Math 221: Transition to Theoretical Mathematics - 01 (Professor Alvarado)

Homework 1 Allison Tanguay

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I worked with/used the following sources: Ryan Alvarado, textbook, ...

1. Let $U := \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. List out the elements in each set (write its elements within braces).

   (a) $S := \{x \in U : x^2 < 46\}$
   Answer: $S = \{0, 1, 2, 3, 4, 5, 6\}$

   (b) $T := \{x \in U : x^2 - 3x = -2\}$
   Answer:

$$x^2 - 3x = -2$$
$$x^2 - 3x + 2 = 0$$
$$(x-2)(x-1) = 0$$
$$x = 2 \text{ or } x = 1$$
$$T = \{1, 2\}$$

2. Let $A := \{1, 2, 3, 4, 5\}$. Write each of the following sets in the form $\{x \in A : p(x)\}$, where $p(x)$ is a condition on x. (See problem 1 to get an idea of what $p(x)$ could be.)

   (a) $B := \{2, 3, 4\}$
   Answer: $B = \{x \in A : (x-2)(x-3)(x-4) = 0\}$

   (b) $C := \{1, 2, 4, 5\}$
   Answer: $C = \{x \in A : x \neq 3\}$

3. Let $A := \big\{a, \{a\}, \{a, \{a\}\}\big\}$. How many elements are in the set A? For each of the following parts, answer true or false. Explain briefly your answer.

   Answer: There are three elements in the set A.

   (a) $a \in A$
   Answer: True since the element a is in the set A.

   (b) $a \subseteq A$
   Answer: False because a is not a set and therefore cannot be a subset of A.

   (c) $\{a\} \subseteq A$
   Answer: True since the element a is in the set A, so $\{a\} \subset A$.

   (d) $\{a\} \in A$
   Answer: True since the set $\{a\}$ is an element of the set A.

4. Give an example of three sets, A, B, and C, such that $A \in B$, $B \in C$ and $A \notin C$.

   Answer: Let
$$A = \{a\}$$
$$B = \{\{a\}\} = \{A\}$$
$$C = \{\{\{a\}\}\} = \{B\}$$

5. Let $U := \{-3, -2, -1, 0, 1, 2, 3\}$. Which of the following sets are equal?

   (a) $A := \{n \in U : |n| < 2\}$
$$A = \{-1, 0, 1\}$$
   (b) $B := \{n \in U : n^2 \leq 1\}$
$$B = \{-1, 0, 1\}$$
   (c) $C := \{n \in U : n^3 = n\}$
$$C = \{-1, 0, 1\}$$
   (d) $D := \{-1, 0, 1\}$

   (e) $E := \{n \in U : n^2 \leq n\}$
$$E = \{0, 1\}$$

   Therefore, the first four sets are equal, i.e. $A = B = C = D$, but E is not equal to the others.

6. Let $S = \{1,2,3,4,5\}$, $T = \{3,4,5,7,8,9\}$, $U = \{1,2,3,4,9\}$, $V = \{2,4,6,8\}$. Find each of the following sets.

   (a) $S \cap U$
   Answer: $S \cap U = \{1,2,3,4\}$.

   (b) $(S \cap T) \cup U$
   Answer: $(S \cap T) \cup U = \{3,4,5\} \cup U = \{1,2,3,4,5,9\}$.

   (c) $(S \cup V) \setminus U$
   Answer: $(S \cup V) \setminus U = \{1,2,3,4,5,6,8\} \setminus U = \{5,6,8\}$.

   (d) $(S \setminus U) \cup (V \setminus U)$
   Answer: $(S \setminus U) \cup (V \setminus U) = \{5\} \cup \{6,8\} = \{5,6,8\}$.

   (e) $(S \cup V) \setminus (T \cap U)$
   Answer: $(S \cup V) \setminus (T \cap U) = \{1,2,3,4,5,6,8\} \setminus \{3,4,9\} = \{1,2,5,6,8\}$

   (f) $(S \times V) \setminus (T \times U)$
   Answer:

$$(S \times V) = \Big\{(1,2),(1,4),(1,6),(1,8),(2,2),(2,4),(2,6),(2,8),(3,2),(3,4),(3,6),(3,8),\\ (4,2),(4,4),(4,6),(4,8),(5,2),(5,4),(5,6),(5,8)\Big\}$$

$$(T \times U) = \Big\{(3,1),(3,2),(3,3),(3,4),(3,9),(4,1),(4,2),(4,3),(4,4),(4,9),\\ (5,1),(5,2),(5,3),(5,4),(5,9),(7,1),(7,2),(7,3),(7,4),(7,9)\\ (8,1),(8,2),(8,3),(8,4),(8,9),(9,1),(9,2),(9,3),(9,4),(9,9)\Big\}$$

$$(S \times V) \setminus (T \times U) = \Big\{(1,2),(1,4),(1,6),(1,8),(2,2),(2,4),(2,6),(2,8),\\ (3,6),(3,8),(4,6),(4,8),(5,6),(5,8)\Big\}$$

   (g) $(V \setminus T) \times (U \setminus S)$
   Answer: $V \setminus T = \{2,6\}$ and $U \setminus S = \{9\}$. Then

$$(V \setminus T) \times (U \setminus S) = \Big\{(2,9),(6,9)\Big\}$$

7. Let $U := \{-4,-3,-2,-1,0,1,2,3,4\}$. If $X := \big\{n \in U : n^2 < 11\big\}$ and $A := \big\{n \in X : \frac{n}{2} \in \mathbb{N}\big\}$, then find $X \setminus A$.
   Answer: $X = \{-3,-2,-1,0,1,2,3\}$ and $A = \{-2,2\}$. Then $X \setminus A = \{-3,-1,0,1,3\}$.

8. Which of the following sets are partitions of $A = \{1, 2, 3, 4, 5\}$? Explain briefly why or why not.

   (a) $A_1 := \{\{1, 3\}, \{2, 5\}\}$
   Answer: A_1 is not a partition of A because $4 \in A$ and 4 is not an element of either of the sets in A_1. (The union of the sets in A_1 is not equal to A.)

   (b) $A_2 := \{\{1, 2\}, \{3, 4, 5\}\}$
   Answer: A_2 is a partition of A because $\{1, 2\} \cup \{3, 4, 5\} = A$ and $\{1, 2\} \cap \{3, 4, 5\} = \emptyset$.

   (c) $A_3 := \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}\}$
   Answer: A_3 is not a partition of A because the sets in A_3 are not mutually disjoint. For example, $\{1, 2\} \cap \{2, 3\} \neq \emptyset$.

9. Suppose $A := \{e, 0\}$ and $B := \{0, 1\}$. Write out the indicated sets by listing their elements.

   (a) $A \times B$
   Answer: $A \times B = \Big\{(e, 0), (e, 1), (0, 0), (0, 1)\Big\}$

   (b) $B \times A$
   Answer: $B \times A = \Big\{(0, e), (0, 0), (1, e), (1, 0)\Big\}$

   (c) $A \times A$
   Answer: $A \times A = \Big\{(e, e), (e, 0), (0, e), (0, 0)\Big\}$

   (d) $A \times \emptyset$
   Answer: $A \times \emptyset = \emptyset$