multiplying each frequency component with the respective reflectivity from the cavity, the reflected electric field, $E_{ref}$ and the intensity of the reflected light, $P_{ref}$ can be calculated.

$$E_{ref} = E_0 [F(\omega)J_0 \exp(i\omega t) + F(\omega + \Omega)J_1 \exp(i(\omega + \Omega)t)F(\omega + \Omega)J_1 \exp(i(\omega - \Omega)t)]$$

$$P_{ref} = |E_{ref}|^2 = |E_0F(\omega)J_0|^2 + |E_0F(\omega + \Omega)J_1|^2 + |E_0F(\omega - \Omega)J_1|^2$$

$$+ |E_0|^2 J_0J_1[e^{-i\Omega t}(F(\omega)F*(\omega + \Omega) - F*(\omega)F(\omega - \Omega))]$$

$$+ e^{i\Omega t}(F*(\omega)F(\omega + \Omega) - F(\omega)F*(\omega - \Omega))]$$
Defining $K \equiv F(\omega)F^*(\omega + \Omega) - F^*(\omega)F(\omega - \Omega)$, $P_c \equiv |E_0|^2 J_0^2$, and $P_s \equiv |E_0|^2 J_1^2$, the equation is simplified to

$$P_{ref} = P_c[F(\omega)]^2 + P_s[|F(\omega + \Omega)|^2 + |F(\omega - \Omega)|^2] + 2\sqrt{P_cP_s}[Im(K) \sin(\Omega t) + Re(K) \cos(\Omega t)]$$

(3.6)

When the cavity is near resonance with $\omega$, the center frequency component is almost completely transmitted whereas the sidebands are nearly all reflected, i.e., $F(\omega) \approx 0$, and $F(\omega \pm \Omega) \approx -1$. Substituting -1 for the sideband reflectivity and approximating to first order in $F(\omega)$, $P_{ref}$ further simplifies to

$$P_{ref} = 2P_s - 4\sqrt{P_cP_s}Im(F(\omega)) \sin(\Omega t)$$

(3.7)

By mixing the reflected intensity signal with the RF modulation signal, an output proportional to $\Im(F(\omega))$ can be obtained. Since the reflected light always picks up a phase shift, this signal goes to zero only when the reflectivity vanishes, and it can be used to lock the cavity into resonance using a PZT offset voltage.

2nd Frequency Doubling Cavity

The second doubling cavity is placed on the rotating table and it uses the BaB$_3$O$_4$ crystal to frequency double the 508nm light into 254nm light. Unlike
KNbO$_3$, BaB$_3$O$_4$ crystal is not temperature stabilized because it works well at room temperature. As in the case for the 1st doubling cavity, the cavity length is precisely adjusted via a PZT controlled mirror in order to achieve high light intensity inside the crystal. However, instead of PDH, this cavity uses Hansch-Couillaud (HC) method. Unlike PDH which uses the frequency dependence of the reflectivity of the cavity, HC uses the phase difference between the reflected and transmitted component of the light.

Since HC is also a very widely used locking mechanism, it is also explained in detail.$^{37}$ Because the BaB$_3$O$_4$ crystal preferentially absorbs light of a certain linear polarization, $E_\perp$, the amplitude of the reflected wave in this preferential direction of the crystal, $E_{\perp}^{ref}$, is equal to the incident wave times the reflectivity of the incident mirror, $E_i^{\perp} \sqrt{R_1}$. However, the component that doesn’t get absorbed by the crystal, $E_\parallel$, gets transmitted into the cavity, and the reflected wave is calculated to be:$^{38}$

$$E_{\parallel}^{ref} = E_\parallel \left[ \sqrt{R_1} - \frac{T_1}{\sqrt{R_1}} \frac{R e^{i\delta}}{1 - R e^{i\delta}} \right]$$

(3.8)

where δ is the phase shift the lightwave picks up during a round trip inside the cavity. By multiplying the numerator and the denominator of the second term by $1 - R e^{-i\delta}$, it becomes clear that the reflected wave picks up an imaginary part if the cavity is not exactly on resonance. This imaginary part persist in the total reflected wave. By defining $\hat{x}$ to be in the perpendicular direction and $\hat{y}$ to point along the parallel direction,

$$\vec{E}_{\text{tot}}^{ref} = E_{\perp}^{ref} \hat{x} + E_{\parallel}^{ref} \hat{y}$$

$$= \sqrt{R_1} E_i^{\parallel} \hat{x} + E_\parallel \left[ \sqrt{R_1} - \frac{T_1 R}{\sqrt{R_1}} \frac{\cos \delta - R + i \sin \delta}{1 - R e^{i\delta}} \right] \hat{y}$$

$$= \sqrt{R_1} E_i^{\perp} \hat{x} + E_\parallel [\sqrt{R_1} + \gamma + i \beta \sin \delta] \hat{y}$$

When the cavity is on resonance, $\gamma = -\sqrt{R_1}$ and $\sin \delta = 0$, so $\vec{E}_{\text{tot}}^{ref}$ is linearly polarized along $\hat{x}$ direction. However, off resonance, the presence of the
imaginary part in the \( \hat{y} \) direction causes the reflected wave to be elliptically polarized. Since the cavity can be locked on resonance by minimizing the elliptical polarization of the reflected wave, the circularly polarized component of the light represents the error signal. In order to obtain a signal proportional to the elliptical polarization, the reflected wave is sent through a 1/4 wave plate and then a polarizing beam splitter. The effect of the 1/4 wave plate is to cause a 90° phase shift in the slow axis. By choosing the slow axis to be the \( y \) axis, the output beam, \( \vec{E}_{\text{tot}}^{\text{ref}} \), becomes:

\[
\vec{E}_{\text{tot}}^{\text{ref}} = \sqrt{R_1}E_i\hat{x} + iE_\parallel[\sqrt{R_1} + \gamma + i\beta \sin \delta]\hat{y}
\]

When the output beam goes through the polarizing beam splitter, it redirects the beam depending on the polarization of the light. It has a definite axis determined by its crystal axis, and a light wave with its polarization axis parallel to the crystal axis is transmitted unhindered, and the light with the other linear polarization gets deflected by some angle. By defining the crystal axis as \( x' \) and the perpendicular axis as \( y' \) where \( \hat{x} = \frac{1}{\sqrt{2}} (\hat{x'} + \hat{y'}) \), and \( \hat{y} = -\frac{1}{\sqrt{2}} (\hat{x'} - \hat{y'}) \), the power of the transmitted beam, \( P_{\text{trans}} \) and the deflected beam, \( P_{\text{def}} \) can be calculated.

\[
\sqrt{2}\vec{E}_{\text{tot}}^{\text{ref}} = \left( \sqrt{R_1}E_i^\perp - iE_i^\parallel(\sqrt{R_1} + \gamma + i\beta \sin \delta) \right)\hat{x'} + \left( \sqrt{R_1}E_i^\parallel + iE_i^\parallel(\sqrt{R_1} + \gamma + i\beta \sin \delta) \right)\hat{y'}
\]

\[
P_{\text{trans}} = \frac{1}{\sqrt{2}} \left( \sqrt{R_1}E_i^\perp - iE_i^\parallel(\sqrt{R_1} + \gamma + i\beta \sin \delta) \right)^2
\]

\[
P_{\text{def}} = \frac{1}{\sqrt{2}} \left( \sqrt{R_1}E_i^\parallel + iE_i^\parallel(\sqrt{R_1} + \gamma + i\beta \sin \delta) \right)^2
\]

Finally, by measuring \( P_{\text{trans}} \) and \( P_{\text{def}} \) with photodiodes and taking the
difference, a signal roughly proportional to \( \sin \delta \) is obtained.

\[
\Delta P = P_{\text{def}} - P_{\text{trans}} = -4 \cdot \frac{1}{2} \left( \sqrt{R_1 E^\perp E^\parallel} \beta \sin \delta \right) = \frac{2E^\perp E^\parallel T_1 R \sin \delta}{(1 - R)^2 + 4R \sin^2(\delta/2)}
\]

To first order in \( \delta \), \( \Delta P \approx \frac{2E^\perp E^\parallel T_1 R}{(1-R)^2} \delta \). Therefore, near resonance, this signal is used to lock the cavity in resonance by feeding back \( \Delta P \) to the cavity PZT driving voltage.

### Noise Eater / Frequency Lock System

The above configuration is designed to stabilize the frequency and the intensity of the light. The photodiode detects the intensity of the light, and a fixed amount of voltage is subtracted from the photodiode signal. This offset voltage sets the amount of light that gets through the system. Any excess amount of light constitutes an error signal and is discarded using the using the Pockels Cell.

The error signal (photodiode signal - offset voltage) is amplified and used to drive the Pockels Cell. Since the Pockels Cell is a device that creates birefringence proportional to the applied voltage, change in the intensity results in the change in the birefringence, and the linear polarizer that is placed after the Pockels Cell converts the birefringence change to the change in the output.
intensity. The present set up uses $\sim 120V$ offset voltage.

The frequency lock uses regular absorption spectroscopy. The light is sent through a Hg vapor cell and the output intensity is recorded using the Frequency-Lock photodiode. Unlike the Saturated Absorption spectroscopy used in Cs Laser, the signal is Doppler broadened. Moreover, because the difference in the buffer gas used in the locking cell, its resonance frequency is pressure shifted compared to the resonance frequency of the experimental cell. Therefore, the laser frequency is locked not to the peak of the absorption spectrum of the locking cell, but to the side, which corresponds to the spectrum peak in the experimental cell. This is achieved by feeding back the difference between the locking cell absorption spectrum signal and a offset voltage to the IR laser PZT driving voltage. Since the change in the laser intensity causes the absorption spectrum to be rescaled, the offset voltage is multiplied by the intensity of the laser so that the offset voltage can be rescaled accordingly (see Figure 3.13).

### 3.2.2 Experimental Techniques

In order to generate a 254nm light of stable intensity, several steps are required. After turning on the IR laser diode and 1st stage temperature control and waiting for $\sim 1$hr, the laser diode is turned on. When a single mode is chosen by adjusting the IR stage PZT bias voltage, the first cavity PZT is ramped and the error signal is optimized by adjusting the second stage PZT offset voltage. After locking the first cavity, $\sim 700mW$ of 508nm light is obtained, and it is used to generate an error signal in the second cavity. The error signal is optimized by adjusting the $1/4$ wave plate in front of the polarization beam
After locking the second cavity, the coupling between the optical fiber and the green light is optimized because the transmission rate of the optical fiber is very sensitive to the angle and position of the incident green light. The second cavity photodiode generally records >50mV for optimal blue light generation. When a sufficient amount of 254nm light is generated, the Noise Eater circuitry can be turned on and the Pockels Cell bias voltage can be applied at \( \sim 130\text{V} \) using the trim pot placed in the feedback circuit.
3.3 Rotating Apparatus

Although most of the apparatus is rotating, the tower, components inside the tower, and the rotating table are grouped as the rotating apparatus because they are affected the most by the rotation. ‘Stabilizing the rotation’ generally refers to minimizing the effect of rotation on these components.

3.3.1 Rotating Table

The rotating table holds most of the optics and the magnetically shielded tower. The octagonal optics table is custom-made by Newport with stainless steel in a honeycomb structure. This stainless steel structure is chosen over granite because of the lower cost and higher resistance to low frequency vibrations. Since stainless steel is less resistant to torsional stress than granite, the 1/2 inch thick 4’×4’ aluminum has been inserted to disperse the contact forces from the twelve brass rollers and to minimize the torsional strain. However, because the aluminum plates were not perfectly flat, not all of the rollers were in contact with the plate. Since three points define a plane, usually only three
rollers are in contact with the aluminum plates, which creates a torsional strain on the plates and results in further distortion. The deformation of the plates and the shifting in the weight distribution among the rollers causes the point of contact between the aluminum plate and the optics table to shift depending on the angle of rotation which results in a significant flexing of the optics table. Therefore, the Sorbothane pads were added in Spring ’07 in order to maintain constant point of contact between the aluminum plate and the optics table.

The motor that powers the rotation is mounted on the northwest corner of the lab, and its power is transferred through ropes which are tied to the aluminum rods bolted onto the aluminum plate. The direction of the rotation is controlled by the ±5V digital output from the Lock-In Detector, and the amount of rotation is controlled by the switching mechanism on the motor.

3.3.2 Tower

![Rotating Table Schematics](image)

Figure 3.15: Rotating Table Schematics
The tower is where the measurement of the Larmor Procession Frequency occurs. The major components of the tower are the experimental vapor cells, the magnetic coils, magnetic shields, and the Cs and Hg signal detectors.

**Experimental Vapor Cells**

Since Hg and Cs cannot be in the same vapor cell due to their reactivity, the Hg experimental cell is placed off the tower axis. In order to offset the effect of inhomogeneity of the magnetic field, two Hg cells are used and their signals are averaged to reduce its effect. All three vapor cells are coated, evacuated cylindrical glass cells. The details of the cells have been explained in ch 3.1 of [7].

**Magnetic Coils**

The magnetic coils are designed to produce a DC magnetic field and an oscillating transverse field. The DC field is at 45° from the vertical with the strength of \( \sim 5.7 \text{mG} \). The field leans toward the Cs laser setup and away from the second Hg frequency doubling cavity. The driving circuitry for the magnetic field allows a fine adjustment of the field through a small input voltage, which changes the DC field \( \sim 2.84 \mu \text{G}/\text{V} \). This voltage is controlled by a Lock-in Detector so that the Cs signal can be used to servo-lock the magnetic field.

**Magnetic Shields**

There are three rotating shields and an outer stationary shield. The inner shields are mounted in a PVC pipe for additional structural support. These shields are made out of MPP (molypermalloy), which are generally used for inductor cores for their ability to confine the magnetic field. The outer shielding is made out of \( \mu \)-metal because its high magnetic permeability (\( \sim 30 \times \text{steel} \)) redirects the external magnetic field.

The three inner shields and the PVC pipe are attached to the table by an aluminum pipe and they rotate with the table. The stationary shield is suspended from beams bolted to the wall and the ceiling.
Cs Signal Detector

After the Laser light passes through the Cs cell, it obtains an intensity modulation, the frequency of which corresponds to the Larmor frequency of the Cs atoms. The laser light is guided via an optical cable to the photodetectors. The optics cable is separated into different quadrants and the intensity in each quadrant is detected separately in order to detect beam position and movement. The intensity at each cable is summed by an Op-Amp circuit and sent to the Lock-In detector for frequency measurement.

![Quadrant Optical Cable](image)

Figure 3.16: Quadrant Optical Cable

Hg Signal Detector

Because photomultiplier tubes (PMTs) are much more sensitive in the UV range, two PMTs are installed to detect 254nm light transmission through both of Hg vapor cells. Both PMTs are powered by a high voltage supply that currently puts out $\sim300V$. The UV light coming out of the Hg cells are guided via mylar tubes, and the current generated by the electron cascade inside the PMTs are converted into voltage in an Op-Amp circuit. The two signals are separately sent to different Lock-in detectors, and the frequency informations are recorded independently.

3.3.3 Improvements

Unlike the apparatus used in most precision measurements, our apparatus moves during the experiment. Although the data is not collected during the rotation, the table movement must not be so violent that it unlocks the lasers.
The tower should not tilt or twist during the rotation. Improvements to the tower and the rotating table have been made to increase their mechanical stability and the Cs detection circuit has been modified to minimize the effect of the tower movement.

**Mechanical Additions**

New braces were added to the tower assembly on Nov.16. The braces are designed to minimize the twisting of the tower, and were installed in between the four gussets (see Figure 3.15). Four more braces were added on Apr.16 above the existing braces to further improve the stability.

The rotating table developed making some grinding and screeching sounds while rotating. The cause has been speculated as the brass rollers and they ahve been replaced by plastic rollers. (See Ch 4.5)

**Improved Cs signal detection**

The Cs signal is more susceptible to the tower tilt than the Hg counterpart because the light is carried by a quadrant optics cable. Although this configuration enables the detection of a beam movement, bending of the optical cable during the table rotation may cause the transmission rate of the fiber to change. Therefore, a vertical mount for the cables has been constructed to minimize distortion of the cables during rotation. Secondly, since the signals from different quadrants picked up a different phase factor in the original detector circuit, a new circuit is constructed with three variable capacitors to allow the phase matching of the quadrant signals.
Chapter 4

System Characterization and Stabilization

Most of the year has been spent characterizing the system, searching for the optimal parameter for the laser stabilization and minimizing the adverse effects from the table rotation. This chapter presents the data that either shows the improvements in the apparatus or characterizes the problem that needs to be addressed. The undesirable responses in the apparatus have been grouped into five broad and interconnected categories, and the following sections reflect the attempts to understand and to minimize the adverse effect of each.

4.1 Table Dependent Phase Shift

Both Cs and Hg signals acquire shifts in phase when the apparatus changes direction. Since the experiment looks for a sinusoidal change in the difference of the signal taken in the experimental and control direction, the measurement is unaffected by the shift as long as it stays consistent. However, Figure 4.1 indicates that both Cs and Hg Table Dependent Phase Shift (TDS) is not constant. Therefore, the TDS presents a source of error in the experiment, and there has been a considerable effort to find its cause. Since Cs and Hg magnetometers have $\sim 3\mu G$/deg sensitivity to the magnetic field, a common mode shift is treated as magnetic. Since the TDS of the Cs in Figure 4.1a
Figure 4.1: Magnetometer Phase in Two Different Table Positions: Jun.19 and Sep.25, 2007
and Hg in Figure 4.1b differ by a factor of five, it indicates that the shift is non-magnetic in nature.

To better understand the nature of the TDS, the phase dependence on the angle of rotation is measured by rotating the table in small increments.

![Graph](a) Jun.19, 2007

![Graph](b) Jul.18, 2007

Figure 4.2: Cs Phase vs. Rotation Angle

The plot shows a hysteresis associated with rotation, which indicates that the system does not return to the same state as it rotates back. As a result, the rotation makes the apparatus unstable and susceptible to drift.

Since the electronics and laser systems in our experiment do not exhibit any short term drift when the table stays at one position, twisting and tilting of the tower have been suggested as probable sources of the shift. Although four 1/2” thick gussets and eights clamps have already been placed on the tower, new 1/2” thick braces were added on Nov.16, to stabilize the tower further, and the TDS is measured again. Unfortunately, the data does not show any obvious increase in the stability of the signal compared to the previous data runs (Figure 4.1). Four more braces have been added on top of the old ones.
in April, but not enough data has been taken afterwards, and their effects remains to be seen.

(a) Cs

(b) Hg

Figure 4.3: Magnetometer Phase in Two Different Table Positions: Nov.16, after the new Gusset installation

4.2 Slow Drift

A slow, linear drift in the apparatus does not affect our measurement of LLI violating term because the experiment measures the difference in the Larmor frequency along the ‘sensitive’ direction and the control direction (see Ch 2.3). However, a non-linear drift in the system poses a problem, as in the case of ‘warm up’ effect in the Hg cell.
Figure 4.4: Hg Experimental Vapor Cell Warmup Characteristic

Here, the modulation amplitude (R) refers to the magnitude of the intensity absorbed by the atoms in the cell. Generally, R corresponds to the intensity of the light because the transition probability of the atoms increases as the light intensity increases. However, in this non-linear increase in R, there is no corresponding increase in the laser intensity (see Figure 4.5).

The most likely source of this slow ramping is the change in the number of atoms in the beam path. In the absence of incident radiation, most Hg atoms are trapped on the inner surface of the vapor cell, but they are liberated as the scattered laser beam transfers energy to the trapped atoms. As a result, the concentration of free Hg atoms increases, and more atoms are in the beam path. Therefore, more atoms are excited and the modulation amplitude rises. The increase in R eventually slows down after few hours as the new equilibrium condition is reached in the Hg population.

Because the ramping behavior is due to the interaction between the vapor cell wall and the atoms, the only way to affect the time constant in the ramping
curve is to change the experimental vapor cell. However, since there is no
guarantee that the newly obtained cell will work better, and also because we
are not setup to make good Hg cells in Amherst, the idea is not pursued. To
prevent the ramping behavior from affecting our measurement, the laser is
turned on and allowed to warm up for 3~4hrs before collecting data.

4.3 Detector sensitivity to beam movement

Detector stability refers to the sensitivity of the photodetectors to the beam
movement. Because the lasers are mounted on the rotating table and the
detectors are attached to the tower, tilting, bending and twisting of the tower
with respect to the optics table can cause the beam to hit different positions
of the photodetector. Therefore, minimizing the phase sensitivity to the beam
movement can minimize the TDS.

Although the Hg phase is observed to be insensitive to small beam move-
ments, the Cs signal shows a strong correlation to the beam movement. The
difference is not surprising because the Hg detector uses a single photomulti-
plier tubes for each cell whereas the Cs beam is transferred through a quadrant
optical fiber and detected using four photodiodes. In the Cs signal detection
circuit, the separate signals from each quadrants are summed and sent to a
Lock-in for detection. This can link phase change to beam movement when
the four quadrant signals have different relative phases because a change in
the intensity distribution affects the amplitude of the signal from each quadrant. Therefore, a new circuit was constructed to allow the phase matching of the quadrant signals using variable capacitors, and the phase difference in the electronic circuitry have been matched to $\sim 20^\circ$. Since the quadrant intensity changes $\sim 10\%$, the effect of phase mismatch is limited to $2^\circ$.

Although the new detection circuit minimizes the error originating from summing the signals of different phases, a beam movement dependent phase shift is also observed on the signal from a single quadrant (Figure 4.6a). Even though the beam movement causes both R and phase to change, R and phase are unrelated since affecting R without steering the beam has negligible effect on the phase. (Figure 4.6b). The interaction between the light and the curved wall of the glass vapor cell have been proposed as a possible source, but it could not be tested since the cells cannot be accessed without disassembling the magnetic shields.

### 4.4 Laser Stability

Laser stability is critical to the experiment because the response of the atoms depends on the intensity and frequency of the light. As mentioned in Ch 3.1.3, the stability of Cs laser has been improved by replacing the diode block and the laser cover. Although the Hg laser stability is also improved by reducing a fast noise in the intensity, a recently acquired problem in the temperature control of the first doubling cavity has prevented its operation.

The Hg laser output used to show a 28kHz intensity fluctuation in the 254nm light, and the noise eater required most of the dynamic range to correct for it. As a result, the noise eater was unable to correct for the occasional moderately sized noise that it was designed to correct for, and the noise pulses created sudden drops in the intensity. This caused problems in the frequency lock circuit by creating an abrupt change in the absorption spectrum. As a result the phase showed $>100^\circ$ pulses that correspond to the intensity drop, and the frequency lock failed frequently. The cause of this pulse was identified as the green cavity PZT, where the cavity mirror exhibited a resonance in
(a) R & Phase Change Caused by Beam Movement

(b) R & Phase Change Caused without Beam Movement

Figure 4.6: R & Phase Change
28kHz. Applying a low-pass filter on the PZT driving voltage reduced the amplitude of the intensity fluctuation, and replacing it with another PZT with heavier mirror also improved the output.

Unfortunately, before the improvement could be tested in a data run, the temperature control for the first doubling cavity failed. Because the first frequency doubling is a sharp function of the crystal temperature, maintaining the crystal temperature 64.8°C is essential. Although the TED has been replaced, the problem persists, and it is currently under repair.

4.5 Table Stability

The table affects the experiment by flexing. Table flexing effectively causes beam movement by changing the relative positions of the optical components. It is likely caused by the rollers pushing up on different parts of table. The effect of flexing has been addressed by adding three Sorbothane pads between the table and the aluminum plate. Currently, its effect is below the sensitivity of our position sensitive photodetector (∼1µm).

![Table Distortion During Rotation](image)

Figure 4.7: Table Distortion During Rotation

As the table rotates, the rollers vibrate and make noise. A noise of a moderate intensity does not affect the measurement because the data is taken 180 seconds after rotation. However, a loud noise can throw the frequency of the laser outside the dynamic range of the locking circuit and cause it to unlock. The noise level has increased recently, possibly due to an increasing distortion of the table, and caused the Cs laser to unlock. Therefore, the brass rollers have been replaced with the Delrin counterparts, which is a light-weight,
low friction plastic resistant to deformation. The roller replacement reduced the table vibration considerably and almost no noise can now be heard during rotation.
Chapter 5

Data Run

There has been two data runs. The first one went from 10/24/07 to 10/27/07, and the second one was taken from 1/25/08 until 1/28/08. This chapter explains the data collection procedure and the data analysis method, and presents the results from the two data runs.

5.1 Data Collection Procedure

The running procedure has not been changed since 2005. Since it has been explained in detail in [20], only the general idea is given here.

Before starting the data run, a large change is applied to the DC magnetic field until the phases of the magnetometers saturate. A similar field change is made in the opposite direction. The range of the responsive region in phase is recorded, and the lock-in phase is set to be zero at the average of the maximum and minimum phase. Since the response region is symmetric around the linear regime where the phase has linear relationship with the magnetic field, the coarse magnetic field is adjusted to be near the center of the linear response region.

Once the magnetic field is set to the linear response region of the magnetometers, the data run program is executed. The program first adjusts the gradient field to ensure that the two Hg cells experience the same magnetic field. It then measures the sensitivities of the magnetometers by applying
±0.2V to the fine control of the magnetic coil, which is calibrated to output 2.84μG/V. Since the phase change varies linearly with the shift in magnetic field when the oscillating transverse field is near the resonance frequency, the measurement gives the slope of phase vs. magnetic field graph, which can be used to convert phase shift into an equivalent magnetic field.

Once the calibration is completed, the magnetic field is locked to the Cs phase, and actual data collection begins by collecting and integrating the data in the first experimental position. After 30 seconds, the data collection stops, the table rotates to the control position, and the system waits for the Hg atoms to settle. The waiting time can be adjusted, but since the Hg atoms have \( \sim 30 \text{ sec} \) relaxation time, the system generally waits for 180 seconds. When the atoms settle, two 30 sec points are taken consecutively, then the table rotates back, waits for 180 seconds, and then repeats the cycle again. Since the change in laser intensity and various electronic drifts can cause the sensitivity and transverse field to drift, the calibration cycle is repeated every few hours.

### 5.2 Data Analysis Method

Once the data is collected, the points that correspond to the calibration cycle are set aside after calculating the slope of the field strength vs. phase. Also, the points that differ from the mean by more than three standard deviation in relevant parameters are rejected. Examples of the relevant parameters are the noise eater signal and the Hg frequency error signal, where a high fluctuation in their values indicate a system failure. Once the values that correspond to instability in the laser systems and large disturbances are rejected, the difference among four measurement is calculated as shown in Ch 2.3. The resulting LLI violating signal in phase angles is converted to magnetic field shift using the calibration information. Finally, the data is fitted to a sine wave with the period of a sidereal day, 23.93447 hours.
5.3 10/24-10/27 Result

5.3.1 Data & Sine Fit

The best fit sinusoidal function with the period of sidereal day is obtained using mathematica. The fitted values for A, B, and C in the equation in the form \( A\cos(\omega t) + B\sin(\omega t) + C \) are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Value (V)</th>
<th>Standard Error ∆ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.0003</td>
<td>0.0002</td>
</tr>
<tr>
<td>B</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
<tr>
<td>C</td>
<td>-0.0124</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table 5.1: 10/24-10/27 Data Fit Parameters

The data points are rejected based on the vertical component of the magnetic field, the noise eater pre-Amp voltage, and the standard deviation obtained during the integration. The data points that show no change in the vertical magnetic field is rejected because it indicates that the table was not
rotating. The noise eater pre-Amp voltage is used as a rejection parameter, because it is indicative of the instability of the Hg laser. Finally, the standard deviations of the data points taken during each 30 second integration have been compared and rejected, because large standard deviation indicates that the phase has been drifting during the integration period. Of the original 295 data points, 79 have been rejected and 216 points are plotted above. The data shows a standard deviation of 0.001V.

5.3.2 Analysis

Since \( \sqrt{(|A| + \Delta A)^2 + (|B| + \Delta B)^2} \) is the maximum amplitude of oscillation, it puts an upper bound on the sinusoidal variation in the data. The fit parameters give 0.6mV as the maximum sidereal variation in the signal, which is converted to Gauss using the magnetic coil response, 2.84\( \mu \)G/V. The resulting Lorentz violating term in the effective magnetic field is 1.7nG. This corresponds to \( K_n \) of 1.3\( \mu \)Hz, which is which is 12 times larger than the result obtained in [12]. The result is far from our target sensitivity of 14pG, and the standard deviation shows how far the current status is from the goal. In order to obtain sensitivity of 14pG, the standard error must be less than \( 4.9 \times 10^{-6} \)V. Since the standard error decreases as the inverse of the square root of the number of measurements, this indicates that the standard deviation of our signal, 0.001V, requires 40,000 data points to reach the target. 40,000 points corresponds to over nine months of data collection, because it takes \( \sim 10 \)min to collect a data point. Therefore, our priority has been to stabilize the system enough to obtain the desired standard error with two or three months’ data collection.

5.4 1/25-1/28 Data Run - No Table Rotation

5.4.1 Modification in the Data Run

In order to measure the intrinsic stability of the magnetometers, some points have been taken without rotating the table. The intrinsic instability of the
magnetometers gives 0.0006V, or 1.7nG, for the standard deviation in the data, which requires only 15,000 points to get the standard error to the desired level. Also, eliminating the rotation speeds up the experiment 20 fold by removing the waiting time in between the rotations. This allows the 15,000 points to be taken in less than 6 days. Therefore, in an ideal condition, removing the table rotation makes the experiment feasible at the current level of stability. However, as the following data shows, the slow drift in the system becomes a limiting factor.

5.4.2 1/25-1/28 Data

Both Cs and Hg laser were unstable during the data run. Hg laser was
unlocked after \( \sim 45 \) hrs, and it was not recovered until after the data collection. Cs laser showed a slow drift in the PZT voltage which eventually led to redesigning of the diode block (see Ch 3.1.3). Due to the slow drifts in the signal, the data puts the upper bound of the Lorentz violating term at 6mV. This represents a factor of ten reduction in sensitivity from 10/24-10/27 data. These slow drift in the signal could have been eliminated by rotating the table, and it shows the importance of rotating the table.
Chapter 6

Conclusion

The current sensitivity of the apparatus is at 1.3µG, ∼100 times less than the target sensitivity of 14pG. However, there is still room for improvement. Short data runs like the one in Figure 5.4.1 taken without table rotation yield a standard deviation of ∼1.7nG. With a factor of two improvement on the magnetometers and elimination of the effects of table rotation, the data collection may only take about a month to complete. Therefore, a minor improvement on the laser systems and the elimination of the adverse effect of table rotation will lead to reaching the target.

Recently, the table rotation has become more stable and quieter during rotation thanks to the new Delrin rollers. Also, four new braces have been added to the tower. Their effect on the signal has not been measured yet, but an improvement in the signal is expected. If the addition of the new braces leads to a significant increase in the sensitivity, more structural support can be added to improve it further.

As for the Hg magnetometers, the 28kHz noise in the intensity of Hg laser intensity may be further reduced using a more sensitive low-pass filter on the PZT driving voltage, and the Hg cell can be replaced to reduce the time constant in the ramping behavior. Also, all three experimental vapor cells can be replaced with square cells to minimize the birefringence.

One possible future improvement might involve, replacing the aluminum plates in the rotating assembly with granite ones. This might eliminate table
bending. Another possibility would be to construct a parallel experiment with two identical apparatus, with one in the experimental position and the other in the control position. This would eliminate the need for table rotation, and the data can be taken twice every minute instead of every 10 minutes because we don’t have to wait for the Hg atoms to settle.
Notes

1. Group is a closed set under a binary operation (multiplication) with an identity element.
2. [15], pp 13-14
3. [21], pg 197
4. [8], pg 129-130
5. [15], pg 13
6. For complete derivation, see [15], pg 15
7. The following derivation follows problem 8.3 in pg. 147 of [21]
8. [21], pg. 200
9. [21], pg. 201
10. I thank Professor Jagannathan for helping me understand the meaning of background field.
11. [1], pg. 189
12. More on this in [1], pg. 188
13. [13], pg. 408: The difference of a factor of c comes from the charge unit. Here I have used coulomb unit. The book uses esu.
14. [10], pg. 239
15. [1], pg.191
16. [18], pg. 10
17. [1], pg.192
18. [21], Ch12, 13
19. [5]
20. The corresponding gamma matrices are defined in pp. 216, 224 in [9]. The wavefunction is the Dirac spinor that describes the state of a spin 1/2 system.
21. [7], pg. 9
22. [14], pg 7
23. For a detailed explanation of Larmor precession see [8] pp. 179-181
25. For a detailed derivation, see [20] pp 10-11
26. [14]
27. [17], pg 13
28. [16]
29. [20], pp 40-41
30. [20], pp 96-100
31. See [7].pg 18
33. See the Chapter 3.1.3 for further discussion on the cover
34. See Ch 3.1.3 for further explanations on the diode block
The derivation closely follows [2], the derivation is based on [11], [3], pg329, http://www.mumetals.com/
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