Assignment 8

Due Wednesday, November 3, 2010

Topics: We have discussed several strange behaviors exhibited by very small particles and discussed the failure of the Bohr model to accurately represent the hydrogen atom. To explain the observations, we need to develop a new quantum mechanical model for small objects such as photons, electrons, and atoms. The solution to this problem comes from the Schrödinger equation – the basic equation of quantum mechanics – and its solutions for the hydrogen atom. This week, we will explore the profound implications of this equation for the behavior of electrons and the structure of atoms. The solutions to the Schrödinger equation can be interpreted as electronic wave functions. In other words, they are a direct manifestation of the wave nature of the electron. The wave functions can also be interpreted as a probability distribution, or a description of the where the electron is likely or unlikely to be found. Each solution is identified by a set of quantum numbers n, ℓ, and m_ℓ, which determine the energy, shape, size, and orientation of the wave function or "orbital".

Reading & Problems: Zumdahl, 6^{th} edition

Date	Lecture	Reading	Problems
Wednesday, 10/27	The Uncertainty Principle	Section 12.5	Chapter 12: #45
Friday, 10/29	The Schroedinger Eqn	Section 12.5, 12.7	Chapter 12: #52, 54,
Monday, 11/1	H- atom & orbitals	Sections 12.8-12.9	Chapter 12: #56
Wednesday, 11/3	H- atom & orbitals	Sections 12.8-12.9	

Additional problems:

A1. At what speed would an electron have to travel to have a wavelength equal to the circumference of the first Bohr orbit of the hydrogen atom? What would then be its kinetic energy?

A2. (a) Calculate the minimum uncertainty in the speed of a speck of dust of mass 1.0 μg with a position known to within 1.0 μm. (b) Suppose that you have located the speck to within 0.10 nm (about 1 atomic diameter). What is the minimum uncertainty in its speed? Based on your answers, do you expect the speck to be governed by classical mechanics or quantum mechanics?

A3. Use the hydrogen atom orbital applet at http://www.falstad.com/qmatom/ to simulate the following orbitals: 5*s*, 5*px*, 4*dxy*. Sketch a cross section or "slice" of each orbital (choose a view where all the lobes are visible). Be sure to show clearly the spatial orientation of the orbitals by indicating the coordinate axes. Use shading or +/- to indicate the sign of the wave function, and indicate any nodes that are present with dashed lines.

A4. On separate graphs, draw the radial part of the wave function (R(r)) corresponding to each sketch. Be sure to label the axes and sign of the wavefunction.

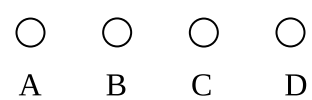
Challenge Problem:

J. J. Thomson's cathode ray tube experiments were the first to demonstrate the existence of the subatomic particles known as electrons. Later, J. J.'s son George Thomson was among the first to show electrons also have wave-like properties. It is interesting to consider why J. J. Thomson did not observe the wave properties of the electrons, whereas George Thomson did.

1. Why J. J. did NOT observe diffraction.
   a. A beam of electrons with a kinetic energy of 1.2×10^{-17} J is generated in a cathode ray tube. What are the (i) velocity and (ii) wavelength of the electrons in the beam?
   b. If this beam of electrons is passed through a slit of width 0.100 cm, what do you expect to observe? Recalling your results from lab, what conditions must be met for diffraction to be observed?
   c. For electrons with the wavelength calculated in 1a, what approximate slit width is necessary to observe diffraction?
   d. Write a summary sentence using your answers above to explain why JJ did not observe diffraction.

2. Why George DID observe diffraction.
   In George's experiments, instead of a large slit, a beam of electrons with the same velocity as those in part 1a is directed at a nickel crystal, and diffraction is observed. The electrons scattered by the crystal interfere in exactly the same way as in the two slit experiment you observed in lab.
   a. Explain why, in the two slit experiment, one would expect to observe intensity maxima ('bright spots') at points other than directly in front of the slits.
   b. The Bragg diffraction equation that you derived in lab stated that in a two slit experiment, an intensity maximum is observed when $n\lambda = d \sin\theta$. In George's experiments with an electron beam and nickel crystal, the first intensity maximum (i.e. n=1) occurs at an angle, θ, of 41.2^{o}. Using the wavelength for the electron calculated in 1a, calculate d, the distance between the "two slits".
   c. To what physical dimension of the nickel crystal does d correspond? Is the value you calculated in part 2b physically reasonable? Is it consistent with your answer to part 1? Explain.

3. What would Heisenberg say about this?
   a. Alas, uncertainty makes us never sure about our answers, or does it? If the uncertainty in the velocity of an electron from 1a is 2.69×10^{5} m/s, what is the uncertainty in its position?
   b. Assume for the moment that the nickel crystal consists of only 4 Ni nuclei, arranged as shown below. Is it possible to specify which pair of Ni nuclei (say, atoms B and C) any given electron has passed between? Explain.



   c. What would you expect to observe if a single electron rather than a beam were fired at the same Ni crystal?