Amherst College
Economics 58
Fall 2010

Solutions to PS # 5

1. a. $E(v^2) = \sum_{i=1}^{2} p(x_i) f(x_i) = 0.5(-1)^2 + 0.5(1)^2 = 1$

b. $E(h^2) = 0.5(-k)^2 + 0.5(k)^2 = k^2$

c. $If\ U(W) = \ln(W),\ r(W) = -\frac{U''(W)}{U'(W)} = \frac{\frac{1}{W^2}}{\frac{1}{W}} = \frac{1}{W}, W > 0$

d. $p = 0.5E(h^2)r(W) = 0.5k^2 \frac{1}{W} = \frac{k^2}{2W}$

$When\ W = 10,$

$k = 0.5 \Rightarrow p = 0.0125$

$k = 1 \Rightarrow p = 0.05$

$k = 2 \Rightarrow p = 0.2$

$When\ W = 100,$

$k = 0.5 \Rightarrow p = 0.00125$

$k = 1 \Rightarrow p = 0.005$

$k = 2 \Rightarrow p = 0.02$

Risk premium is higher when the level of initial wealth is lower.
Greater the size of risk faced (larger the *k*), higher will be the risk premium.
Because *k* enters as a quadratic, increasing *k* and *W* in the same proportion will increase *p*.

2. a. $U'(W) = a - 2bW \quad so \quad a > 2bW$.

b. $U''(W) = -2b \quad r(W) = \frac{-U''}{U'} = \frac{2b}{a - 2bW}$ This increases as *W* increases.

c. Next period's wealth is $kW(1+r) + (1-k)W = W(1+kr)$.

d. $U(W) = aW(1+kr) - bW^2(1+rk)^2$ This is random because *r* is random.

$E[U(W)] = aW(1+k\bar{r}) - bW^2[(1+k\bar{r})^2 + k^2\sigma_r^2]$. The first order condition for a maximum is:

$aW\bar{r} - bW^2(2\bar{r} + 2k(\bar{r}^2 + \sigma_r^2)) = 0$. Dividing by W and solving for *k*:

$$k = \frac{(a - 2bW)\bar{r}}{2bW(\bar{r}^2 + \sigma_r^2)} = \frac{a\bar{r}}{2bW(\bar{r}^2 + \sigma_r^2)} - \frac{\bar{r}}{\bar{r}^2 + \sigma_r^2}.$$

e. Clearly, $\frac{\partial k^*}{\partial W} < 0, \frac{\partial k^*}{\partial \sigma_r^2} < 0$. The first of these is inconsistent with data from the real world. Why is $\partial k^* / \partial \bar{r}$ of indeterminate sign?

3. **a.** Find the maximum value of U through differentiation:

$$\frac{\partial U}{\partial c} = \frac{(s+p)p_c - pp_c}{(s+p)^2} w - \frac{sp_c}{(s+p)^2} z + \frac{sp_c}{(s+p)^2} c - \frac{s}{s+p} = 0$$

Multiply by $\frac{(s+p)^2}{s}$ yields

$$(w - z + c)p_c = s + p$$

b. Differentiation of first order condition with respect to, say, *w* yields:

$$\frac{\partial(wp_c - zp_c + cp_c - s - p)}{\partial w}$$

$$= p_c + wp_{cc}\frac{\partial c}{\partial w} - zp_{cc}\frac{\partial c}{\partial w} + cp_{cc}\frac{\partial c}{\partial w} + p_c\frac{\partial c}{\partial w} - p_c\frac{\partial c}{\partial w} = 0$$

or

$$\frac{\partial c}{\partial w} = \frac{-p_c}{p_{cc}(w - z + c)}$$

Since both numerator and denominator are negative, the fraction is positive. Other derivatives are calculated in a similar way.

c. The equilibrium here is a Nash equilibrium where individual's choice of *c* is optimal given firm's choice of *a* and vice versa.

d. Unemployment benefits might increase the value of leisure (*z*) thereby reducing *c* and increasing unemployment. Subsidized job search would effectively increase *c* thereby possibly reducing unemployment.