

Syllabus for the Comprehensive Examination in Linear Algebra (MATH 271/2)
Department of Mathematics and Statistics, Amherst College

Basic Definitions

- Vector space
- Subspace
- Span of a finite subset
- Linear independence of a finite set of vectors
- Basis and dimension
- Linear transformation
- Kernel or null space
- Image or range
- Inverse of a matrix or linear transformation
- Determinant
- Characteristic polynomial
- Eigenvalues and eigenvectors of a matrix
- Diagonalizability

Computational Techniques

- Determine when a subset is a subspace
- Basic matrix manipulations
- Row operations on matrices
- Solving systems of linear equations
- Find the inverse of a matrix
- Find a basis of a given subspace
- Find the nullity, rank, and determinant of a matrix
- Find the null space $N(T)$ and range $R(T)$ of a linear transformation T
- Given bases of V and W , find the matrix of a linear transformation $T : V \rightarrow W$
- Given a matrix
 - Compute its characteristic polynomial
 - Find its eigenvalues and eigenspaces

Basic Results to Know

- $\dim N(T) + \dim R(T) = \dim V$
- $\text{nullity}(A) + \text{rank}(A) = \text{number of columns of } A$
- Criteria for A^{-1} to exist
- Criteria for A to be diagonalizable

Write short proofs for problems involving subspaces, linear maps, linear independence, spanning sets, null spaces and ranges.