L13 & L14

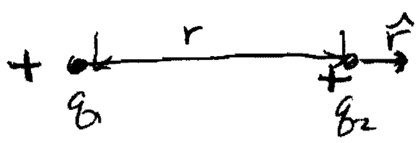
Ch. 22 Electric Charge and Force

1) Electric charge
2) Charging by Friction / Induction
3) Coulomb's Law.

Last time:

Coulomb's Law

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$



$\vec{F}$ is (+) and in direction of $\hat{r}$

? Estimate force to rip off electron in Hydrogen atom = 10 nN

What is covalent bond strength?

$\sim 500\,kJ/mole = 8E{-}19\,J/molecule$

$= 800E{-}21\,\frac{J}{molecule} = 800\,pN \cdot nm$

$\cong 1\,nN \cdot nm$

? What is permittivity of free space, ϵ_0?

→ constant that sets force.

→ constant that is derived from speed of light

→ Quantum Mechanics...

The vacuum in Quantum Mechanics is
<u>not</u> empty space.

- it contains particles and photons that pop in and out of existance
- electrons and positrons might exist then annihilate due to the energy-time uncertainty principle
- Generally called vacuum fluctuations.

$\downarrow$ depend on ε_0

(?) Unit analysis and quantum mechanics.
the permittivity has a basis in quantum mechanics as well.

- Now you may not know quantum mechanics but this is a perfect opportunity to introduce unit analysis.

* Unit analysis - a technique for the estimator or BSer

- if you don't know a formula you can most likely obtain or guess it by thinking about the units of the values involved.

→ Say you forgot the formula for gravitational potential energy energy ~~is in J = Nm = kgm²/s²~~ you can do a unit analysis to obtain it.

you know it must involve

$m \rightarrow M$

$g - L/T^2$

$h \rightarrow L$

$U \rightarrow ML^2/T^2$

$$ML^2/T^2 = (M)(L/T^2)(L)$$

$$U \propto mgh$$

→ perhaps w/ a constant

? Assuming ϵ_0 is setting vacuum fluctuations. Can we estimate its size using unit analysis.

$$\epsilon_0 \to \frac{C^2}{N\,m^2} = \frac{C^2 s^2}{kg\,m^3} = \frac{C^2}{J\,m}$$

$e \to 1.6\,E{-19}\ C$

~~$\alpha \to$ fine structure constant~~

$h \to 6.63\,E{-34}\ J \cdot s$

$c \to 3E8\ m/s$

$$\epsilon_0 \propto \frac{e^2}{hc} \propto 1.28\,E{-13}\ \frac{C^2}{N m^2}$$

? How big is an atom?

$$r_{Bohr} = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = \frac{4\pi\,\epsilon_0 h^2}{4\pi^2\, m_e e^2}$$

$$= \frac{\epsilon_0 h^2}{\pi m_e e^2}$$

$\epsilon_0 \to C^2/Nm^2 = C^2 s^2/kg\,m^3$

$e \to C$

$h \to J \cdot s = Nms \qquad kg\,m^2/s$

$m_e \to kg = 9.1\,E{-31}\ kg.$

$$(C^2)\left(\frac{C^2 s^2}{kg\,m^3}\right)\left(\frac{kg^2 m^4}{s^2}\right)/(kg) \Rightarrow$$

$$\propto \frac{\epsilon_0 h^2}{m_e e^2} = 1.6E{-10}\ m \quad \approx 2\,Å$$

Ch. 23 Electric Field

1) Electric field of point charges
2) Electric field of charge distribution.
3) Electric Dipole.

* Electric field – the "disturbance" a charge distribution generates to exert forces on other charges.

→ Contact vs. Non contact forces

Contact	Non contact
Normal	Electric force
Friction	Gravity
Applied Force	Weak / Strong Force

↓

"action-at-a-distance

↓

How does this occur?

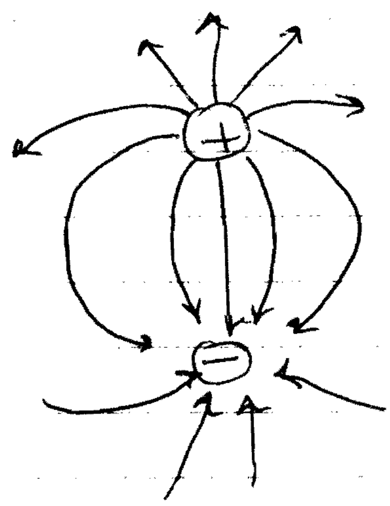
→ Fields allow "action-at-a-distance" to "act locally. They convey the force across space.

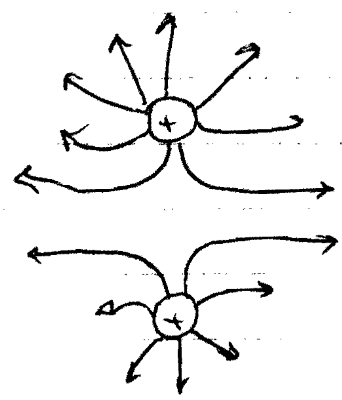
→ any charge distribution will set an electric field that will permeate space and exert a force on all other charges it interacts with.

$\vec{F} = q_1 \vec{E}$

$\vec{E} = k \frac{q_2}{r^2} \hat{r}$ $\qquad$ q_2 or q'

convention field points away from (+) charges and towards (−)





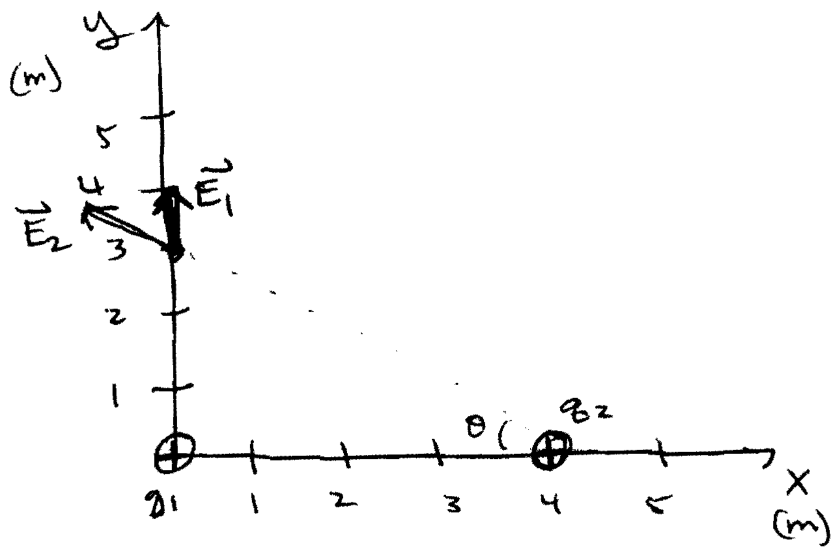
→ Pedagogical choice – we could have spent a lot of time calculating the force for different charge distributions in the last chapter. I will forgo doing that and calculate the electric field for different charge distributions instead →
This is what is generally done anyway.

1) $\vec{E}$ due to point charges

$$\vec{E} = k \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

sum over all charges in distribution that you want to calculate $\vec{E}$ for.

Ex)



$q_1 = 8\ nC$ ~~$\Delta y = 3$~~

$q_2 = 12\ nC$ ~~$\Delta x = 4$~~

$\vec{E}(y = 3m) = ?$

$\cancel{\vec{E} = k \frac{q_1}{\Delta y}}$

$\cancel{\vec{E} = \vec{E}_1 + \vec{E}_2}$

$\cancel{= k \frac{(8nC)}{}}$

$\cancel{= k \frac{q_1}{r_1^2} \hat{r}_1 + k \frac{q_2}{r_2^2} \hat{r}_2}$

$\cancel{= k \frac{(8nC)}{(3)^2} \hat{y} + k (12nC)/5^2}$

$$\vec{E}_{TOT} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_1 = E_1\,\hat{y} = k\frac{q_1}{r_1^2}\,\hat{y} = \left(8.99\text{E}9\ \frac{Nm^2}{C^2}\right)\frac{(8\,nC)}{(3m)^2}\,\hat{y}$$

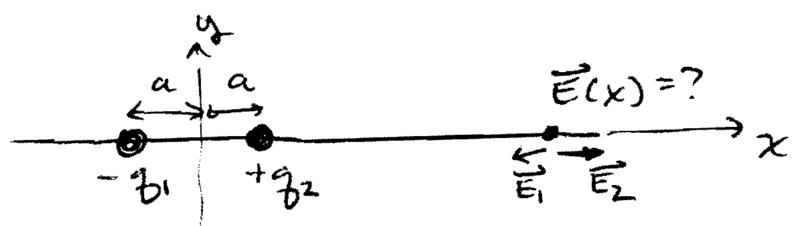
$$\vec{E}_2 = -E_2\cos\theta\ \hat{x} + E_2\sin\theta\ \hat{y}$$

$$= k\frac{q_2}{r_2^2}\cos\theta\ \hat{x} + k\frac{q_2}{r_2^2}\sin\theta\ \hat{y}$$

$$= \left(8.99\text{E}9\ \frac{Nm^2}{C^2}\right)\left[\frac{(12\,nC)}{(5m)^2}\right]\left[-\frac{4}{5}\,\hat{x} + \frac{3}{5}\,\hat{y}\right]$$

$$\vec{E}_{TOT} = (8\,\hat{y}) + (3.5\,\hat{x} + 2.5\,\hat{y}) = \left(-3.5\,\hat{x} + 10.5\,\hat{y}\right)\frac{N}{C}$$

Ex)



for $x > a$

$q_1 = q_2 = q.$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= -k\frac{q_1}{r_1^2}\,\hat{x} + k\frac{q_2}{r_2^2}\,\hat{x}$$

$$= \left[\frac{-kq}{(x+a)^2} + \frac{kq}{(x-a)^2}\right]\hat{x}$$

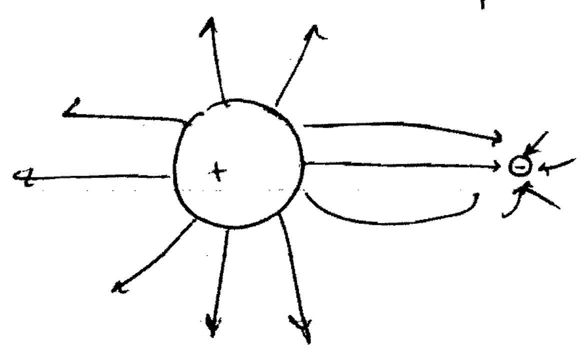
$$= kq\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right] = kq\frac{4ax}{(x^2-a^2)^2}\,\hat{x}$$

for $x > a.$

What about conducting spheres?

- conductor has all charge at the outside and free to move.

→ assume conducting spheres act like pt. charges far from sphere...



charge on sphere (+) if more lines emanate than terminate on sphere...

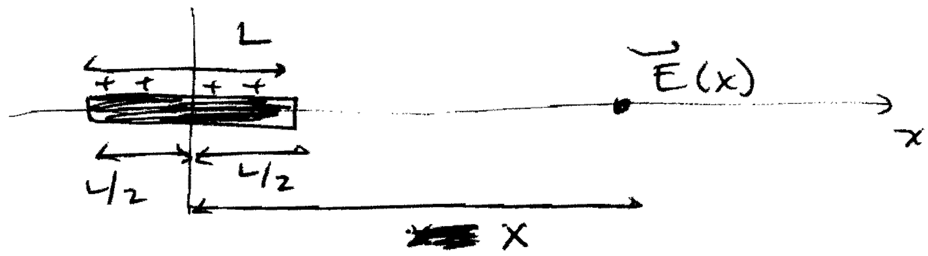
2) $\vec{E}$ from continuous charge distribution.

$$d\vec{E} = k\frac{dq}{r^2}\hat{r}$$

→ Here is where multivariable will help...

→ If you don't know multivariable you will need to memorize each case and how it works.

Ex) $\vec{E}$ of a finite line charge far from charge. Length = L and total charge = Q



→ set origin at center of line charge...

$$\int d\vec{E} = \int k \frac{dq}{r^2}\,\hat{r}$$

$$= \int k \frac{dq}{x^2}\,\hat{x}$$

$$= \int_{x-L/2}^{x+L/2} k \frac{\lambda\, dx}{x^2} \qquad \longrightarrow \quad \lambda = \text{line charge density} = \frac{Q}{L}$$

$$= -\frac{k\lambda}{x}\Bigg|_{x-L/2}^{x+L/2}$$

$$= -k\lambda\left(\frac{1}{x+L/2} - \frac{1}{x-L/2}\right)$$

$$= k\lambda\left(\frac{1}{x-L/2} - \frac{1}{x+L/2}\right)$$

$$= \frac{kQ}{L}\left(\frac{L}{\left(x^2-\left(\frac{L}{2}\right)^2\right)}\right)$$

$$= \frac{kQ}{\left(x^2-(L/2)^2\right)} \qquad x > L/2$$

$$= \frac{kQ}{x^2} \qquad \text{if } x \gg L/2$$