

**Syllabus for the Algebra Comprehensive Exam**  
**Department of Mathematics and Statistics, Amherst College**

**Sets, Functions, and Integers**

- One-to-one, onto, and bijective maps
- Equivalence relations and equivalence classes
- Division algorithm, gcd and lcm, primes and unique factorization

**Groups**

- Uniqueness of identities and inverses
- The order of an element

**Subgroups**

- Lagrange's Theorem and its consequences
- Cosets
- Normal subgroups
- Quotient groups

**Group Homomorphisms**

- Kernels and images
- Isomorphisms
- The Fundamental Theorem of Group Homomorphisms:  $G/\text{Ker}(\phi) \simeq \text{Im}(\phi)$

**Permutations**

- $S_n$  and disjoint cycle decomposition
- Transpositions and  $A_n$

**Rings**

- Commutative rings; Rings with unity; Fields
- Polynomial rings

**Ideals**

- Ideals
- Quotient rings

**Ring Homomorphisms**

- Kernels and images
- Isomorphisms
- The Fundamental Theorem of Ring Homomorphisms:  $R/\text{Ker}(\phi) \simeq \text{Im}(\phi)$

**Quotient Rings and Fields**

- Criteria for  $R$  to be a field
- Maximal ideals
- Criteria for  $R/M$  to be a field

**Polynomial Rings  $k[x]$ , for a Field  $k$**

- The division algorithm
- Every ideal in  $k[x]$  is principal
- Irreducible polynomials and maximal ideals in  $k[x]$