Physics 400, Fall 2011: Problem Set 3

Due Thursday Oct. 6, 11:59 pm

1. Nordlund 6.1
2. Nordlund 6.4
3. Nordlund 6.7

4. (a) Using what you know about the geometry of DNA, actin filaments, and microtubules, determine the areal moment of inertia $I$ for each of these molecules. Be careful and remember that microtubules are hollow. Make sure that you comment on the various simplifications that you are making when you replace the macromolecule by some simple geometry.

(b) Given that the elastic modulus of actin is 2.3 GPa, take as your working hypothesis that $E$ is universal for the macromolecules of interest here and has a value 2 GPa. In light of this choice of modulus, compute the stress needed to stretch both actin and DNA with a strain of 1%. Convert this result into a pulling force in piconewtons.
6.1 Centrifugal Acceleration

\[ \omega = \frac{40,000 \text{ revolutions}}{\text{minute}} = \left( \frac{40,000 \text{ rev}}{\text{min}} \right) \left( \frac{2 \pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{60 \text{ sec}}{1 \text{ min}} \right) \]
\[ = 4189 \text{ rad sec}^{-1} \]
\[ \alpha = \omega^2 r = \left(4189 \text{ rad sec}^{-1} \right)^2 (0.15 \text{ m}) = 2.63 \times 10^6 \text{ m sec}^{-2} \approx 2.7 \times 10^5 \text{ g} \]

6. The centrifugal force (and the corresponding potential energy) depends on the distance from the axis, but the change in position will be so small that we can treat \( r \) as a constant in our expression for \( \text{cent. (r)} \), the centrifugal force.

Then
\[ \Delta V_{\text{cent.}} = F(r) \cdot \Delta r = m \omega^2 r \Delta r \]
- In this case, \( \Delta r = 2R \) is the diameter of the particle.
- Particle mass and density are related by
\[ m = \rho \left( \frac{4}{3} \pi R^3 \right), \text{ with particle density} \]
\[ \rho \approx 1.25 \text{ pater} \]
- So, \( \Delta V_{\text{cent.}} = m \omega^2 r \Delta r \]
\[ = (1.25 \rho w) \left( \frac{4}{3} \pi R^3 \right) w^2 2R \]
\[ = (1.25 \times 1000 \text{ kg m}^{-3}) \left( \frac{4}{3} \pi R^3 \right) (4189 \text{ rad sec}^{-1})^2 (0.15 \text{ m}) \]
\[ = \left( \frac{4}{3} \pi R^3 \right) (2R) \omega^2 \]
\[ \text{collecting} \]
\[ \text{factor} \]
\[ = \left( 2.76 \times 10^6 \text{ kg m}^{-2} \right) R^4 \]

we want
\[ \Delta V_{\text{cent.}} = \frac{k \cdot T}{8} \]
\[ \approx 4.1 \times 10^{-21} \text{ J} \]

So,
\[ (2.76 \times 10^6 \text{ kg m}^{-2}) R^4 = 4.1 \times 10^{-21} \text{ J} \]
\[ \implies R = 19.6 \text{ nm} \]
[about the size of a ribosome]
This particle will have a mass
\[ m = \rho \left( \frac{4}{3} \pi R^3 \right) \]
\[ = \left( 1250 \frac{kg}{m^3} \right) \left[ \frac{4}{3} \pi \left( 14.6 \times 10^{-2} m \right)^3 \right] \approx 3.94 \times 10^{-2} kg \]

(2) There will be a buoyant force in the direction opposite the sum of the centripetal force [I assume the gravitational force is so small it can be ignored].

The buoyant force is caused when the surrounding fluid, under the influence of an external force field, is competing for the same space as the particle.

So, the answer in part (2) was only correct when finding the centripetal potential energy change alone. We should also have included the buoyant force.

As in the simple gravitational case, all we have to do is go back and use the reduced density of the particle
\[ \rho = (1.25 - 1.00) \rho_w. \]
Lipid Aggregates

- Membranes can form spherical double-layer structures, the smallest of which has a diameter \( \approx 50 \text{ nm} \).

- Find the dimensions of a typical lipid molecule. Determine how many lipid molecules fit in:
  1. On the smallest spherical membrane
  2. In the exercise 1.1 largest membrane of diameter \( \approx 50 \mu\text{m} \).

Assume 60% of the membrane area is lipid.

From a minor census of E. coli, the area of a membrane lipid is \( \approx 0.5 \text{ nm}^2 = \text{A lipid} \).

The area of a sphere of diameter \( d \) is

\[
A = 4\pi r^2 = 4\pi \left(\frac{d}{2}\right)^2 = \pi \frac{d^2}{4} = \pi d^2
\]

For a sphere formed from a bilayer, there will be lipid heads on both the inner and outer surface, so the total surface area is

\[
A_{\text{tot}} = 2\pi d^2
\]

If all of this area was lipid, the number of lipid molecules would be

\[
N = \frac{A_{\text{tot}}}{\text{A lipid}}. \quad \text{If only 60% of the membrane is lipid, it's really } \quad N = 0.6 \frac{A_{\text{tot}}}{\text{A lipid}}
\]

For \( d = 50 \text{ nm} \), we have

\[
N = 0.6 \frac{2\pi (50 \text{ nm})^2}{0.5 \text{ nm}^2} = 0.6 \frac{50^2}{0.5} = 18,850 \approx 19,000 = 1.9 \times 10^4
\]

For \( d = 50 \mu\text{m} = 50,000 \text{ nm} = 5 \times 10^4 \text{ nm} \),

\[
N = 0.6 \frac{2\pi (5 \times 10^4 \text{ nm})^2}{0.5 \text{ nm}^2} = 0.6 \frac{(2\pi)(5 \times 10^4)^2}{0.5} = 18,850 \times 10^6 \approx 1.9 \times 10^{10}
\]
Lipid Flips in membranes

1) \( N_F = \frac{N_{flipped}}{N_{tot}} = e^{-\frac{\Delta G}{RT}} \)
   \( \text{[See p.145, par. 2]} \)
   \( = \exp\left[-\frac{\Delta G}{2.4\times10^5}\right] \)
   
   For \( \Delta G = 5 \text{ kJ/mole} \rightarrow 0.134 \)
   \( \Delta G = 15 \text{ kJ/mole} \rightarrow 2.4\times10^{-3} \)
   \( \Delta G = 30 \text{ kJ/mole} \rightarrow 5.86\times10^{-4} \)
   \( \Delta G = 60 \text{ kJ/mole} \rightarrow 3.43\times10^{-5} \)

   For a total of \( N_{tot} = N_F + N_u = 10^6 \), we can write:
   \( N_F = N_u e^{-\frac{\Delta G}{RT}} \), or
   \( N_u = \frac{N_F}{e^{-\frac{\Delta G}{RT}}} \)
   
   and
   \( N_{tot} = N_F + N_u = N_F \left(1 + \frac{1}{e^{-\frac{\Delta G}{RT}}}\right) \)
   \( N_F = \frac{N_{tot}}{1 + \frac{1}{e^{-\frac{\Delta G}{RT}}}} \)

   For \( \Delta G = 5 \text{ kJ/mole} \), \( N_F = 10^6 \frac{1}{0.134 + 1} = 1.2\times10^5 \)
   \( \Delta G = 15 \text{ kJ/mole} \), \( N_F = 10^6 \frac{1}{2.4\times10^{-3} + 1} = 2420 \)
   \( \Delta G = 30 \text{ kJ/mole} \), \( N_F = 6 \)
   \( \Delta G = 60 \text{ kJ/mole} \), \( N_F \approx 3.4\times10^{-5} \approx 0 \)

2) From
   N Sapoj et al. (2009) Biophys. J. 96(3), 349a
   (or sec. 6.4.3.3 of Nordlund)
   Flips in pure, naked model lipid membranes take
   longer hours \( (\sim \frac{1}{10^4} \text{ flip/hr}) \)
   and flips in reconstituted membranes
   (almost natural with proteins and other
   molecules) take minutes to seconds
   (rate about 1 flip/10 sec \( \sim 0.1 \text{ flip/hr} \))
If there are $10^9$ lipids in the membrane, the rate for

in pure membranes is $\boxed{(10^9 \times 10^{-4} \text{ s}^{-1}) = 10^5 \text{ per second}}$.

and in reconstituted membranes $\boxed{(10^9 \times 10^{-5}) = 10^4 \text{ per second}}$. 
(a) Treat all molecules as rods. DNA and actin filaments have solid-disc cross-section. The microtubule cross-section is an annular-disc.

From the lecture slides, you know the geometric moments for both of these geometries:

\[ I = \frac{\pi R^4}{4} \]

\[ I = \frac{\pi}{4} (R_2^4 - R_1^4) \]

Radius of DNA \( \approx 1 \text{ nm} \) \( \rightarrow I = \frac{\pi}{4} \text{ nm}^4 \)

Radius of actin \( \approx 2.5 \text{ nm} \) \( \rightarrow I = \frac{\pi}{4} 4 \text{ nm}^4 \)

Reduction:

Microtubule: \( R_2 \approx 28 \text{ nm} \)
\( R_1 \approx 10.5 \text{ nm} \)
\( \int I = \frac{\pi}{4} (R_2^4 - R_1^4) \approx 64 \pi \text{ nm}^4 \)

(b) There's no bending, just stretching. So, relevant formula is

\[ \frac{\text{stress}}{\text{strain}} = E \text{ (Young's modulus)} \]

or stress = \( E \left( \frac{\Delta L}{L} \right) \)

In all cases the stress is the same:

\[ \text{stress} = (2 \times 10^8 \text{ Pa}) \times 10^{-2} = 2 \times 10^6 \text{ Pa} \]

\[ \approx 20 \text{ kN/m}^2 \]

What varies is the area:

\( \text{Area} = \pi r^2 \)

\[ A_{\text{DNA}} = \pi \approx 3 \text{ nm}^2 \]

\[ A_{\text{Actin}} = \pi (4 \text{ nm})^2 \approx 50 \text{ nm}^2 \]

\[ A_{\text{Microtubule}} \approx \pi \left( (12.5 \text{ nm})^2 - (10.5 \text{ nm})^2 \right) \approx 150 \text{ nm}^2 \]

Which determines the force = stress \( \times \) area.
\( F_{\text{DNA}} = (20 \text{ pN/nm}^2) \cdot (n \text{ nm}^2) \approx 60 \text{ pN} \)
\( F_{\text{ATP}} = (20 \text{ pN/nm}^2) \cdot (50 \text{ nm}^2) = 10^3 \text{ pN} \)
\( F_{\text{unwinding}} = (20 \text{ pN/nm}^2) \cdot 400 \text{ nm}^2 \approx 8 \times 10^3 \text{ pN} \)