Seeking a Dirac Monopole in a Spinor Condensate

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Abstract

Magnetic monopoles have not been observed in nature, but topologically analogous Dirac monopoles have been predicted in the vorticity field of a Bose-Einstein condensate. We attempt to experimentally realize this singular structure by manipulating the external magnetic fields applied to a spinor condensate in an optical trap. We investigate the technicalities and the sources of error in the experiment. We pay particular attention to the calibration of our various magnetic coils and, in this process, we develop a protocol for in situ measurement and cancellation of background DC fields. We also observe several Feshbach resonances, and use these as spectroscopic tools to investigate the quality of the magnetic fields. Our results indicate that our experiment suffers from AC magnetic field noise. Further progress on this experiment hinges on our ability to measure and cancel the magnetic field noise.
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Chapter 1

Introduction

Magnetic monopoles are hypothetical particles containing net magnetic charges. Theoretical developments in such diverse branches of physics as particle physics, quantum mechanics, and cosmology, provide a compelling argument for the existence of these particles [1–3]. However, experimental evidence of the existence of magnetic monopoles is still lacking, and hence there is great incentive for experimenters and theorists alike to study analogous topological structures in more tractable systems. A topological analog of a magnetic monopole has previously been predicted for superfluid $^3$He system [4, 5], but to date there has been no experimental observation.\(^1\)

Our investigations into topological monopoles takes place in dilute Bose-Einstein condensates of $^{87}$Rb with a hyperfine spin degree of freedom. This system combines superfluid and magnetic orders [7], and is amenable to the creation of a variety of topological defects through variation of an external

\(^{1}\text{NOTE ADDED IN PROOF: Amo et al., in a paper recently submitted to arXiv, experimentally demonstrate that half-solitons in a polariton quantum fluid behave like magnetic monopoles [6].}\)
magnetic field. It has been demonstrated theoretically and experimentally that, through adiabatic modification of the external magnetic field, it possible to imprint vortices [8, 9] as well as pointlike defects [3, 10] in the \( F = 1 \) ground state of a spinor condensate (Sections 1.4, 2.1). In particular, this thesis will attempt to follow the method prescribed by Pietilä and Möttönen [3], aiming to imprint a pointlike defect in the spin texture of a condensate. This defect in the spin texture leads to a Dirac monopole in the vorticity field of the condensate.

In this thesis, we chronicle our attempt to create a Dirac monopole in a spinor \(^{87}\text{Rb}\) condensate, and discuss the difficulties with the experiment. We begin with an overview of the theoretical underpinnings of our experiment, briefly explore our apparatus, examine our experimental setup, and then scrutinize our various magnetic coils to find our primary difficulties. In addition, we shall also briefly explore several Feshbach resonances, using them as valuable spectroscopic tools to obtain information about the accuracy and the precision of our applied magnetic fields.

## 1.1 Magnetic Monopoles

Before proceeding further, we need review some essential facts about magnetic monopoles and magnetic fields. A magnetic monopole, similar to an electric charge, is the source of a radial magnetic field \( \mathbf{B} \),

\[
\mathbf{B} = \frac{q \mathbf{r}}{r^3},
\]  

(1.1)
where \(g\) is the pole strength. The divergence of \(\mathbf{B}\) at the monopole is singular.

In Maxwell’s formulation, the magnetic field is given by \(\mathbf{B} = \nabla \times \mathbf{A}\), where \(\mathbf{A}\) is magnetic potential. With this formulation, we get,

\[
\nabla \cdot \mathbf{B} = 0.
\]

(1.2)

For a vector potential defined globally, this seems to preclude the existence of a magnetic monopole.

### 1.2 Dirac Monopole

Dirac, in a seminal paper in 1931 [1], overcame this quandary by proposing that the magnetic monopole exist in conjunction with an infinitesimal, semi-infinite line with the monopole at its terminus. The vector potential \(\mathbf{A}\) is now defined everywhere except along this semi-infinite line. To illustrate this, let us look at a sample vector potential,

\[
\mathbf{A}(\mathbf{r}) = -\frac{g}{r} \left( \frac{\mathbf{n} \times \mathbf{r}}{r^3} - \mathbf{n} \cdot \mathbf{r} \right),
\]

(1.3)

where \(\mathbf{\hat{r}}\) is the unit vector along the radial direction. This vector potential \(\mathbf{A}\) is singular along the semi-infinite line defined by \(\mathbf{n}\) [11]. We find that \(\nabla \times \mathbf{A} = \frac{g \mathbf{r}}{r^3}\), which is the magnetic field produced by a monopole at the origin (Eq. 1.1). Such a semi-infinite line of singular potential is known as a Dirac string, and the monopole at its terminus is known as a Dirac Monopole. The Dirac string contains the return magnetic flux into the monopole, and thus
maintains the validity of Maxwell’s equations.

Dirac’s formulation for the magnetic monopole has an important consequence - it establishes that the quantization of charge is contingent upon the existence of a magnetic monopole (and vice versa) [1]. As such, any insight into the nature of the Dirac monopole would have far-reaching implications throughout physics.

The concept of a Dirac monopole, while first proposed in the context of magnetic fields, now also appears in various other branches of physics. In particular, we can show that, for Bose-Einstein Condensates, there can exist a velocity profile that is analogous to the vector potential of the Dirac magnetic monopole [10]. To proceed with this, let us first review the basic properties of a Bose-Einstein condensate (BEC).

1.3 Bose-Einstein Condensates - an introduction

We can gain insight into the phenomenon of Bose-Einstein condensation in a dilute gaseous system by examining the thermal de Broglie wavelengths of the particles in the system:

\[ \lambda_T = \frac{h}{\sqrt{2 \pi m k_B T}}, \] (1.4)

where \( h \) is Planck’s constant, and \( m \) is the mass of the particles. Here, the de Broglie wavelength functions as a measure of the spatial extent of the wave functions of the individual particles. At high temperatures, the de Broglie
wavelength is much smaller than the interatomic separation in the system, and the system behaves classically. As the temperature decreases, however, the wavelength increases, and, at a critical temperature, when the wavelength is on the order of the interatomic separation, the wave functions overlap and the particles become indistinguishable, and Bose-Einstein condensation occurs.

It is also useful to look at the phase-space density, \( \varpi \), which is defined as the number of particles contained within the quantum mechanical volume of each particle, \( \lambda_T^3 \): \[ \varpi = n\lambda_T^3, \] (1.5)

where \( n \) is the number density of the particles. Here, \( \varpi \) is a measure of the overlap of wave functions of adjacent atoms. As \( \varpi \) becomes greater than 1, the wave functions overlap, and at \( \varpi > 2.612 \) (for a gas confined by rigid walls) [12], condensation occurs. In this condensate, all the overlapping atoms, over the extent of the condensate, are phase coherent [13], and can be described in terms of a single macroscopic wave function, which is the order parameter of the condensate.

For a weakly interacting Bose gas, the order parameter for a Bose-Einstein condensate is the macroscopic wave function, \( \Psi(\mathbf{r}, t) \), the time evolution of which is described by the Gross-Pitaevskii (GP) equation [12],

\[ i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + U_0 |\Psi(\mathbf{r}, t)|^2 \right) \Psi(\mathbf{r}, t) \] (1.6)

where \( m \) is the mass of each atom, \( V \) is the confining potential, and \( U_0 \) is a non-linear pseudopotential which describes the effective interatomic in-
action. Here, we can see that the Gross-Pitaevski equation is analogous to the Schrödinger equation for the wave function of a single particle, with an additional non-linear term in the Hamiltonian.

1.3.1 Quantization of circulation

In order to gain insight into the dynamics of the condensate, it is instructive to reformulate the order parameter $\Psi$ in terms of its amplitude, $f$, and its phase, $\phi$, as

$$ \Psi = f e^{i\phi}. \quad (1.7) $$

Then, rewriting Eq. 1.6 as a pair of hydrodynamic equations, we can calculate the superfluid velocity of the condensate [12],

$$ \mathbf{v} = \frac{\hbar}{m} \nabla \phi. \quad (1.8) $$

The vorticity field of the condensate, $\Omega$, is then defined as

$$ \Omega = \nabla \times \mathbf{v}, \quad (1.9) $$

and is zero everywhere,

$$ \Omega = \nabla \times \mathbf{v} = \frac{\hbar}{m} \nabla \times (\nabla \phi) = 0. \quad (1.10) $$

This shows that the velocity field is irrotational, provided that $\phi$ is globally defined [12]. If $\phi$ has a singularity, however, a Bose-Einstein condensate can
exhibit rotation. For the general case, let us consider $\Delta \phi$ to be the change of phase around a closed contour, i.e.,

$$\Delta \phi = \oint \nabla \phi \cdot dl. \quad (1.11)$$

Then, after the change of phase $\Delta \phi$, our condensate order parameter becomes

$$\Psi = fe^{i(\phi + \Delta \phi)} = fe^{i\phi}e^{i\Delta \phi}. \quad (1.12)$$

However, $\Psi$ is a single valued function and cannot change in value after traveling around a closed contour. From Eqs. 1.7 and 1.12, we get

$$\Psi = fe^{i\phi} = fe^{i\phi}e^{i\Delta \phi} \Rightarrow e^{\Delta \phi} = 1,$$

which is only true if $\Delta \phi$ is an integer multiple of $2\pi$,

$$\Delta \phi = \oint \nabla \phi \cdot dl = 2\pi l \quad (1.13)$$

where $l$ is an integer. Now, the circulation $\Gamma$ around a closed contour is given by

$$\Gamma = \oint \mathbf{v} \cdot dl. \quad (1.14)$$

From Eqs. 1.8 and 1.13, we get,

$$\Gamma = \frac{2\pi \hbar}{m} l = \frac{\hbar}{m} l \quad (1.15)$$
Thus, the circulation $\Gamma$ is quantized in units of $\hbar/m$.

For a purely azimuthal flow in a trap invariant under rotation about the $z$-axis, the structure of the flow pattern around a singularity in the phase is that of a vortex line. In addition, in a uniform condensate, it is energetically favorable for a vortex line with multiple quanta of circulation to split up into multiple vortices, each with a single quantum of circulation. A detailed derivation of these results can be found in the book by Pethick and Smith [12].

1.4 Spinor Condensates

In our formalism so far, we have assumed that the atoms in the Bose-Einstein condensate do not possess a spin degree of freedom. The trapping potential in an optical trap is independent of the magnetic state of the atom, thus introducing a spin degree of freedom, which appears in spinor condensates [14, 15].

We shall concern ourselves here with the magnetic sublevels ($m_F = 0, \pm 1$) of the $F = 1$ (Eq. 1.20) hyperfine manifold in $^{87}$Rb.$^2$

The order parameter, $\Psi$ of a spinor condensate is characterized by the three components $\psi_1, \psi_0, \psi_{-1}$, each corresponding to each magnetic sublevel.

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_0 \\ \psi_{-1} \end{pmatrix}$$  \hspace{1cm} (1.16)

$^2$A detailed picture of the hyperfine manifold and Zeeman splitting in $^{87}$Rb is presented in Chapter 2.
In the Gross-Pitaevski approximation [12], we can write the condensate order parameter as

$$\Psi = \sqrt{n(r)} \zeta(r) \quad (1.17)$$

where $n(r) = \sum_i |\psi_i(r)|^2$ is the total density of the particles in all hyperfine states, and $\zeta(r)$ is a three-component spinor,

$$\zeta(r) = \begin{pmatrix} \zeta_1(r) \\ \zeta_0(r) \\ \zeta_{-1}(r) \end{pmatrix}, \quad (1.18)$$

normalized by the relation $\zeta(r)\zeta^\dagger(r) = 1$.

The superfluid velocity of the condensate is then defined as

$$\langle v_s \rangle = \frac{1}{m} \langle \zeta | -i\hbar \nabla | \zeta \rangle = \frac{-i\hbar}{m} \zeta^\dagger \nabla \zeta. \quad (1.19)$$

where $(\nabla \zeta)_i = \nabla \zeta_i$. Eq. 1.19 reduces to Eq. 1.8 for the case of a one-component condensate ($\zeta = e^{i\phi}$).

The spin texture of the condensate, defined as the expectation value of the spin of the condensate, is then given by

$$\langle F \rangle = \zeta^\dagger F \zeta. \quad (1.20)$$

where $F = (F_x, F_y, F_z)$ is the vector formed by the spin 1 matrices in the basis
defined along the $z$-axis. Here,

\[
F_x = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix}
\]

\[
F_y = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & -i & 0 \\
i & 0 & -i \\
0 & i & 0
\end{pmatrix}
\]

\[
F_z = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1
\end{pmatrix}
\]

(1.21)

The spin texture $\langle F \rangle$ gives the direction the spin points at at any given point in the condensate.

For a spin-1 condensate, all the degenerate but physically distinguishable spinors are related by a gauge (phase) transformation $e^{i\phi}$ and spatial spin rotations $e^{-iF_x \alpha} e^{-iF_y \beta} e^{-iF_z \tau}$, where $(\alpha, \beta, \tau)$ are Euler angles [7, 10]. We can write the global ground state spinor $\zeta$ as

\[
\zeta = e^{i\phi} e^{-iF_x \alpha} e^{-iF_y \beta} e^{-iF_z \tau} \begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}
\]

(1.22)

where $\phi$ denotes the macroscopic condensate phase, and $(1, 0, 0)^T$ is the ground state eigenspinor for a magnetic field applied along $\hat{z}$. We exponentiate the
matrices using a Taylor expansion to get

\[
\begin{align*}
\begin{pmatrix}
  e^{-i\alpha} & 0 & 0 \\
  0 & 0 & 0 \\
  0 & 0 & e^{i\alpha}
\end{pmatrix}
\end{align*}
\]

(1.23)

\[
\begin{align*}
\begin{pmatrix}
  \frac{1}{2}(1 + \cos \beta) & \frac{1}{\sqrt{2}} \sin \beta & \frac{1}{2}(1 - \cos \beta) \\
  \frac{1}{\sqrt{2}}(\sin \beta) & \cos \beta & -\frac{1}{\sqrt{2}} \sin \beta \\
  \frac{1}{2}(1 - \cos \beta) & \frac{1}{\sqrt{2}} \sin \beta & \frac{1}{2}(1 + \cos \beta)
\end{pmatrix}
\end{align*}
\]

(1.24)

\[
\begin{align*}
\begin{pmatrix}
  e^{-i\tau} & 0 & 0 \\
  0 & 0 & 0 \\
  0 & 0 & e^{i\tau}
\end{pmatrix}
\end{align*}
\]

(1.25)

Eq. 1.22 now resolves to the form

\[
\zeta = e^{i(\phi - \tau)} \begin{pmatrix}
  e^{-i\alpha} \cos^2(\beta/2) \\
  \sin \beta/\sqrt{2} \\
  e^{i\alpha} \sin^2(\beta/2)
\end{pmatrix}
\]

(1.26)

The combination of \(\phi - \tau\) in Eq. 1.26 for the macroscopic condensate phase shows an equivalence between phase change and spin rotation, a spin-gauge symmetry \[7\] — a macroscopic phase change can be undone by the spin rotation \(\tau\) of the same magnitude. This symmetry indicates that all distinct configurations of \(\zeta\) (including the macroscopic phase) are determined by rotations about the Euler angles \[7\]. We can therefore replace \(\phi - \tau\) in Eq. 1.26 with
the Euler angle $\tau'$. We can now work out the spin texture of the condensate (Eq. 1.20) to be

$$\langle F \rangle = \cos \beta \hat{z} + \sin \beta \cos \alpha \hat{x} + \sin \beta \sin \alpha \hat{y}$$

(1.27)

### 1.5 Creation of a Dirac Monopole

To create a Dirac monopole in the vorticity field of the spinor condensate, we apply an external magnetic field which is a combination of a quadrupole field and a bias field,

$$\mathbf{B}(r, t) = B_1(x\hat{x} + y\hat{y}) + B_2\hat{z} + B_0(t)\hat{z},$$

(1.28)

where, since $\nabla \cdot \mathbf{B} = 0$, we get $2B_1 + B_2 = 0$ for the quadrupole field. $B_0(t)$ is a uniform bias field applied along the $\hat{z}$ direction. For simplicity’s sake, let us define new coordinates given by $(x', y', z') = (x, y, 2z - B_0(t)/B_1)$ [3]. We can see that $(x', y', z') = (0, 0, 0)$ when the field $\mathbf{B} = 0$. Now, assuming $B_1 < 0$, $B_0 > 0$, the unit vector along the direction of the magnetic field is,

$$\mathbf{b} = \frac{1}{\sqrt{x'^2 + y'^2 + z'^2}} (-x'\hat{x}' - y'\hat{y}' + z'\hat{z}')$$

(1.29)

$$= -\cos \phi' \sin \theta' \hat{x}' - \sin \phi' \sin \theta' \hat{y}' + \cos \theta' \hat{z'},$$

where $(r', \theta', \phi')$ refer to the spherical coordinates in our new coordinate system.

In the weak field limit, the condensate order parameter is an eigenstate of
the linear Zeeman operator $g_F \mu_B |B| \hat{B} \cdot \mathbf{F}$. For a ferromagnetic $^{87}\text{Rb}$ condensate in the global ground state, $g_F < 0$, and the local Zeeman energy is minimized for $B_1 < 0$, $B_0 > 0$ [12]. Then, the spins are aligned with the applied magnetic field. From Eqs. 1.27 and 1.29, we find the values for the Euler angles, $\beta = \theta'$ and $\alpha = \pi + \phi'$. For simplicity, let us assume that $\tau' = \alpha$. This assumption leads to no loss in generality, as will be shown presently. We now have

$$
\zeta = \begin{pmatrix}
\cos^2(\theta'/2) \\
e^{i(\pi+\phi')} \sin \theta'/\sqrt{2} \\
e^{2i\phi'} \sin^2(\theta'/2)
\end{pmatrix}
$$

(1.30)

where we have used the relation $e^{i(2\phi'+2\pi)} = e^{2i\phi'}$. Then, using $(\nabla \zeta)_i = \nabla \zeta_i$ in the spherical coordinates, we get for the superfluid velocity (Eq. 1.19),

$$
\langle \mathbf{v}_s \rangle = \frac{\hbar}{m} \frac{1 - \cos \theta'}{r' \sin \theta'} \hat{\phi}'.
$$

(1.31)

For the vorticity, $\langle \mathbf{\Omega} \rangle = \nabla \times \langle \mathbf{v}_s \rangle$ we get,

$$
\langle \mathbf{\Omega} \rangle = \frac{\hbar}{m r'^2} \hat{r}'.
$$

(1.32)

For $\tau' \neq \alpha$, when we follow the preceding steps, we calculate $\langle \mathbf{v}_s \rangle = -\frac{\cot \theta'}{r'} \hat{\phi}'$, which also gives us the vorticity $\langle \mathbf{\Omega} \rangle = \frac{\hbar}{m r'^2} \hat{r}'$.

We can draw an explicit analogy between this vorticity field (Eq. 1.32) and the magnetic field produced due to a magnetic monopole (Eq. 1.1). The superfluid velocity of this condensate plays the role of the vector potential of the magnetic field.
1.6 The Dirac String

In the preceding section, we have shown that we can create a monopole in the vorticity field of a spinor condensate by the application of the magnetic field $\mathbf{B}$. However, we now have a contradiction between Eqs. 1.10, and 1.32.

In this section, we shall demonstrate that, for the superfluid velocity in Eq. 1.31, the monopole has a Dirac string emanating in the direction $-\mathbf{\hat{z}}$, and that the Dirac string takes the form of a doubly quantized vortex. We shall follow the method presented by Volovik and Mineev in [5].

Eq. 1.31 gives the superfluid velocity

$$\langle \mathbf{v}_s \rangle = \frac{\hbar}{m} \frac{1 - \cos \theta'}{r' \sin \theta'} \mathbf{\hat{\phi}}'. \quad (1.31)$$
The circulation in the spinor condensate for the superfluid velocity $\langle v_s \rangle$ is given by

$$\langle \Gamma \rangle = \oint \langle v \rangle \cdot dl$$  \hspace{1cm} (1.33)

Let us now assume that $a = \sqrt{x'^2 + y'^2}$. Then, $r' = \sqrt{a^2 + z'^2}$, $\cos \theta' = \frac{z'}{\sqrt{z'^2 + a^2}}$ and $\sin \theta' = \frac{a}{\sqrt{z'^2 + a^2}}$. Additionally, $dl = a\, d\phi' \, \hat{r}'$. Then, our equation for the circulation simplifies to

$$\langle \Gamma \rangle = \frac{\hbar}{m} \oint \frac{\sqrt{a^2 + z'^2} - z}{a\sqrt{a^2 + z'^2}} \, a\, d\phi'$$

$$= \frac{\hbar}{m} 2\pi \left( 1 - \frac{z'}{\sqrt{z'^2 + a^2}} \right)$$  \hspace{1cm} (1.34)

Now, if we take the limit as $a \to 0$, we see that $\langle \Gamma \rangle = 0$ for $z' > 0$ and $\langle \Gamma \rangle = 4\pi \hbar/m$ for $z' < 0$, i.e., there is a doubly quantized vortex line along the
semiaxis $z' < 0$.

Equivalently, we can also use a geometric argument to demonstrate that the Dirac string takes the form of a doubly quantized vortex [9, 16]. For this argument, we revert to the coordinate system $(x, y, z)$, expressed in terms of the cylindrical coordinates $(r, \phi, z)$.

Consider an atom at the position $(r, \phi, z)$. Let us begin by looking at an atom at $z = 0$ and $\phi = 0$, with its spin pointing along the direction of the magnetic field, $\hat{b}$. The magnetic field experienced by the atom is given by (Eq. 1.28), which we reproduce here:

$$B(r, t) = B_1(x\hat{x} + y\hat{y}) + B_2z\hat{z} + B_0(t)\hat{z}, \quad (1.28)$$

Note that $B_1 < 0$ and $B_0(0) > 0$. Now, if $B_0 \gg B_1 x$, $\hat{b} \approx \hat{z}$. For $B_0 = 0$, $\hat{b} = \hat{r}$; and, for $B_0 \ll B_1 x$, $\hat{b} \approx -\hat{z}$. This indicates a rotation about the axis $\hat{n}(\phi) = \sin \phi \hat{x} - \cos \phi \hat{y}$. \quad (1.35)

To verify that $\hat{n}$ is the axis of rotation, we apply the right hand rule for rotations (Fig. 1.3). The axis of rotation is the same for all atoms, irrespective of $z$.

Now, let us assume that an atom at $(r, \phi, z)$ is rotated about $\hat{n}$ by an angle $\beta$. The operator for this rotation about $\beta$ is $e^{i\beta \hat{n}}$, where $\mathbf{F}$ is as defined in
Figure 1.3: Geometry of the field applied to create a Dirac monopole at $z = 0$. The atomic spins rotate about the axis $\hat{n}$.

Eq. 1.21. Then, we get

$$F \cdot \hat{n} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -ie^{-i\phi} & 0 \\ e^{i\phi} & 0 & -ie^{-i\phi} \\ 0 & e^{i\phi} & 0 \end{pmatrix} \quad (1.36)$$

Using a Taylor expansion for the rotation operator, we get

$$e^{-iF \cdot \hat{n}\beta} = \begin{pmatrix} \frac{1}{2}(1 + \cos \beta) & -\frac{1}{\sqrt{2}}e^{-i\phi} \sin \beta & \frac{1}{2}e^{-2i\phi}(1 - \cos \beta) \\ \frac{1}{\sqrt{2}}e^{i\phi}(\sin \beta) & \cos \beta & -\frac{1}{\sqrt{2}}e^{-i\phi} \sin \beta \\ \frac{1}{2}e^{2i\phi}(1 - \cos \beta) & \frac{1}{\sqrt{2}}e^{i\phi} \sin \beta & \frac{1}{2}(1 + \cos \beta) \end{pmatrix}. \quad (1.37)$$

Now, let us take the limit as $r \to 0$. In this case, for a transition from
$B_0 > B_2z$ to $B_0 < B_2z$, we get $\beta \to \pi$. Eq. 1.37 simplifies to

$$e^{-iF_y\beta} = \begin{pmatrix} 0 & 0 & e^{-2i\phi} \\ 0 & -1 & 0 \\ e^{-2i\phi} & 0 & 0 \end{pmatrix}, \quad (1.38)$$

which, when it acts on $|1, 1\rangle$, gives,

$$e^{-iF_y\beta} |1, 1\rangle = \begin{pmatrix} 0 & 0 & e^{-2i\phi} \\ 0 & -1 & 0 \\ e^{-2i\phi} & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ e^{2i\phi} \end{pmatrix} = e^{2i\phi} |1, -1\rangle \quad (1.39)$$

We see a macroscopic phase shift of $2\phi$, which gives a vortex with winding number 2, i.e., a doubly quantized vortex. We also see that the atoms now transition to the $|1, -1\rangle$ state. For an interpretation of this result in terms of the Berry’s phase of the atoms, we refer the reader to [9].
Chapter 2

Apparatus

This chapter provides an overview of our BEC apparatus, and briefly discusses the experimental techniques that we use to produce condensates. We begin with an introduction to $^{87}\text{Rb}$, proceed with a discussion of the various elements of our apparatus, and conclude with the additions to the apparatus in the past year. For more detail on these various components of our experiment, refer to [12], and to previous theses [17–22].

2.1 An introduction to $^{87}\text{Rb}$

Our atom of choice, $^{87}\text{Rb}$, is an alkali metal with 37 protons, 37 electrons, and 50 neutrons. In its nucleus, $^{87}\text{Rb}$ has one unpaired proton, which, in the ground state, occupies the $2p_{3/2}$ nuclear orbital [23], and has spin $3/2$. Similarly, $^{87}\text{Rb}$ has one unpaired electron, which, in the ground state, occupies the $s$-orbital, and has total electronic spin $1/2$. Since the sum of electronic and nuclear spins of the atom is an integer ($3/2 \pm 1/2$), $^{87}\text{Rb}$ is a composite
boson.

### 2.1.1 Hyperfine splitting in $^{87}$Rb

In $^{87}$Rb, the nuclear spin is coupled to the electronic angular momentum via a hyperfine interaction — the magnetic dipole created by the electronic angular momentum interacts with the magnetic field created by the magnetic dipole due to the nuclear spin. The good quantum number for this interaction is the quantum number for the total angular momentum, $F$. For the $^{87}$Rb ground state $^52S_{1/2}$, the nuclear spin is given by $I = 3/2$, the orbital angular momentum is given by $L = 0$, and the electronic spin is given by $S = 1/2$. This gives us the total angular momentum $F = I + J$, where $J = L + S$, such that $F \in \{1, 2\}$. Similarly, for the excited state $^52P_{3/2}$, using the usual primed notation for excited states, we get $F \in \{0', 1', 2', 3'\}$. The transitions between the $^52S_{1/2}$ ground states and $^52P_{3/2}$ excited states are called D2 transitions. Our experiment makes use of three important D2 transitions for our cooling, trapping and imaging techniques - the cycling, optical pumping, and repump transitions. We illustrate these transitions in Fig. 2.1.

For the $^52S_{1/2}$ manifold, the hyperfine energy shift is given by the relation [24],

$$
\Delta E_{hfs} = \frac{1}{2} A_{hfs} (F(F + 1) - I(I + 1) - J(J + 1)) = A_{hfs}(-1/4 \pm 1), \quad (2.1)
$$

where $A_{hfs}$ is the magnetic dipole constant, experimentally determined to be $h \cdot 3.417$ in terms of the Planck’s constant $h$. 

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Figure 2.1: Hyperfine structure of the D2 transition between the ground state, $5^2S_{1/2}$, and the first excited state, $5^2P_{3/2}$, with the frequency splittings as indicated [24]. The transitions used in our experiment are labelled. Energy levels are not drawn to scale.

2.1.2 The Zeeman effect

The application of an external magnetic field serves to further lift the degeneracy in the hyperfine manifolds. Here, the magnetic moment associated with the total angular momentum of the atom interacts with the external magnetic field, producing an energy splitting that corresponds to the alignment of the magnetic moment with the external field. Through this process of Zeeman splitting, we can observe $m_F \in \{-F, -F+1, ..., F\}$ magnetic sublevels in each state with total angular momentum $F$. We use the notation $|F, m_F\rangle$ to refer to an atom in the $m_F$ magnetic sublevel in the $F$ hyperfine manifold.

By Earnshaw’s theorem, in a current-free space, the magnitude of a magnetic field cannot have a maximum. We can therefore only trap atoms in
those Zeeman states which are attracted towards a magnetic minimum. In $^{87}\text{Rb}$, the states $|1, -1\rangle$, $|2, 2\rangle$, and $|2, 1\rangle$ satisfy this requirement, and hence can be magnetically trapped to create a condensate [12].

To calculate the precise energy shift between the various Zeeman levels in the $5^2S_{1/2}$ ground manifold for an applied field $B$ along the $z$-direction, we use the Breit-Rabi formula [24],

$$E_{BR} = -\frac{\Delta E_{hfs}}{2(2I + 1)} + g_I \mu_B m_F B + (-1)^F \frac{\Delta E_{hfs}}{2} \sqrt{1 + \frac{4}{2I + 1} m_F x + x^2} \quad (2.2)$$

with

$$x = \frac{(g_J - g_I) \mu_B B}{\Delta E_{hfs}},$$

and where $\mu_B$ is the Bohr magneton, $\Delta E_{hfs}$ is the hyperfine energy shift (Eq. 2.1) and $g_J$ and $g_I$ and the electronic and nuclear Landé factor respectively. For weak magnetic fields, we can approximate Eq. 2.2 to its lowest order to find that the Zeeman levels split linearly according to

$$\Delta E = \mu_B g_F m_F B_z \quad (2.3)$$

### 2.1.3 Landau-Zener transitions

Our attempt to create a Dirac monopole uses the $|1, 1\rangle$ state. However, the $|1, 1\rangle$ state is not magnetically trappable (Fig. 2.2). As a solution, we first create a condensate in the magnetically trappable $|2, 2\rangle$ state, and then transfer the condensate to the $|1, 1\rangle$ state.
Figure 2.2: Zeeman splitting of the $F = 1$ and $F = 2$ hyperfine manifolds of $^{87}\text{Rb}$. Blue indicates magnetically trappable states. Splitting between the hyperfine states is not to scale.

To understand how this transfer works, let us first imagine a two-level system with states $|1\rangle$ and $|2\rangle$, with the energy of the states dependent linearly a common parameter $q$ and the states degenerate at $q = q_c$ (Fig. 2.3). Then, we can write the Hamiltonian of the system as $H_0(q)$.

Let us now introduce a perturbation $H_1$, which is independent of $q$, which couples the levels together. This disturbs the degeneracy of the system, and gives us a new Hamiltonian $H = H_0 + H_1$, with the new eigenstates $|a\rangle$ and $|b\rangle$ (Fig. 2.3). The energy splitting between the states $|a\rangle$ and $|b\rangle$ is non-zero at $q_c$, and thus there is no degeneracy. Further, we find that at $q \ll q_c$, $|a\rangle \sim |1\rangle$. 

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and at $q \gg q_c$, $|a\rangle \sim |2\rangle$, and vice versa for $|b\rangle$. Now, if we adiabatically ramp $q$ through $q_c$, from a value $q \ll q_c$ to $q \gg q_c$, we force the atoms to transition from state $|1\rangle$ to $|2\rangle$ [17, 25]. This transition is known as a Landau-Zener transition [26, 27].

![Diagram](https://via.placeholder.com/150)

**Figure 2.3:** Atoms in $|a\rangle$ ($|b\rangle$) transition from $|1\rangle$ ($|2\rangle$) to $|2\rangle$ ($|1\rangle$) as we adiabatically ramp $q$ through $q_c$. Energy splitting between the states $|a\rangle$ and $|b\rangle$ at the avoided crossing, $q_c$, is $E_0$.

We can transfer atoms from the $|2, 2\rangle$ state to the $|1, 1\rangle$ state by coupling the two states using a microwave driving field. To accomplish this, we set a magnetic field $B_c$, and then calculate the energy difference between these two states using the Breit-Rabi equation (Eq. 2.2) for this magnetic field.
We then apply a microwave driving field with its magnetic component along the vertical direction, with the frequency of the microwave resonant with the $|2, 2\rangle \rightarrow |1, 1\rangle$ transition. We transfer atoms to the $|1, 1\rangle$ state when we then apply a uniform magnetic field in the North-South direction, and adiabatically ramp the field through $B_c$.

In our experiment, we apply a microwave driving field of frequency 6843.008 MHz, and ramp down our magnetic field (along the N-S direction) from 4.3 G to 3.6 G in 25 ms. With this procedure, we get a transfer efficiency of $\approx 100\%$.

### 2.2 Experimental Details

We use a dual magneto-optical trap system [19, 22, 28, 29], and magnetic trapping, with a time-orbiting potential (TOP) trap, together with evaporative cooling to produce Bose-Einstein condensates [19, 28, 30]. We then transfer the atoms to a purely optical trap [15, 31], where we can experiment with a $|1, 1\rangle$ condensate. Finally, we release the atoms from the optical trap, and image the atoms, and derive our data using these images. A schematic of our primary apparatus is presented in Fig. 2.4.

**Dual Magneto-Optical Trap**

We initiate our experiment by passing a current through a getter, a sample containing $^{87}$Rb, thus sublimating $^{87}$Rb atoms, and producing a low background vapor pressure of $^{87}$Rb. We then use a combination of magnetic fields and laser light to capture these gaseous atoms in a magneto-optical trap [12, 32, 33],
which we refer to as the Collection MOT (see Fig. 2.4). Our collection MOT utilizes a pair of anti-Helmholtz coils in combination with 3 orthogonal pairs of counter-propagating laser beams (MOT beams). The anti-Helmholtz coils produce a quadrupole magnetic field gradient. Since atomic energy levels depend on this field gradient, the radiation pressure due the lasers depends on the position of the atoms. This produces an effective force towards the center of the trap, spatially confining and cooling the atoms [12, 22, 32].

We then transfer them towards our science MOT by applying a net force on the atoms through brief pulses of the cycling laser directed towards the science MOT. Upon collecting the requisite number of atoms, we drive the atoms to the $|2, 2\rangle$ state (Fig. 2.2) and transfer them to the magnetic trap [19, 21].

**Magnetic Trapping**

In this stage of cooling, we use a pair of anti-Helmholtz coils, with their symmetry axis in the vertical direction, to create a quadrupole field gradient with its minimum at the center of the trap. We superimpose a rotating bias field in the horizontal plane on this quadrupole field gradient, and thus create a time-averaged orbiting potential (TOP) trap [19, 28, 30, 34].

**Evaporative cooling**

With the thermal atoms in the TOP trap, we evaporatively cool the atoms to produce a Bose-Einstein condensate. In this process, we apply a resonant radiofrequency (RF) field to selectively remove the highest energy atoms in the thermal cloud, thus reducing the average kinetic energy and the temperature.
of the system. This allows for Bose-Einstein condensation [19].

**Optical trapping**

After creating a $|2, 2\rangle$ condensate through evaporative cooling, we transfer atoms to the Far-Off Resonance Trap (FORT) [18, 35], henceforth referred to as the optical trap. The optical trap utilizes a 1064nm ND:YVO$_4$ laser to produce an ac electric field gradient. This field gradient induceds an atomic electric dipole; the dipole then interacts with the field gradient thus trapping the atoms [31, 36]. The trapping is independent of the internal state of the atoms, and can trap all atoms in the hyperfine ground state. The optical trap therefore enables us to freely manipulate the spin structure of our condensate. This allows for the creation of a Dirac monopole in the condensate.

**Coil control**

To manipulate the magnetic field experienced by the condensate, we change the current through the magnetic coils — three bias coils (North-South, Up-Down, and East-West), and a ‘quadrupole’ coil. Digital to Analog Converter (DAC) voltages control the currents through these coils through a feedback circuit that uses a sense resistor (for the bias coils) and a special Hall-effect sensor (for the quadrupole coil).\(^1\)

We shall go into more detail about the geometry and calibration of these coils in later chapters.

\(^1\)We use a NI PCI-6733 Digital to Analog Converter (DAC) in our apparatus to convert the programmed voltages into analog output. This DAC has a -10V range, and a 16-bit resolution.
Microwave synthesizer

Our apparatus employs two microwave sources, a HP8672A HP Synthesizer and a frequency doubled Agilent E4422B Synthesizer, selected with $1\mu$s time resolution by a set of computer-controlled switches. The signal from these sources is amplified by a Lockheed-Martin 20W amplifier, and then carried via a semi-rigid cable to a sawed-off wave-guide (Fig. 2.4), which couples the microwave to the atoms. We employ the microwave to transfer the atoms from $|2,2\rangle$ state to the $|1,1\rangle$ state using Landau-Zener transitions, and also for microwave spectroscopy to calibrate our magnetic fields.

Imaging

Our apparatus uses absorptive imaging to collect data about the condensate. For this process of imaging, we release the condensate from the magnetic or optical trap, and allow it to undergo ballistic expansion. While the condensate falls, we apply a field of 1G along the North-South direction, thus providing a quantization axis for light absorption for the imaging process. Optionally, we can also use the quadrupole coils to apply a field gradient along the vertical direction, using the Stern-Gerlach effect to separate the different Zeeman states of the condensate (Fig. 2.2) along the EW direction.

We image using a probe laser that is resonant with the $2 \rightarrow 3'$ cycling transition (Fig. 2.1). To image atoms in the $F = 1$ manifold, we first apply a brief pulse of repump light. The probe laser then drives the cycling transition, and the atoms absorb and rescatter photons. This casts a shadow onto a charge-coupled device (CCD) camera, which records and rasterizes the beam.
profile. The camera also takes two additional pictures, the ‘light’ frame with the probe light on, and the ‘dark’ frame with the probe light off. A computer then processes these images, extracting information about the column density distribution of the condensate, and numerically integrating this distribution to calculate the number of atoms in the condensate, which gives the data for our various experiments [21, 37].

We can shine the probe laser either vertically (to get the ‘top view’), or horizontally (to get the ‘side view’). In addition, we can also use extraction imaging to take ‘top view’ pictures of the condensate in real time, which is useful for studying real-time dynamics of vortices and any other interesting features in the condensate [38, 39].

2.2.1 Changes to the apparatus

The magnetic coils in our experiment suffer from background AC fields of 60Hz (powerline cycle). To ensure reproducibility in our various experiments, we added a synchronization circuit, hereafter referred to as the sync circuit, that synchronizes our experiment with a fixed point in the 60Hz cycle\(^2\) [40], and with a fixed point in the 2kHz cycle (to synchronize with the rotating field of the TOP trap). With the sync circuit in place, we stop everything, wait until the 2kHz and 60Hz signals agree to \(\sim 100\mu s\), and then start the experiment at a fixed, reproducible point.

\(^2\)The sync circuit was built by Nate Thomas, a junior at Amherst College, under the guidance of Prof. Hall.
Figure 2.4: A schematic of our apparatus on the primary optical table. Excludes the vertical MOT beams, and the magnetic coils in the collection MOT. Adapted from [18].
Figure 2.5: The atoms are trapped in the science MOT enclosed by the quadrupole and bias coils. We change the current through the coils to vary the magnetic field experienced by the condensate. Photograph by R. P. Anderson.
Chapter 3

Experimental Details and Issues

In our attempt to create a Dirac monopole, we follow Pietilä and Möttönen [3]. In this process, we manipulate a spinor condensate in an optical trap by using a quadrupole field adjusted with a bias field. The total field applied is given by,

\[ B(r, t) = B_1(x \hat{x} + y \hat{y}) + B_2 \hat{z} + B_0(t) \hat{z}, \]  

(1.28)

where \( B_1 \) and \( B_2 \) are static magnetic field gradients, with \( B_2 = -2B_1 \), and \( B_0(t) \) is the time-dependent bias field. We produce the static field gradients through quadrupole coils in an anti-Helmholtz configuration. The bias field is produced by a Helmholtz coil configuration in the vertical direction.

To replicate the simulation parameters used by Pietilä and Möttönen, we need fields of \( B_2 = 10 \) G/cm, and \( B_0(t = 0) = 10 \) mG. Since our quadrupole field gradient for the programmed voltage, is 61.25 G/(Vcm) axially, we need to apply 0.1633 V in the quadrupole coils. Similarly, we need to apply \( V_1 = 1.0126 \) V in the Up-Down (UD) coils (Table 4.1). Next, we need to ramp the
UD field linearly from $V_1$ to $-V_1$ in 100ms, while the North-South(NS) and East-West(EW) fields are set to null the background DC field.

3.1 Expectations

In our apparatus, the optical trap is situated at the center of the quadrupole coil configuration. Thus, when we ramp the bias field to zero, we bring the zero of the quadrupole field into the condensate. The zero of the quadrupole field then becomes the center of the spin defect, the vorticity monopole. The associated Dirac string is a doubly quantized vortex filament extending outwards from the monopole, in a direction opposite to the travel of the monopole. As we ramp the bias field to negative values, we move the monopole through the condensate, and beyond. We now have a doubly quantized vortex running through the condensate. Since a doubly quantized vortex is energetically unstable, we expect it to decompose into two singly quantized vortices [12]. We can apply our top imaging technique to observe different stages of this dynamical process — we can observe the decay of a single vortex into two vortices, providing evidence that it was doubly quantized to start with.

3.2 Results

In multiple runs of our experiment for various values of the ramp time and applied field, we were unable to observe any vortices in the condensate. After some rigorous exploration, we discovered several problems that hampered our attempts to create a Dirac monopole — we discovered various issues with the
apparatus as well as our experimental technique. In the following section, we shall attempt to chronicle some of these issues.

### 3.3 Experimental Issues

Our attempt to create a Dirac monopole relies on a high degree of accuracy, precision, and stability in the magnetic fields created in our apparatus. Any fluctuation in the fields produced, along with any inaccuracy in the calibrations for the various coils, can prevent the formation of a monopole.

The magnetic coils in our apparatus are instruments with a linear response
- the field produced by the coils is directly proportion to the voltage applied.

For the magnetic coils, the applied field is determined by three factors in particular — the scale factor, the offset, and the geometry of the coils.

**Scale Factors**

The scale factors in our apparatus are the slopes of the calibration curves for our magnetic coils. We need a high level of accuracy in the scale factor to ensure that the atoms experience the intended magnetic field.

The slopes of the calibration curves of our magnetic coils depends solely on the coil geometry, and is thus stable for long periods of time. To get an accurate value for the scale factor, we only need to recalibrate our magnetic coils every once in a while. We perform this recalibration through microwave spectroscopy measurements.

**Offset**

The offset in the magnetic coils is due to the presence of a background DC field. If the background DC field is not correctly nulled, the position of the zero of the quadrupole field changes. Since our condensate, fixed in the optical trap, is very small (~ 30µm radially), even a small deviation (~ 15 mG for \(B_2 = 10\) G/cm) in the zero of the field could mean that the zero of the quadrupole field misses the condensate entirely.

In our experiment, we attempt to null the background DC field by measuring the components of this field along the UD, EW, and NS directions, and applying *nulling* fields (offsets) in the NS and EW direction, and by calibrating...
ing an offset in the UD direction. In the past, we have measured these offsets through microwave spectroscopy, which is a time-consuming and cumbersome process, and cannot be repeated day-to-day. The background DC field, however, shifts day-to-day, and our initial calibrations soon become inadequate. As a solution, we develop a new kind of measurement of these offsets — we use Majorana transitions of the condensate atoms themselves to make an \textit{in situ} measurement of the background DC field (Section 4.2).

\textbf{Coil geometry}

The magnetic fields produced by our coils is dependent on the geometry of the coils — we get a uniform bias field from each pair of bias coils only if the coils share a common axis, and are identically circular, and also symmetrical about the experimental area. The production of a symmetric quadrupole field is also contingent on the same conditions. Deviations from this geometry can lead to unwanted gradients in the field produced by the magnetic coils.

<table>
<thead>
<tr>
<th></th>
<th>UD Coils</th>
<th>EW Coils</th>
<th>NS Coils</th>
<th>Quadrupole Coils</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (m)</td>
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<td>0.0635</td>
<td>0.0762</td>
<td>Special</td>
</tr>
<tr>
<td>Turns</td>
<td>12</td>
<td>33</td>
<td>33</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 3.1: Geometry of the magnetic coils. The windings for the quadrupole coils are special windings.

\section*{3.4 Noise}

The magnetic coils suffer from the presence of background AC fields, hereafter referred to as \textit{noise}. Since our experiment is not synchronized with this noise,
the effect of this noise shifts from shot-to-shot. In particular, noise with low frequencies and large amplitudes dynamically moves the zero of the quadrupole field around during our experiment, making the creation of a Dirac monopole impossible.

In addition, noise at low (and intermediate) frequencies, also drives transitions between the various states of our hyperfine manifold, and uncontrollably the spin texture of our condensate. This is due to the fact that for low values of the applied field, the Larmor frequency of the spins in the magnetic field is correspondingly low, and becomes comparable to AC noise at this range. The Larmor frequency of a spin in a magnetic field is given by

$$\omega_l = -\frac{eg}{2m}B, \quad (3.1)$$

where $\omega_l$ is the Larmor frequency, $e$ is the charge of an electron, $m$ is the mass of an electron, $g = 2$ is the Landé factor, and $B$ is the magnitude of the applied field. Using this relation, we find that, for instance, the presence of 60 Hz noise (due to the power line cycle) becomes a resonant issue for an applied field of $\sim 8.6 \mu G$. This small value of the applied field for the 60 Hz noise indicates that, for noise at low frequencies, transitions between the hyperfine states is not a major issue; the dynamic instability in the zero of the quadrupole field, however, is a major problem at these frequencies.

From spectroscopic measurements for our quadrupole coils for below 100kHz, we see that we suffer from the harmonics of the line noise, harmonics of 2kHz, harmonics of 10kHz, and an assortment of other frequencies. Noise at
frequencies > 100kHz is not of particular interest to us, since, for the dimensions of our condensate and for an applied field of 10 G/cm axially (i.e., 5G/cm radially), we do not have Larmor frequencies with such large frequencies. If our condensate has a radial extent of 30µm, and we find that the magnitude of the field at the edge of the condensate is 7.5mG, which corresponds to a Larmor frequency of 5.25 kHz.

We need to accurately calibrate the magnetic fields and mitigate the noise to successfully create a Dirac monopole.
Chapter 4

Calibrating the Magnetic Coils

In order to create a Dirac monopole in our spinor condensate, we accurately calibrate our various magnetic coils. We begin by chronicling our methods to calibrate the various bias coils.

4.1 Calibrating the bias coils

We calibrate the bias coils individually by employing the process of microwave spectroscopy. In this process we apply a magnetic field along one of the axes, and then apply a microwave driving field (with its magnetic field parallel to the vertical axis), in order to drive transitions from the $|1, 1\rangle$ state to a spin state in the $F = 2$ manifold.

For a field $B$ oriented along the vertical axis, we can drive transitions from the $|1, 1\rangle$ state to the $|2, 1\rangle$ state by applying a microwave driving field resonant with this transition. The frequency required for this transition can be calculated using the Breit-Rabi equation (Eq. 2.2), which gives us the difference in
energy between the various energy states in the $5^2S_{1/2}$ ground manifold.

For a field pointing along one of the horizontal axes, we drive transitions from the $|1, 1\rangle$ state to the $|2, 2\rangle$ state.

### 4.1.1 Experimental Technique

To calibrate the UD coil, we set the voltage applied to some value $V_z$, and calculate an approximate field value ($B_z$) using our old calibration. We then set $V_x$ and $V_y$ to approximately their nulling values. We calculate the microwave driving frequency for the field $B_z$, and then apply a microwave pulse at $-10\text{ dBm}$ power for 1 ms.\(^1\) We use image the condensate using the Stern-Gerlach technique, and note the number of atoms transferred to the $|2, 1\rangle$ state. We repeat this procedure several times while varying the frequency of the driving field. This gives us a resonance curve for $V_z$. We then fit the resonance curve to a Lorentzian, and obtain the microwave frequency at the center of the curve.\(^2\) Inverting the Breit-Rabi equation (Eq. 2.2) for this microwave frequency, we can get the actual magnetic field produced by the applied voltage.

We repeat this experiment for several values of $V_z$, and perform a linear fit to find the calibration for the UD coils. To calibrate the EW and NS coils, we use a microwave driving frequency resonant with the $|1, 1\rangle \rightarrow |2, 2\rangle$ transition, to perform the above-described experiment. Sample resonance curves for the microwave transition are presented in Figs. 4.2, 4.3, and 4.4. We thus obtain

\(^1\)For our microwave spectroscopy measurements, we quote microwave powers in dBm. These values are the programmed powers for the various synthesizers, and are not indicative of the power applied to the atoms.

\(^2\)In this thesis, we use Lorentzian fits for a variety of different measurements because the Lorentzian fits give us a reasonable center and width. However, these fits are not meant to be definitive, and should only be used as an approximation.
Figure 4.1: Atoms transition from $|1, 1\rangle$ to $|2, 1\rangle$ state after a test microwave pulse of 3.0dBm is applied for 1ms with $V_z = 1.250V$ and microwave frequency 6834.858 MHz.

Figure 4.2: Resonance curve for an applied voltage $V_z = 1.250 V$ with a microwave pulse of -10.0dBm applied for 1ms. Center of the resonance curve is at 0.12427(2)G.

the calibration results presented in Figs. 4.5, 4.6, 4.7. Since the geometry of the coils is presumably stable for long periods of time, the corresponding slopes
in the calibration curves should also remain stable and need not be measured day-to-day.

### 4.2 Background DC fields

We have noted earlier that the offset error in our magnetic coils is due to the presence of background DC fields. Since the DC fields vary day-to-day, time-consuming microwave spectroscopic measurements are not really an option for everyday measurements of the offset. As a solution, we here present a protocol in which we use the condensate atoms as *in situ* magnetometers, using Majorana transitions of the atoms themselves to characterize the background field.
4.2.1 Theory of Majorana Transitions

When the direction of an applied field is changed faster than the atoms can follow adiabatically, the atoms transition to other spin states. These transitions are known as Majorana transitions, after Ettore Majorana [41] who first derived the expression for such a transition in a two-level system:

\[ P_{1/2, -1/2} = \exp\left(-\frac{\omega_l}{\omega_r}\right), \tag{4.1} \]

where \( \omega_l \) is the Larmor rotation frequency given by \( \omega_l = 2\pi \gamma B \) with \( \gamma \) the gyromagnetic ratio and \( B \) the magnitude of the applied magnetic field. \( \omega_r \) is the rotation frequency of the magnetic field, and \( P_{1/2, -1/2} \) is the probability of
transition from the magnetic substate $m = 1/2$ to $m = -1/2$ (or vice versa).

Eq. 4.1 has an important consequence — we can see that for a constant $\omega_r$, if we ramp the applied magnetic field through zero, all the atoms transition from one state to another. For a non-zero magnetic field, the number of atoms transferred increases when $\omega_r$ is increased.

For a multilevel system with total angular momentum $F$, Eq. 4.1 generalizes to [42]:

$$P_{m,m'} = (F + m)!(F + m')!(F - m)!(F - m')!(\sin \frac{\theta}{2})^{4F}$$
$$\times \left( \sum_{v=0}^{2F} \frac{(-1)^v (\cot \frac{\theta}{2})^{2v+m+m'}}{v!(v + m + m')!(F - m - v)!(F - m' - v)!} \right)^2 \quad (4.2)$$

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Figure 4.6: Linear fit of the magnetic field ($B_x$) produced by the EW coils as a function of the voltage applied $V_x$.

where $P_{m,m'}$ is the probability of transition from the magnetic substate $m$ to $m'$, and the value of $\theta$ is obtained from the relation $\sin^2 \left( \frac{\theta}{2} \right) = \frac{P_{1/2,-1/2}}{2}$. For the $F = 1$ manifold in $^{87}\text{Rb}$, the probabilities of transitions from the $|1, 1\rangle$ state to the $|1, -1\rangle$ and $|1, 0\rangle$ states are presented in Fig. 4.9 as a function of the Larmor frequency of the atoms. We can see from Fig. 4.9 that as we decrease the magnitude of the net applied field (and hence the magnitude of the Larmor frequency) to near-zero values, we get preferential transfer to the $|1, -1\rangle$ state. The number of atoms in the $|1, -1\rangle$ state hence gives us information about how close we are to zero-field — with all the atoms transferred to the $|1, -1\rangle$ state when we have zero-field. We use this information to our advantage when we null the DC background fields.
4.2.2 Experimental Technique

Our experimental technique involves the measurement and cancellation of background DC fields along each of the $x$-, $y$- and $z$- axes individually. To accomplish this, we first prepare a condensate in the $|2, 2\rangle$ state in the optical trap, and then use the Landau-Zener technique to transfer the atoms into the $|1, 1\rangle$ state. We then use Majorana transitions to measure and cancel the background DC fields along each of the three axes individually. We should note that our magnetic coils are set up to produce a field in a direction opposite the earth’s magnetic field, and thus, at their nulling values, the control voltages respect $V_x > 0$, $V_y > 0$, and $V_z > 0$. 

Figure 4.7: Linear fit of the magnetic field ($B_y$) produced by the NS coils as a function of the voltage applied $V_y$. 

Slope = 2.3552(1)G/V  
Offset = -0.1832(2)G
Figure 4.8: Spins precess around an external magnetic field with the Larmor frequency $\omega_l$, and adiabatically follow the change in the magnetic field.

To cancel the background DC fields along the $z$-axis, we first set the voltage applied in the EW Helmholtz coils to a high value, $V_x = 3.0$ V, producing a field of $3.0 \cdot 1.97 = 5.9$ G (Table 4.1) along the $\hat{x}$ direction. This preserves the spin polarization of the atoms against unintentional passage through zero field. We then fix the voltage in our NS coils to approximately the nulling value, obtained from microwave spectroscopy (or previous Majorana transition measurements). Then, we set the voltage in the UD coils to produce a test field $B_z$, which is approximately the nulling value. With this setup, we ramp the field produced by the EW coils, $B_x$, to 0.01 G in 200ms. Since the background DC field along the $x$-axis is greater than 0.01 G, the net field is now in the $-\hat{x}$ direction, and we have ramped through the zero of the x-field. We then
Figure 4.9: Probability of transition from $|1,1\rangle$ to the states in the $F=1$ manifold. For a fixed $\omega_r$, for high values of the Larmor frequency, atoms remain in the $|1,1\rangle$ state. As we lower the Larmor frequencies, atoms are preferentially transferred to the $|1,0\rangle$ state initially. Closer to zero, however, atoms are preferentially transferred to the $|1,1\rangle$ state.

image the condensate after separating the components using the Stern-Gerlach effect, and note the number of atoms in each state in the $F=1$ manifold.

We then repeat this step for various values of $V_z$ so that we get a resonance curve around the point of maximum transfer. The center of the resonance curve gives us the required value for the nulling field (Fig. 4.11). We get the same value when we ramp $V_y$ with $V_x$ fixed (to within $\sim 5$ mG).

Our choice of 200ms for the ramp time ensures our resonance curve remains narrow. If we decrease the ramp time, the effective value of $\omega_r$ increases, and we get much a much wider resonance curve, increasing the uncertainty in our measurement.

We perform similar experiments to obtain the nulling values for $V_x$ and $V_y$. 

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Figure 4.10: Majorana transitions for various applied values of $V_z$, with $V_y$ slightly off its nulling value. In (a), $V_z$ is off from its nulling value, and very few atoms are transferred to other spin states. In (b), $V_z$ is closer to its nulling value, and the majority of the transfer is to the $|1,0\rangle$ state. In (c), $V_z$ is very close to its nulling value, and almost all the atoms transferred to $|1,-1\rangle$.

In Fig. 4.12 and Fig. 4.13, we present sample resonance curves for $V_x$ and $V_y$ respectively.

Our data from microwave and Majorana spectroscopy provide us accurate calibrations for the scale factors and offsets for our bias coils. The resolution of noise, however, still remains a pertinent question. The AC noise in our system creates fluctuations in the current through our bias coils, thus preventing us from gaining precise control over the net magnetic field. In addition, since our
Figure 4.11: Resonance curve of the number of atoms in $|1, -1\rangle$ state as a function of the field applied in the z-direction. We need to apply .53174(5) G through the UD coils to cancel the background DC field along the z-axis.

Microwave spectroscopy experiments take place over a short time duration, they are particularly vulnerable to errors due to shot-to-shot fluctuations in the applied field. This creates some inaccuracy in the calibration itself. Our Majorana spectroscopy experiments, however, take place over long time durations, and any fluctuations due to noise are averaged. Our values for the offset are thus consistent from shot-to-shot.
Figure 4.12: Resonance curve of the number of atoms in $|1, -1\rangle$ state as a function of the field applied in the y-direction. We need to apply 0.1822(1) G through the NS coils to cancel the background DC field along the y-axis.

### 4.3 Introduction of 60Hz Synchronization

Our bias fields are set up to only produce magnetic fields of magnitude less than 15 G. Consequently, the errors due to AC noise are quite small. Our quadrupole coil, however, can be set up to work in the Helmholtz mode to produce bias fields of up to $\sim$1000 G. At these large field values, our experience is that AC noise becomes amplified. While this provides us an opportunity to better characterize the noise, it also poses a problem for any attempts at calibrating the coil in the Helmholtz mode. Henceforth, we shall work with the quadrupole coils exclusively in the Helmholtz configuration. We shall refer to this configuration as the “axial coils.”
Figure 4.13: Resonance curve of the number of atoms in $|1, -1\rangle$ state as a function of the field applied in the x-direction. We need to apply 0.08671(5) G through the EW coils to cancel the background DC field along the x-axis.

With the hypothesis that a large fraction of noise is due to 60 Hz powerline cycle, we introduced a synchronization system in our apparatus (Section 2.2.1). This synchronization has the effect of eliminating the variability in the magnetic field due to the 60Hz signal and its harmonics over a short timescale, allowing us to perform the microwave spectroscopy calibrations.

In Figs. 4.14, 4.15, and 4.16, we present data (with and without sync) for microwave spectroscopy for the EW coils, the NS coils, and the axial coils.\(^3\) For the UD coils, which can only produce fields less than 5 G, we lack the

\(^3\)In Figs. 4.14, 4.15, and 4.16, we use the microwave frequency as our independent variable so as to achieve a better resolution for the figures. We can invert the Breit-Rabi equation (Eq. 2.2) for the $|1, 1\rangle \rightarrow |2, 1\rangle$ transition for the axial coils, and the $|1, 1\rangle \rightarrow |2, 2\rangle$ transition for the NS and EW coils to obtain the corresponding magnetic fields.
Figure 4.14: Data for microwave spectroscopy for EW coils with and without sync. With sync on, the width of the curve is reduced by $\sim 700\mu$G.

sufficient resolution to observe this effect.

### 4.4 Calibration of axial coils

We calibrate the axial coils through microwave spectroscopy, using a microwave driving field of 1ms pulse length and $-6$dBm power, to transfer atoms from the $|1, 1\rangle$ state to the $|2, 1\rangle$ state. Our spectroscopy technique for the axial coils uses a similar procedure to the technique for the UD coils, with two important exceptions. Firstly, during this calibration, the UD coil is set to null the background DC field, and the axial coil provides the only magnetic field along the vertical direction. Secondly, we employ the 60Hz sync for the
Figure 4.15: Data for microwave spectroscopy for NS coils with and without sync. With sync on, the width of the curve is reduced by $\sim 200\mu\text{G}$.

We perform this experiment for values of applied field up to $\sim 400\text{G}$. At the larger values of the applied field, we find that the field produced is very erratic (Fig. 4.18). This is consistent with our hypothesis that our apparatus suffers from AC field noise. Our calibration at these field values is therefore only approximate.

Having measured the microwave transition frequency at a variety of programmed fields, we obtain the linear fit presented in Fig. 4.19. Our calibration results for our various magnetic coils is presented in Table 4.1.\textsuperscript{4}

\textsuperscript{4}The uncertainties presented here (and in our other measurements) are statistical uncertainties, and are not obtained through any systematic error analysis. The actual uncertainties are likely much higher.
Figure 4.16: Data for microwave spectroscopy for axial coils (Section 4.4) with and without sync. With sync on, the width of the curve is reduced by $\sim 100\mu G$.

<table>
<thead>
<tr>
<th></th>
<th>UD Coils</th>
<th>EW Coils</th>
<th>NS Coils</th>
<th>Axial Coils</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope (G/V)</td>
<td>0.5244(7)</td>
<td>1.96991(8)</td>
<td>2.3553(1)</td>
<td>214.02(2)</td>
</tr>
<tr>
<td>Offset (G)</td>
<td>-0.530(1)</td>
<td>-0.0831(2)</td>
<td>-0.1832(2)</td>
<td>-0.05(2)</td>
</tr>
</tbody>
</table>

Table 4.1: Calibration of the bias coils.
Figure 4.17: Sample resonance curve for axial coils, with applied field of \(~1.0\text{G}\)
with synchronization with 60Hz powerline cycle enabled.
Figure 4.18: Erratic behavior in the axial coils for large field values.
Figure 4.19: Linear fit of the magnetic field produced by the axial coils in common mode as a function of the applied voltage.
Chapter 5

Inelastic Loss Spectroscopy

using Feshbach Resonances

Inelastic losses through Feshbach resonances are a valuable spectroscopic tool. The location and widths for various Feshbach resonances have been theoretically predicted and experimentally explored [43], and thus provide a valuable marker with which we can test both the calibration and the stability of the magnetic fields produced by the magnetic coils. We explore several Feshbach resonances occurring in the $|1,1\rangle$ magnetic sublevel to help characterize the magnetic field produced by our axial coils.

A Feshbach resonance occurs when a free scattering state of two colliding atoms couples resonantly with a quasi-bound molecular state with nearly zero energy, i.e., when the interatomic potential energy in a quasibound state becomes degenerate with the kinetic energy of two colliding atoms [12, 44–46]. When the corresponding magnetic moments of the quasi-bound state and the
unbound states are different, the energy difference between these states can be controlled via the application of an external magnetic field. When the states become degenerate, we allow the atoms to tunnel between the unbound and the quasi-bound states.

In a BEC, atoms in quasi-bound molecular states are highly susceptible to two-body or three-body loss mechanisms. These losses occur because of a release of internal energy in the form of kinetic energy, either when the colliding atoms end up in a lower internal state, or when a molecule is formed. These inelastic losses are greatly enhanced as we approach near a Feshbach resonance, and loss coefficient assumes the shape of a Lorentzian [46].

5.1 Experimental Technique

Our experimental technique for the inelastic loss spectroscopy measurements involves a two step process. As a preliminary step, we test the calibration of our axial coil against the known positions of several Feshbach resonances. We then take a closer look at the Feshbach resonance at the theoretical location of 392.9 G [43].

For our preliminary step, we start with $|1, 1\rangle$ condensate in the optical trap. We then apply 3.0 V along the EW coils, so as to preserve spin polarization in our optical trap. We then apply nulling voltages, obtained via Majorana spectroscopy, along our NS and UD bias coils to null background DC fields along the $y$– and $z$– axes respectively. We then turn on the axial coil and set it to our test voltage.
We hold the condensate in the axial field for 150 ms, turn off the field, and image our condensate from the side. We note the number of atoms remaining. We then repeat the procedure for several values of the applied field around the test value. This procedure should give us a Lorentzian curve if we hit the Feshbach resonance.

In Table 5.1, we present our applied field values for the Feshbach resonances we observed. We also present the field values for the corresponding (presumed) Feshbach resonances observed by Marte et al [43]. The close agreement in the field values suggest that we did, indeed, observe the same Feshbach resonances, and that our calibration for the axial coils is quite accurate. However, we should also note that the difference between our observed values and the values obtained by Marte et al increases as we increase the applied field. This is consistent with our results from microwave spectroscopy measurements — our apparatus suffers from AC noise. The effects of this AC noise are amplified at higher field values.

<table>
<thead>
<tr>
<th>$B_{\text{obs}}$ (G)</th>
<th>$B_{\text{th}}$ (G)</th>
<th>$B_{\text{exp}}$ (G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>319.30 (2) G</td>
<td>319.7</td>
<td>319.30</td>
</tr>
<tr>
<td>391.48 (2) G</td>
<td>392.9</td>
<td>391.49</td>
</tr>
<tr>
<td>406.20 (2) G</td>
<td>406.6</td>
<td>406.23</td>
</tr>
<tr>
<td>632.40 (4) G</td>
<td>632.5</td>
<td>632.45</td>
</tr>
<tr>
<td>685.37 (4) G</td>
<td>685.8</td>
<td>685.43</td>
</tr>
</tbody>
</table>

Table 5.1: Observed Feshbach Resonances. Column $B_{\text{obs}}$ contains our measurements for the resonances; columns $B_{\text{th}}$ and $B_{\text{exp}}$ respectively contain the theoretical and experimental field values obtained by Marte et al [43].

$^1$We calculate the observed fields presented in Table 5.1 by multiplying the voltage applied by the slope of the calibration curve (Table 4.1). When we incorporate the observed offset, our values differ from the values observed by Marte et al by a larger extent. This offset is puzzling, and warrants further investigation.
In Fig. 5.3, we present the Lorentzian fit to the Feshbach resonance centered at 391.46 G. The width of the resonance curve is found to be 80 mG. Theoretically, however, the width of the resonance is only 0.3 mG [43]. This disconnect between the theoretical and observed width suggests that we have significant instrumental magnetic field noise. One potential source of concern is the time it takes for the axial coils to stabilize to the set field value — when we ramp the axial field to the test field, there could be fluctuations in the field for a few milliseconds. This effect could greatly increase the width of the resonance curve.

To overcome this source of concern, we perform another experiment where we ramp the axial coil to ∼1 G below (above) the Feshbach resonance, and wait for the field to stabilize. We then apply a bias voltage of 6 V (0 V) in the UD coil to increase (decrease) the field to the test value. Since our ramps for our bias coils are much slower than the ramps for the axial coils, we cannot simply set the axial field to one below (above) the resonance. We then change the axial field so that the a bias voltage of 3 V gives the resonance. We then change the bias voltage from 6 V to the desired point on the resonance curve.

We examine the resonance at 391.49 G (Fig. 5.3) with this procedure, and present the resulting resonance curve in Fig. 5.7. We see that the resonance curve is now ∼ 30 mG wide. Since our bias coils are stable to better than 10 mG, this width now indicates that our axial coils suffer from AC noise. In addition, we also observe that the widths of our Feshbach resonances increase as we increase the applied field. This is consistent with the introduction of a field gradient in addition to noise, possibly as a result of imperfect coil
geometry. However, we were unable to find any other evidence of the existence of such a gradient.

While we were able to find the Feshbach resonances listed in Table 5.1, we were unable to find the narrower Feshbach resonances previously observed at 249 G and 306 G [43]. This suggests that our noise is sufficient to create fluctuations in the field that completely wash out these narrow resonances.
Figure 5.1: Atoms in $|1,1\rangle$ for various values of field applied: (a) For 405.94 G applied, condensate does not lose atoms. (b) For 406.05 G applied, condensate loses some atoms. (c) For 406.16 G applied field, most of the atoms are lost.
Figure 5.2: Feshbach resonance at 319 G.
Figure 5.3: Feshbach resonance at 391 G.
Figure 5.4: Feshbach resonance at 406 G.
Figure 5.5: Feshbach resonance at 632 G.
Figure 5.6: Feshbach resonance at 685 G.
Figure 5.7: Resonance curve near 391.49 G, obtained by fixing the axial field, and manipulating the UD bias field. If we take the circled data points to be representative of the width, we find that the resonance curve is \( \sim 30 \text{ mG} \) wide. However, since the curve appears to be saturated, our actual width could be lower than 30 mG.
Chapter 6

Conclusion

We have attempted to create a Dirac monopole in the spin texture of a Bose-Einstein condensate. We have investigated some of the difficulties that require improvements in the apparatus to address. In this process, we have recalibrated our various magnetic coils, and have developed a protocol that uses Majorana transitions to perform \textit{in situ} measurement of the background DC field offset. In addition, we have also implemented a synchronization protocol that allows us to start our experiment at a fixed, reproducible point in the 60 Hz powerline cycle. Most recently, we have tested our calibrations to observe some Feshbach resonances, which, in turn, has provided us valuable spectroscopic data about the noise and stability of our magnetic coils.

Our experiments in this thesis have opened up several new promising avenues for future experiments. The most important of these is the development of a protocol to perform \textit{in situ} measurements of the background AC field — the 60Hz cycle and its harmonics in particular [40]. To accomplish this, we can
perform a series of microwave spectroscopy measurements for a fixed applied voltage for a magnetic coil, while varying the point at the 60Hz cycle at which we begin our experiment. The magnitude of the field observed for the various points in the 60Hz cycle would give us a time series, which we could fit to a function containing DC, 60, 120, 180 and 240Hz (and other 60Hz harmonics as appropriate) with various amplitudes and phases as fit parameters. In subsequent experiments, we can then use an arbitrary waveform generator to subtract this fit from the field produced for that magnetic coil, thereby cancelling the background fields [40]. A successful cancellation of the 60Hz (and harmonics) noise would make the creation of a Dirac monopole much more accessible.

A future attempt to create a Dirac monopole could also rely on using a combination of our EW and NS coils to produce the required quadrupole field gradient [3]. This experiment would require that we preserve the cylindrical symmetry of the net magnetic field through precise aligning of the magnetic fields produced by these coils.

Future work could also involve research on how the evolution of vortices is affected by the application of a field near our observed Feshbach resonances. Further investigation into the nature of these observed Feshbach resonances is also warranted.
Bibliography


