Assignment 7-Solutions

Chapter 12: #22, 24, 27, 112, 36, 38, 42b, 42d, 129, 28, 30, 32, 130

Light and Matter

22. The wavelength is the distance between consecutive wave peaks. Wave *a* shows 4 wavelengths and wave *b* shows 8 wavelengths.

$$\text{Wave } a\text{: } \lambda = \frac{1.6 \times 10^{-3}\text{ m}}{4} = 4.0 \times 10^{-4}\text{ m}$$

$$\text{Wave } b\text{: } \lambda = \frac{1.6 \times 10^{-3}\text{ m}}{8} = 2.0 \times 10^{-4}\text{ m}$$

Wave *a* has the longer wavelength. Because frequency and photon energy are both inversely proportional to wavelength, wave *b* will have the higher frequency and larger photon energy since it has the shorter wavelength.

$$\nu = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\text{ m/s}}{2.0 \times 10^{-4}\text{ m}} = 1.5 \times 10^{12}\text{ s}^{-1}$$

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34}\text{ J s} \times 3.00 \times 10^{8}\text{ m/s}}{2.0 \times 10^{-4}\text{ m}} = 9.9 \times 10^{-22}\text{ J}$$

Because both waves are examples of electromagnetic radiation, both waves travel at the same speed, c, the speed of light. From Figure 12.3 of the text, both of these waves represent infrared electromagnetic radiation.

24. $$E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34}\text{ J s} \times 3.00 \times 10^{8}\text{ m/s}}{25\text{ nm} \times \frac{1\text{ m}}{1 \times 10^{9}\text{ nm}}} = 8.0 \times 10^{-18}\text{ J/photon}$$

$$\frac{8.0 \times 10^{-18}\text{ J}}{\text{photon}} \times \frac{6.02 \times 10^{23}\text{ photons}}{\text{mol}} = 4.8 \times 10^{6}\text{ J/mol}$$

27. The energy to remove a single electron is:

$$\frac{208.4\text{ kJ}}{\text{mol}} \times \frac{1\text{ mol}}{6.022 \times 10^{23}} = 3.461 \times 10^{-22}\text{ kJ} = 3.461 \times 10^{-19}\text{ J} = E_w$$

Energy of 254-nm light is:

$$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\text{ J s})(2.998 \times 10^{8}\text{ m/s})}{254 \times 10^{-9}\text{ m}} = 7.82 \times 10^{-19}\text{ J}$$

$E_{photon} = E_K + E_w$, $E_K = 7.82 \times 10^{-19}\text{ J} - 3.461 \times 10^{-19}\text{ J} = 4.36 \times 10^{-19}\text{ J}$ = maximum KE

Additional Exercises

112. Energy to make water boil = $s \times m \times \Delta T = \frac{4.18\ J}{°C\ g} \times 50.0\ g \times 75.0°C = 1.57 \times 10^4\ J$.

$$E_{photon} = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34}\ J\ s \times 2.998 \times 10^8\ m/s}{9.75 \times 10^{-2}\ m} = 2.04 \times 10^{-24}\ J$$

$$1.57 \times 10^4\ J \times \frac{1\ s}{750.\ J} = 20.9\ s; \qquad 1.57 \times 10^4\ J \times \frac{1\ photon}{2.04 \times 10^{-24}\ J} = 7.70 \times 10^{27}\ photons$$

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Hydrogen Atom: The Bohr Model

36. Ionization from $n = 1$ corresponds to the transition $n_i = 1 \rightarrow n_f = \infty$ where $E_\infty = 0$.

$$\Delta E = E_\infty - E_1 = -E_1 = R_H\left(\frac{1}{1^2}\right) = R_H,\ \Delta E = 2.178 \times 10^{-18}\ J = E_{photon}$$

$$\lambda = \frac{hc}{E} = \frac{(6.6261 \times 10^{-34}\ J\ s)(2.9979 \times 10^8\ m/s)}{2.178 \times 10^{-18}\ J} = 9.120 \times 10^{-8}\ m = 91.20\ nm$$

To ionize from $n = 3$, $\Delta E = 0 - E_3 = R_H\left(\frac{1}{3^2}\right) = 2.178 \times 10^{-18}\ J\left(\frac{1}{9}\right)$

$$\Delta E = E_{photon} = 2.420 \times 10^{-19}\ J;\ \lambda = \frac{hc}{E} = 8.208 \times 10^{-7}\ m = 820.8\ nm$$

38. $\Delta E = -2.178 \times 10^{-18}\ J\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = -2.178 \times 10^{-18}\ J\left(\frac{1}{5^2} - \frac{1}{1^2}\right) = 2.091 \times 10^{-18}\ J = E_{photon}$

$$\lambda = \frac{hc}{E} = \frac{6.6261 \times 10^{-34}\ J\ s \times 2.9979 \times 10^8\ m/s}{2.091 \times 10^{-18}\ J} = 9.500 \times 10^{-8}\ m = 95.00\ nm$$

Because wavelength and energy are inversely related, visible light ($\lambda \approx 400$–700 nm) is not energetic enough to excite an electron in hydrogen from $n = 1$ to $n = 5$.

$$\Delta E = -2.178 \times 10^{-18}\ J\left(\frac{1}{6^2} - \frac{1}{2^2}\right) = 4.840 \times 10^{-19}\ J$$

$$\lambda = \frac{hc}{E} = \frac{6.6261 \times 10^{-34}\ J\ s \times 2.9979 \times 10^8\ m/s}{4.840 \times 10^{-19}\ J} = 4.104 \times 10^{-7}\ m = 410.4\ nm$$

Visible light with $\lambda = 410.4$ nm will excite an electron from the $n = 2$ to the $n = 6$ energy level.

42. For one electron species, $E_n = \frac{-R_H Z^2}{n^2}$, where $R_H = 2.178 \times 10^{-18}$ J, and Z = atomic number (nuclear charge).

The electronic transition is $n = 1 \rightarrow n = \infty$ ($E_\infty = 0$). This is called the ionization energy (IE). Because $E_\infty = 0$, the IE is given by the energy of state $n = 1$ ($\Delta E = E_\infty - E_1 = -E_1 = R_H Z^2/1^2 = R_H Z^2$).

b. He^+: Z = 2; IE = 1311.6 kJ/mol × 2^2 = 5246 kJ/mol (Assume $n = 1$ for all.)

d. C^{5+}: Z = 6; IE = 1311.6 kJ/mol × 6^2 = 4.722×10^4 kJ/mol

Additional Exercises

129. a. Because wavelength is inversely proportional to energy, the spectral line to the right of B (at a larger wavelength) represents the lowest possible energy transition; this is $n = 4$ to $n = 3$. The B line represents the next lowest energy transition, which is $n = 5$ to $n = 3$, and the A line corresponds to the $n = 6$ to $n = 3$ electronic transition.

b. Because this spectrum is for a one-electron ion, $E_n = -2.178 \times 10^{-18}$ J (Z^2/n^2). To determine ΔE and, in turn, the wavelength of spectral line A, we must determine Z, the atomic number of the one electron species. Use spectral line B data to determine Z.

$$\Delta E_{5 \rightarrow 3} = -2.178 \times 10^{-18}\ \text{J} \left(\frac{Z^2}{3^2} - \frac{Z^2}{5^2}\right) = -2.178 \times 10^{-18} \left(\frac{16Z^2}{9 \times 25}\right)$$

$$E = \frac{hc}{\lambda} = \frac{6.6261 \times 10^{-34}\ \text{J s}(2.9979 \times 10^8\ \text{m/s})}{142.5 \times 10^{-9}\ \text{m}} = 1.394 \times 10^{-18}\ \text{J}$$

Because an emission occurs, $\Delta E_{5 \rightarrow 3} = -1.394 \times 10^{-18}$ J.

$$\Delta E = -1.394 \times 10^{-18}\ \text{J} = -2.178 \times 10^{-18}\ \text{J} \left(\frac{16\,Z^2}{9 \times 25}\right),\ Z^2 = 9.001,\ Z = 3;\ \text{the ion is } Li^{2+}.$$

Solving for the wavelength of line A:

$$\Delta E_{6 \rightarrow 3} = -2.178 \times 10^{-18}(3)^2\left(\frac{1}{3^2} - \frac{1}{6^2}\right) = -1.634 \times 10^{-18}\ \text{J}$$

$$\lambda = \frac{hc}{|\Delta E|} = \frac{6.6261 \times 10^{-34}\ \text{J s}(2.9979 \times 10^8\ \text{m/s})}{1.634 \times 10^{-18}\ \text{J}} = 1.216 \times 10^{-7}\ \text{m} = 121.6\ \text{nm}$$

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Light and Matter

28. Planck's discovery that heated bodies give off only certain frequencies of light and Einstein's study of the photoelectric effect support the quantum theory of light. The wave-particle duality is summed up by saying all matter exhibits both particulate and wave properties. Electromagnetic radiation, which was thought to be a pure waveform, transmits energy as if it has particulate properties. Conversely, electrons, which were thought to be particles, have a wavelength associated with them. This is true for all matter. Some evidence supporting wave properties of matter are:

    1. Electrons can be diffracted like light.

    2. The electron microscope uses electrons in a fashion similar to the way in which light is used in a light microscope.

    However, wave properties of matter are only important for small particles with a tiny mass, e.g., electrons. The wave properties of larger particles are not significant.

30. a. 10.% of speed of light = $0.10 \times 3.00 \times 10^8$ m/s = 3.0×10^7 m/s

$$\lambda = \frac{h}{mv},\ \lambda = \frac{6.63 \times 10^{-34}\ \text{J s}}{9.11 \times 10^{-31}\ \text{kg} \times 3.0 \times 10^{7}\ \text{m/s}} = 2.4 \times 10^{-11}\ \text{m} = 2.4 \times 10^{-2}\ \text{nm}$$

Note: For units to come out, the mass must be in kg since 1 J = 1 kg m^2/s^2.

b. $$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J s}}{0.055\ \text{kg} \times 35\ \text{m/s}} = 3.4 \times 10^{-34}\ \text{m} = 3.4 \times 10^{-25}\ \text{nm}$$

This number is so small that it is insignificant. We cannot detect a wavelength this small. The meaning of this number is that we do not have to worry about the wave properties of large objects.

32. $\lambda = \frac{h}{mv},\ v = \frac{h}{\lambda m}$; for $\lambda = 1.0 \times 10^2$ nm = 1.0×10^{-7} m:

$$v = \frac{6.63 \times 10^{-34}\ \text{J s}}{9.11 \times 10^{-31}\ \text{kg} \times 1.0 \times 10^{-7}\ \text{m}} = 7.3 \times 10^{3}\ \text{m/s}$$

For λ = 1.0 nm = 1.0×10^{-9} m: $$v = \frac{6.63 \times 10^{-34}\ \text{J s}}{9.11 \times 10^{-31}\ \text{kg} \times 1.0 \times 10^{-9}\ \text{m}} = 7.3 \times 10^{5}\ \text{m/s}$$

Challenge Problems

130. $\lambda = \frac{h}{mv}$; $v_{rms} = \sqrt{\frac{3RT}{m}}$; $\lambda = \frac{h}{m\sqrt{\frac{3RT}{m}}} = \frac{h}{\sqrt{3RTm}}$

For one atom, $R = \frac{8.3145 \text{ J}}{\text{K mol}} \times \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ atoms}} = 1.381 \times 10^{-23} \text{ J K}^{-1} \text{ atom}^{-1}$

$2.31 \times 10^{-11} \text{ m} = \frac{6.626 \times 10^{-34} \text{ J s}}{\sqrt{m}\sqrt{3(1.381 \times 10^{-23})(373 \text{ K})}}$, $m = 5.32 \times 10^{-26} \text{ kg} = 5.32 \times 10^{-23} \text{ g}$

Molar mass $= \frac{5.32 \times 10^{-23} \text{ g}}{\text{atom}} \times \frac{6.022 \times 10^{23} \text{ atoms}}{\text{mol}} = 32.0 \text{ g/mol}$

The atom is sulfur (S).

Assignment 7 - Solution to Challenge Problem

1. a. The equation for the energy states of the He^+ ion is:

$$E_{photon} = |\Delta E| = (-2.178\times10^{-18} \text{ J})(Z^2)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where $Z = 2$, $n_i = 1$, and $E_{photon} = \frac{hc}{\lambda}$, for λ = 24.3 nm. Solving gives $n_f = 4$.

b. For the second transition, we use the same equation, but now $n_i = 4$ and $n_f = 2$. Note that ΔE will be negative since this is an emission. Solving gives λ = 121.6 nm – ultraviolet radiation (not visible).

c. Transition a: n=1 → n=4; Transition b: n=4 → n=2

d. The discrete lines in the hydrogen atom emission spectrum could not be explained using classical mechanics. Specifically, the spectrum can only be explained based on well defined energy states of the electron within the hydrogen atom. Classical mechanics does not predict these states, but they follow directly from the assumption that the various energy states represent allowed standing waves for the electron in the vicinity of the nucleus of the H atom.