Syllabus for the Multivariable Calculus Portion of the Comprehensive Exam
Department of Mathematics and Statistics, Amherst College

Elementary vector analysis
- Dot or scalar product
- Cross or vector product
- Lines and planes
- Tangent vectors and tangent lines to parametrized curves

Definitions and computations involving functions of several variables
- Partial derivative
- Directional derivative
- Gradient
- Tangent plane to a surface

Maxima and minima of functions of several variables
- Finding critical points
- The second derivative test for local maxima/minima and saddle points
- The method of Lagrange multipliers

Double integrals
- Cartesian and polar coordinates
- Finding area and volume

Triple integrals
- Cartesian, cylindrical and spherical coordinates
- Finding volume

Line integrals of vector fields
- Fundamental Theorem of Calculus for Line Integrals
- Green’s Theorem
Syllabus for the Comprehensive Examination in Linear Algebra (MATH 271/2)
Department of Mathematics and Statistics, Amherst College

Basic Definitions

- Vector space
- Subspace
- Span of a finite subset
- Linear independence of a finite set of vectors
- Basis and dimension
- Linear transformation
- Kernel or null space
- Image or range
- Inverse of a matrix or linear transformation
- Determinant
- Characteristic polynomial
- Eigenvalues and eigenvectors of a matrix
- Diagonalizability

Computational Techniques

- Determine when a subset is a subspace
- Basic matrix manipulations
- Row operations on matrices
- Solving systems of linear equations
- Find the inverse of a matrix
- Find a basis of a given subspace
- Find the nullity, rank, and determinant of a matrix
- Find the null space $N(T)$ and range $R(T)$ of a linear transformation $T$
- Given bases of $V$ and $W$, find the matrix of a linear transformation $T : V \rightarrow W$
- Given a matrix
  - Compute its characteristic polynomial
  - Find its eigenvalues and eigenspaces

Basic Results to Know

- $\dim N(T) + \dim R(T) = \dim V$
- $\text{nullity}(A) + \text{rank}(A) = \text{number of columns of } A$
- Criteria for $A^{-1}$ to exist
- Criteria for $A$ to be diagonalizable

Write short proofs for problems involving subspaces, linear maps, linear independence, spanning sets, null spaces and ranges.
Syllabus for the Algebra Comprehensive Exam
Department of Mathematics and Statistics, Amherst College

Sets, Functions, and Integers
• One-to-one, onto, and bijective maps
• Equivalence relations and equivalence classes
• Division algorithm, gcd and lcm, primes and unique factorization

Groups
• Uniqueness of identities and inverses
• The order of an element

Subgroups
• Lagrange’s Theorem and its consequences
• Cosets
• Normal subgroups
• Quotient groups

Group Homomorphisms
• Kernels and images
• Isomorphisms
• The Fundamental Theorem of Group Homomorphisms: $G/\text{Ker}(\phi) \cong \text{Im}(\phi)$

Permutations
• $S_n$ and disjoint cycle decomposition
• Transpositions and $A_n$

Rings
• Commutative rings; Rings with unity; Fields
• Polynomial rings

Ideals
• Ideals
• Quotient rings

Ring Homomorphisms
• Kernels and images
• Isomorphisms
• The Fundamental Theorem of Ring Homomorphisms: $R/\text{Ker}(\phi) \cong \text{Im}(\phi)$

Quotient Rings and Fields
• Criteria for $R$ to be a field
• Maximal ideals
• Criteria for $R/M$ to be a field

Polynomial Rings $k[x]$, for a Field $k$
• The division algorithm
• Every ideal in $k[x]$ is principal
• Irreducible polynomials and maximal ideals in $k[x]$
Syllabus for the Analysis Comprehensive Exam
Department of Mathematics and Statistics, Amherst College

Mathematical Induction

The Real Numbers
- Rational and irrational numbers
- Real numbers and the axiom of completeness

Sequences
- Convergence
- Convergence of bounded monotone sequences
- Cauchy sequences
- The Bolzano-Weierstrass Theorem for sequences

Point-Set Theory
- Limit points (also called cluster points or accumulation points)
- The Bolzano-Weierstrass Theorem for sets
- Open and closed sets
- Compact sets and the Heine-Borel Theorem

Infinite Series
- Convergence
- $p$-series and geometric series
- Absolute and conditional convergence
- Comparison, ratio, and alternating series tests

Limits and Continuity
- The limit of a function
- The definition of continuity and relation to sequences
- Continuity of sums, products, quotients, and compositions
- The Intermediate Value Theorem
- Boundedness
- Attainment of extreme values
- Uniform continuity

Differentiability and Derivatives
- Limit definition of derivative
- Derivatives at local extreme points
- The Mean Value Theorem and Rolle’s Theorem

Sequences of Functions
- Pointwise and uniform convergence
- Continuity of the limit function
- Proving uniform convergence
Series of Functions
- Pointwise and uniform convergence
- The Weierstrass M-test
- Power series
- Radius and interval of convergence, behavior at endpoints
- Continuity and differentiation of power series

Integration
- Definition of the Riemann integral
- Properties of the Riemann integral
- Integrability of a continuous function over \([a, b]\)
- Integration of sequences and series
- Integration of power series