

# Allocating Misallocation: Decomposing Measures of Aggregate Allocative Efficiency<sup>1</sup>

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<sup>1</sup>This presentation is part of the Collaborative Microproductivity Project (CMP). Any views expressed are those of the authors and not those of the U.S. Census Bureau. The Census Bureau's Disclosure Review Board and Disclosure Avoidance Officers have reviewed this data product for unauthorized disclosure of confidential information and have approved the disclosure avoidance practices applied to this release. (DRB Approval Number CBDRB-FY19-495, CBDRB-FY22-CES004-006).

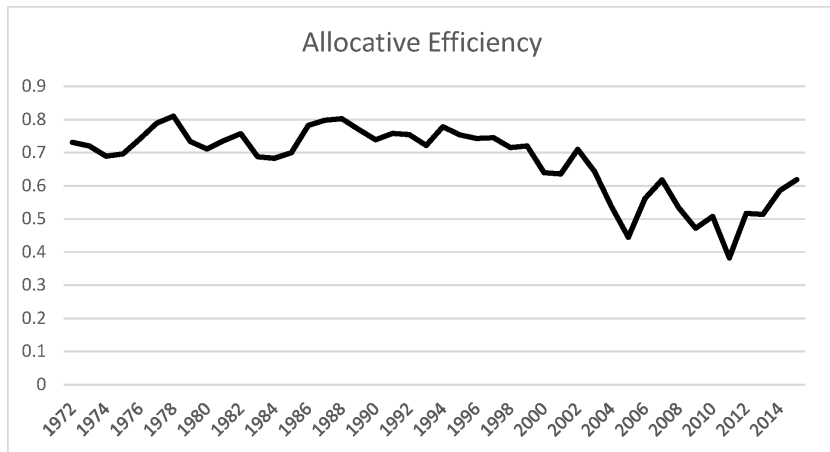
# Motivation

- ▶ Trend 1: Secular decline in measures of dynamism
  - ▶ Job Creation, Job Destruction, and Reallocation Rates (Decker et al., 2014)
  - ▶ Worker Reallocation Rates (Davis and Haltiwanger, 2014)
    - ▶ Geographic, Demographic, and Industry changes do not explain all of the decline (at most 35%)
  - ▶ Entrepreneurship (Decker et al., 2014, 2020)
  - ▶ High Growth Young Firms (Decker et al., 2016)
  - ▶ Migration Rates (Molloy, Smith and Wozniak, 2013)
- ▶ Should we be worried?
  - ▶ Adverse implications for productivity if due to rising frictions and distortions
  - ▶ Alternative hypothesis is that patterns reflect changing business structure
    - ▶ e.g., large, global chains in retail trade are more stable and productive
    - ▶ Rising Markups (De Loecker and Eeckhout, 2020)
  - ▶ Both hypotheses might be true. If latter dominates, why has productivity growth been so anemic since early 2000s (and potentially earlier)? (Gordon, 2016)

# Motivation

- ▶ Trend 2: Increased Dispersion in productivity and growth rate distributions (Decker et al., 2020)
- ▶ Should we be worried?
  - ▶ Shocks or Responsiveness? (Decker et al., 2020)
  - ▶ Fundamentals or Distortions?
    - ▶ Standard macro models interpret revenue productivity dispersion as increased misallocation (Hsieh and Klenow (2009), Bils, Klenow and Ruane (2020))
    - ▶ Measured allocative efficiency (AE) declines in U.S. manufacturing, especially post-2000
    - ▶ Due to rising dispersion in revenue productivity (TFPR) and increasing correlation between TFPR and TFPQ (Blackwood et al. (2021))

# What is Causing the Decline in Measured Allocative Efficiency in the US?



Authors' calculations using Census micro data of U.S. manufacturing, 1972-2015.

# Our Approach

## Decompose Standard Allocative Efficiency Measure

- ▶ Scale vs. Input Mix Distortions
  - ▶ Many sources of measured misallocation, some frictions/distortions impact all inputs equally, others impact factor mix
    - ▶ Pure scale sources: markups, measurement error in revenue
    - ▶ Pure mix sources: heterogeneous technologies
    - ▶ Mix and scale effects: adjustment frictions
  - ▶ We develop a decomposition that distinguishes scale and mix components.
  - ▶ We find both components are important
- ▶ Industry Contributions
  - ▶ Decline in AE is largely a within-industry phenomenon, characteristic of most industries, but...
  - ▶ Much of the overall decline can be explained by a few narrowly defined industries

## To-do List:

- ▶ Shocks vs. Responsiveness and misallocation
- ▶ Construct model with candidate mechanisms
- ▶ Compare decompositions simulated output to decompositions of data

# The Framework

$$Q = \prod_s Q_s^{\theta_s} \quad Q_s = \left( \sum_i Q_{is}^{\rho_s} \right)^{\frac{1}{\rho_s}} \quad Q_{is} = A_{is} \prod_j X_{ijs}^{\gamma_s \alpha_{js}}$$

Plant Profit Maximization:

$$\pi_{is} = \max_{X_{ijs}} \left( 1 - \tau_{is}^R \right) P_{is} Q_{is} - \sum_j (1 + \tau_{ijs}) w_{js} X_{ijs}$$

Define  $R_{is} = P_{is} Q_{is}$ , take FOC's:

$$\left( 1 - \tau_{is}^R \right) \rho_s \gamma_s \frac{R_{is}}{X_{ijs}} = (1 + \tau_{ijs}) w_{js} \implies \frac{R_{is}}{X_{ijs}} = \frac{(1 + \tau_{ijs})}{(1 - \tau_{is}^R)} \frac{w_{js}}{\rho_s \gamma_s}$$

Some algebra and applying a useful definitions:

$$\underbrace{TFPR_{is}^{CS} \equiv \frac{R_{is}}{\prod_j X_{ijs}^{\alpha_{js}}}}_{\text{Cost-share based Revenue TFP}} \propto \underbrace{\frac{\prod_j (1 + \tau_{ijs})^{\alpha_{js}}}{(1 - \tau_{is}^R)}}_{\text{Composite distortion in HK}} \equiv \tau_{is}$$

- Under common assumptions,  $ARPX \propto MRPX$ , and so should be equalized across plants
- Commonly used measure of productivity  $TFPR^{CS}$  variation is solely due to distortions

# Identifying and Decomposing Distortions

- ▶ Can estimate plant-level distortion from  $TFPR^{CS}$ :  $\tau_{is} \propto \frac{R_{is}}{\prod_j X_{ijs}^{\alpha_{js}}}$
- ▶ Under CRS, can obtain estimate of  $A_{is}$  (sometimes called TFPQ):  $A_{is} \propto \frac{R_{is}^{\frac{1}{\rho_s}}}{\prod_j X_{ijs}^{\alpha_{js}}}$ 
  - ▶ Need alternative procedures under NCRS (Blackwood et al., 2021)
- ▶ For individual distortions, need an assumption ( $N_j$  equations,  $N_j + 1$  unknowns)

- ▶ With distortions:  $\frac{R_{is}}{TC_{is}} = \frac{1}{\rho_s \gamma_s} \frac{\left( \sum_j \frac{\alpha_{js}}{(1+\tau_{ijs})} \right)^{-1}}{(1-\tau_{is}^R)}, \frac{\alpha_{js}}{c_{ijs}} = (1 + \tau_{ijs}) \left( \sum_j \frac{\alpha_{js}}{(1+\tau_{ijs})} \right)$ 
  - ▶  $TC_{is} \equiv \sum_j w_{js} X_{ijs} \quad ; \quad c_{ijs} \equiv \frac{w_{js} X_{ijs}}{TC_{is}}$
- ▶ Note, if no distortions,  $\frac{R_{is}}{TC_{is}} = \frac{1}{\rho_s \gamma_s}, \frac{\alpha_{js}}{c_{ijs}} = 1 \quad \forall j$
- ▶ Example to motivate: Only idiosyncratic distortion are markups ( $\tau_{ijs} = 0$ )
  - ▶ Markup:  $\mu_{is} = R_{is} / TC_{is} \propto (1 - \tau_{is}^R)^{-1} \quad ; \quad c_{ijs} = \alpha_{js} \quad \forall j$
  - ▶ Only scale effects, no mix effects

# Scale vs. Mix Decomposition: Our Normalization

Idea: Normalize distortions so that mix only impacted by input distortions, markup by  $\tau_{is}^R$

$$\left( \sum_j \frac{\alpha_{js}}{1 + \tau_{ijs}} \right)^{-1} = 1 \implies \begin{cases} \frac{R_{is}}{TC_{is}} = \frac{1}{\rho_s \gamma_s} \frac{1}{(1 - \tau_{is}^R)} \\ \frac{\alpha_{js}}{c_{ijs}} = (1 + \tau_{ijs}) \end{cases}$$

What does this mean in words?

- ▶ Common distortions across inputs (the full extent or “scale” of the plant) loaded on  $\tau_{is}^R$ .
- ▶ Heterogeneity in plant-level markups is due to dispersion in scale distortion.
- ▶ Variation in input mix loaded on input distortion



# Scale vs. Mix Decomposition

With these assumptions/results in hand, some considerations:

- ▶ Can identify scale vs. mix distortions with readily computable moments.
- ▶ The “distortions” themselves should be taken seriously, not literally (think latent variables approach)
  - ▶ Normalizing means we are building in assumptions, which help with interpretation
- ▶ Many sources of misallocation (e.g., adjustment frictions) have both mix and scale effects.
- ▶ Pure scale sources include: markups, measurement error in revenue (only)
- ▶ Pure mix sources potentially includes specification error (e.g., heterogeneous technologies)
- ▶ Different effects on measured and true AE: some of these (i.e., measurement and specification error) imply decline in measured but not actual AE
- ▶ Also... you can now forget about  $\gamma_s$ , which will now be 1

# Key Equations: Allocative Efficiency Decomposition

With a heave dose of algebra, can show Sectoral TFP :

$$A_s = \frac{Q_s}{\prod_j X_{js}^{\alpha_{js}}} = \left( \sum_i A_{is}^{\frac{\rho_s}{1-\rho_s}} \left( \frac{\tau_{is}}{\tau_s} \right)^{\frac{-\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}} \quad A_s^* = \left( \sum_i A_{is}^{\frac{\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}}$$

Allocative Efficiency:  $AE_s = A_s / A_s^*$

$$AE_s = \left( \frac{1}{N_s} \sum_i \left( \frac{A_{is}}{\widetilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s}} \left( \frac{\tau_{is}}{\tau_s} \right)^{\frac{-\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}}$$

This paper:

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# Key Equations: Allocative Efficiency Decomposition

Sectoral TFP

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$$AE_s^{-rev} = \left( \frac{1}{N_s} \sum_i \left( \frac{A_{is}}{\widetilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s}} \left( \frac{(1 - \tau_s^R)}{(1 - \tau_s^R)} \right)^{\frac{-\rho_s}{1-\rho_s}} \prod_j \left( \frac{(1 + \tau_{ijs})^{\alpha_{js}}}{(1 + \tau_{js})^{\alpha_{js}}} \right)^{\frac{-\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}}$$

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$AE^{-rev}$  also interpretable as AE due to mix distortions only.

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This paper:

$$AE_s^{-mix} = \left( \frac{1}{N_s} \sum_i \left( \frac{A_{is}}{\widetilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s}} \left( \frac{(1 - \tau_s^R)}{(1 - \tau_{is}^R)} \right)^{-\frac{\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}}$$

$AE^{-mix}$  also interpretable as AE due to scale distortions only.

# Key Equations: Allocative Efficiency Decomposition

Sectoral TFP

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This paper:

$$AE_s^{-TFPQ} = \left( \frac{1}{N_s} \sum_i \left( \frac{(1 - \tau_s^R)}{(1 - \tau_{is}^R)} \right)^{\frac{-\rho_s}{1-\rho_s}} \prod_j \left( \frac{(1 + \tau_{ijs})^{\alpha_{js}}}{(1 + \tau_{js})^{\alpha_{js}}} \right)^{\frac{-\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}}$$

# Data

## **CMP Dataset: 1972-2015**

- ▶ Multi-year collaborative project between Census and BLS to produce microproductivity dataset
- ▶ ASM/CM data (ASM sample only, weighted): revenue, expenditures, inputs, industry
  - ▶ Capital stock built using perpetual inventory method
- ▶ External data on rental prices, deflators, etc. from BLS, NBER, BEA
- ▶ Cleaning: trim 1% tails on Average Revenue products, only include plants with positive measured profit

## **Implementation**

- ▶ Key parameters (output elasticities, demand elasticities) estimated within the dataset

## One last detail... Parameters

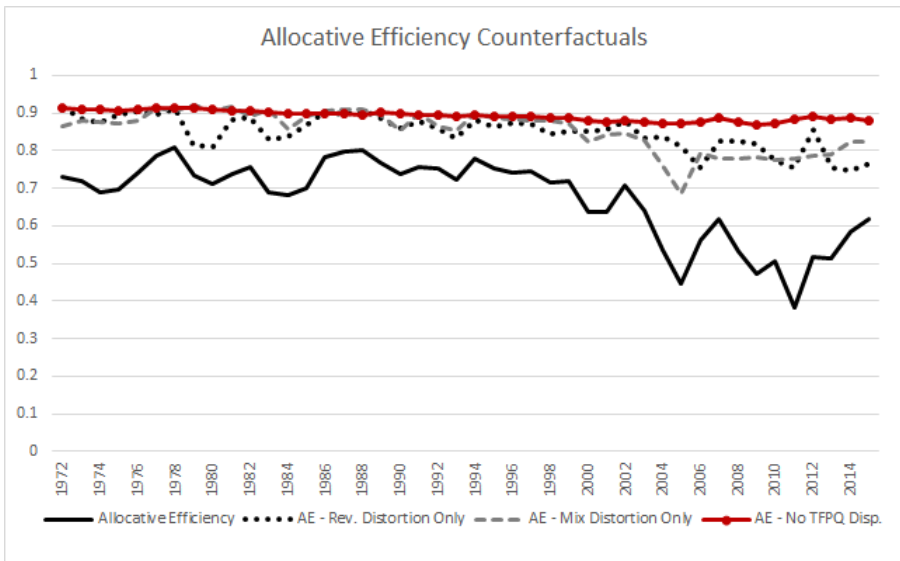
We do not obtain parameter values from external sources, which has the following implications:

1. Internally consistent estimation of  $\alpha_{js}$ 's using cost shares requires  $\tau_{js} = \tau_{ks} \forall j, k$ .
2. Given (1), estimate of  $\rho_s \gamma_s$  from DeLoecker Warzynski method requires:

$$\left(1 - \tau_s^R\right) = (1 + \tau_{sj}) \quad \forall j \implies (1 + \tau_{ijs}) = \frac{\alpha_{js}}{c_{ijs}} \quad (1)$$

3. Our normalization + (1)  $\implies \tau_{js} = 0, \forall j$ .
4. Likewise (1)+(2)+(3)  $\implies \tau_s^R = 0$ .





Scale/mix distortions about equally important. AE much higher without TFPQ dispersion even with idiosyncratic distortions, suggesting correlation between TFPQ and TFPR is important.

# AE and Covariance between “Fundamentals” and “Distortions”

Why does TFPQ dispersion matter so much?

- Most of the literature focused on dispersion in  $TFPR^{CS}$  (distortions)

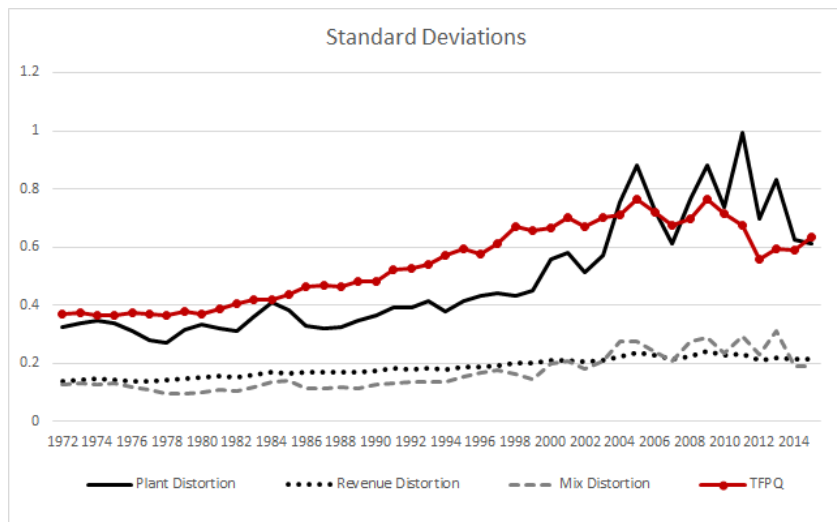
$$AE_s = \left( \frac{1}{N_s} \sum_i \left( \frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s}} \left( \frac{\tau_{is}}{\tau_s} \right)^{\frac{-\rho_s}{1-\rho_s}} \right)^{\frac{1-\rho_s}{\rho_s}}$$

Blackwood et al. (2021):

$$\log AE_s = \log \left( \frac{\tilde{\tau}_s}{\tau_s} \right) + \frac{1-\rho_s}{\rho_s} \log \left[ \text{cov} \left( \left( \frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s}}, \left( \frac{\tau_{is}}{\tau_s} \right)^{\frac{-\rho_s}{1-\rho_s}} \right) + 1 \right]$$

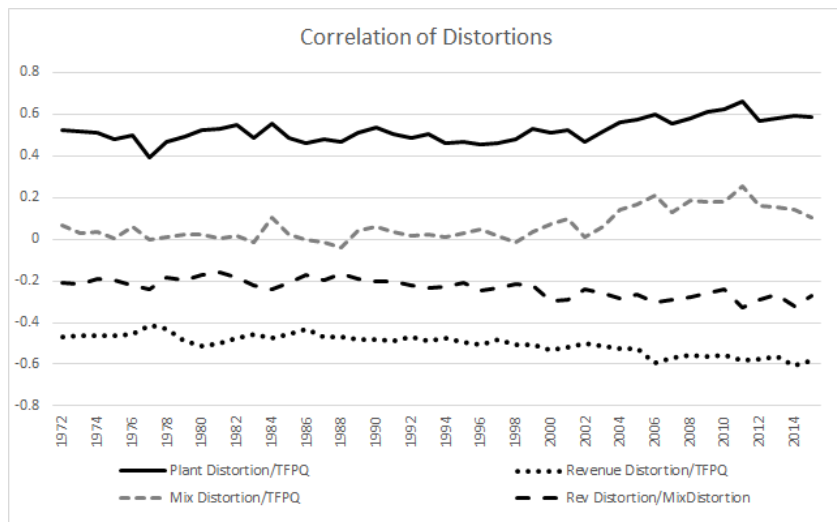
Whole of the covariance structure matters: standard deviations of distortions and fundamentals, AND correlation

# Moments of TFPQ and “Distortions”



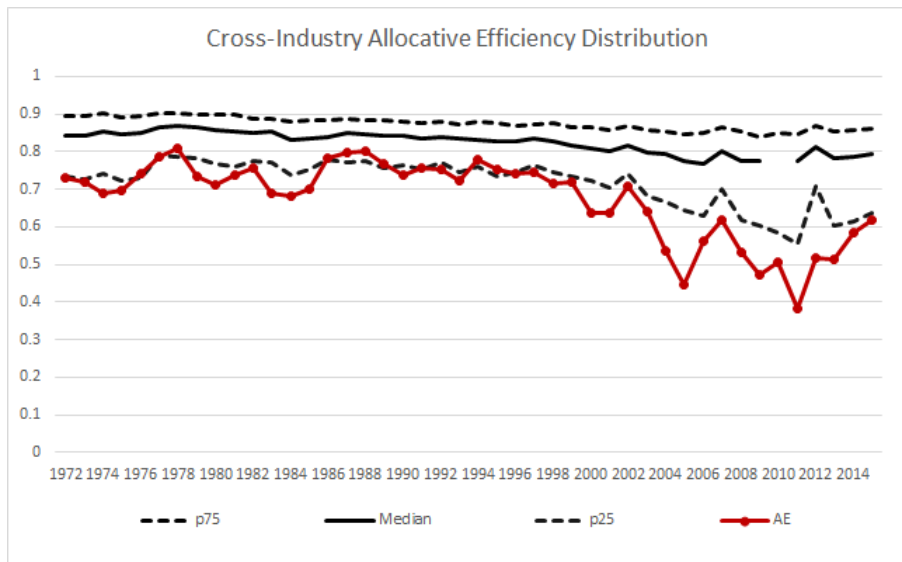
Decomposition shows rising dispersion in both scale and mix distortions.

# Moments of TFPQ and “Distortions”: 1972-2015



Increased correlation between TFPQ and overall distortion has both scale and mix components.

# How Much of This is an Industry Story?



# A Closer Look at Aggregate AE

$$Q = \prod_s Q_s^{\theta_s}$$

Turns out, this implies a similar structure for both aggregate TFP and AE:

$$AE = \prod_s AE_s^{\theta_s}$$

Taking logs:

$$\log(AE) = \sum_s \theta_s \log(AE_s)$$

This is very nice! Can easily be separated/decomposed. Changes too:

$$\Delta \log(AE_t) = \ln(AE_t) - \ln(AE_{t-1})$$

Note: Does not need to be  $t - 1$  to  $t$ , can be any two periods

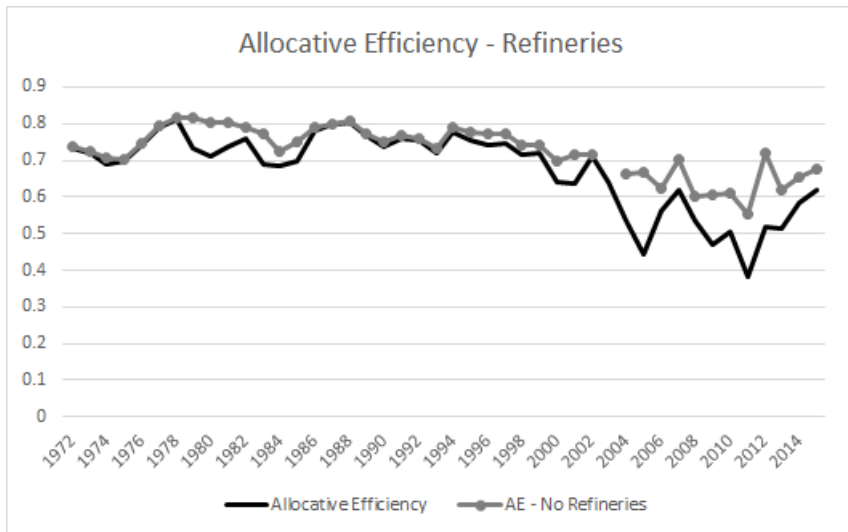
# FHK Industry Decomposition

$$\Delta \ln(AE_t) = \underbrace{\sum_s \theta_{s,t-1} \Delta \ln(AE_{s,t})}_{\text{Within}} + \underbrace{\sum_s (\theta_{s,t} - \theta_{s,t-1}) \ln(AE_{s,t-1})}_{\text{Between}} + \underbrace{\sum_s (\theta_{s,t} - \theta_{s,t-1}) \Delta \ln(AE_{s,t-1})}_{\text{OP Covariance}}$$

	Log Change in AE	Within	Between	OP Cov. Term
Baseline AE	− <b>0.668</b>	− <b>0.496</b>	−0.0647	− <b>0.1073</b>
Revenue Distortions Only	− <b>0.143</b>	−0.1026	−0.0263	−0.0141
Mix Distortions Only	− <b>0.1211</b>	−0.1313	−0.0303	0.0404

**Table:** FHK Decomposition of Allocative Efficiency Change from 1997 to 2011

# No Refineries Counterfactual: 1972-2015





# FHK Industry Decomposition

	Log Change in AE	Within	Between	OP Cov. Term
<b>Total MFG</b>	<b>-0.668</b>	-0.496	-0.0647	-0.1073
Refineries	<b>-0.3329</b>	<b>-0.0993</b>	<b>-0.0592</b>	<b>-0.1745</b>
Automobiles	<b>-0.0715</b>	-0.1191	0.0024	0.0452
Plastics	<b>-0.0238</b>	-0.0181	-0.0006	-0.0051

Table: Industry Decomposition of Allocative Efficiency Change from 1997 to 2011

# Discussion of Industry Analysis

- ▶ On the one hand, declining AE is a broad-based phenomenon in the manufacturing sector
  - ▶ On the other hand, three industries (in particular one) greatly influences the overall decline
- ▶ How to handle these industries?
  1. Determine if they share enough similarities with broader sector to serve as illustrations
  2. Explore the idiosyncratic factors contributing to industry-specific declines
    - ▶ Refineries: Oil prices, fracking, weather, pipeline reversals, regulation, high fixed costs
- ▶ Lessons for aggregation
  - ▶ Does Cobb-Douglas give undue influence to individual industries?
  - ▶ What are costs/benefits to alternative demand structures for macroeconomists?
    - ▶ We want to preserve tractability for this class of models: useful accounting
    - ▶ Can supplement with more flexible models
  - ▶ Input-Output
    - ▶ “Roundabout” production readily implemented (Typically amplifies decline in AE)
    - ▶ Importance of I-O emphasized by Baqaee and Farhi (2020, etc.)

# Summary

## Key Findings:

- ▶ Both scale and mix distortions about equally important for declining AE (literature has focused mostly on scale!)
- ▶ Importance of TFPQ dispersion
- ▶ Declining AE is a broad-based phenomenon, but quantitatively, a few industries dominate

## Tentative Implications for Specific Mechanisms (work in progress):

- ▶ Rising idiosyncratic markups or revenue measurement error can account for at most half of declining AE (scale)
- ▶ Rising heterogeneity in production technology can account for at most half of declining (measured) AE (mix)
  - ▶ Either multiple mechanisms at work or mechanism (e.g., adjustment costs) that has both scale and mix effects
- ▶ Alternate Aggregation
- ▶ Industry specific explanations are important for the aggregate

# Next Steps

- ▶ Explore responsiveness in manufacturing sector
  - ▶ Employment and Materials
- ▶ Construct quantitative model, simulate, compare output
  - ▶ E.g. we know adjustment cost model can generate dispersion in TFPR and lower responsiveness in employment
    - ▶ Correlation between TFPQ and TFPR?
    - ▶ Mix vs. scale distortions?
- ▶ Discuss role of individual industries
- ▶ Explore alternative aggregation