Problem Session 2 for Math 29: Some Discrete Fun

1. A city keeps track of how many noise citations various clubs have received over the past 3 months. Clubs receiving their sixth citation in three months are shut down for at least a month, so no currently operating clubs have more than 5 citations. The c.d.f. of X, the number of citations held by currently operating clubs, is given below. A recent mailer to club owners was sent to all clubs not in good standing determined by their number of citations (not in good standing = more than 3 citations).

X	0	1	2	3	4	5
F(x)	1/21	1/7	2/7	10/21	5/7	1

a. What is the probability a club is in good standing with regards to citations?

b. What is the probability a club had exactly three citations?

c. What is the pmf of X?

X	0	1	2	3	4	5
p(x)						

d. What is the median number of citations held by currently operating clubs?

e. What is the expected value of the number of citations held by currently operating clubs?

f. What is the variance of the value of the number of citations held by currently operating clubs?

g. A new fine is imposed to generate revenue for the city. 50 dollars per citation and a generic 5 dollar processing fee (even if the club has no citations!) must be paid to the city. What are the expected value and variance of the amount each club is going to have to pay the city?

2. A missile defense system has n detectors covering a given region. Assume each detector has its own power source, etc. such that the detectors function independently. The probability a detector detects an incoming missile is .9. The region will report a detected missile if at least one detector detects a missile.

a. What is the distribution of X = number of detectors that detect a missile for a given n? Be sure you check all necessary conditions for that distribution.

b. If $n = 5$ and a missile enters the region, what is the probability of detection by 4 or more detectors (Setup)? What is the probability the region reports a detection (Compute)?

c. What should n be if you want the probability of detection to be at least .999?

d. Detectors can make false detections. You could assume each detector has some fixed probability of reporting a false detection. If false detections exist, how might you adjust the rule that a region reports a detected missile if at least one detector detects a missile to protect against false detections? (This is actually a serious issue in some problems with networks).

3. Suppose you need to make an airline reservation and you decide to do it by phone rather than online. For the particular airline you have selected, suppose that the telephone lines are busy 60 percent of the time. Assume you keep calling until you get through and make your reservation.

a. What is the probability you successfully make your reservation on the second try (implies you did not make it the first try)?

b. If you and a friend are both trying to make reservations, what is the probability that a total of 4 tries will be needed for both of you to successfully make reservations?

c. What is the expected number of tries it takes both you and your friend to get through?

d. Suppose you learn that the airline has a tendency to overbook their flights. They book 39 reservations for a small flight with only 36 seats. Assuming each person's behavior is independent and people show up for reserved seats 95 percent of the time, what is the probability this flight is overbooked? (Setup, can compute if you are curious).

4. A CPA is checking audits on company accounts made by a junior associate. This particular associate only makes 0 or 1 mistakes per audit, and it turns out that the associate makes minor mistakes on 20 percent of his audits and major mistakes in an additional 5 percent.

a. If the CPA starts randomly checking the audits on company accounts, what is the probability he will have to go through exactly three good audits before finding one with a mistake?

b. What is the probability he will have to go through exactly 8 audits before finding one with a major mistake?

c. What is the mean and variance of the number of audits he will have to go through before finding three with a mistake?

d. If he focuses just on major mistakes and hasn't found any in the first 10 audits, what is the probability he goes 5 more before the first one with a major mistake?