Answers set 1

1

The # of people is $2.73 \times 10^8$

# of square miles is $3.54 \times 10^6$

So the density is

$$n = \frac{2.73 \times 10^8}{3.54 \times 10^6} = 77.6 \frac{\text{people}}{\text{square mile}}$$

The web says

deal from Denver to

Cheyenne = 78 miles

Oklahoma City = 507 miles

And the # of people living in the two circles is $n \times \pi r^2$

Cheyenne 2.5 million

Oklahoma City 67.4 million
the definition of a light year is the distance light travels in a year. so by this definition, light goes 1 million light years in 1 million years so its velocity is 1 million light yrs / 1 million years = 1

I can think of 2 ways to find a + b.

Method 1: small steps over 10,000 years.

day 2000

First:

\[
\frac{N}{3} = \frac{4.7 \times 10^9}{(1000)^3} = a + 1000 b
\]

Next:

\[
\frac{N}{3} = \frac{480 \times 10^9}{(4000)^3} = a + 4000 b
\]

i.e.

\[
4.7 = a + 1000 b
\]

\[
7.5 = a + 4000 b
\]

using these I get

\[
a = 4.03
\]

\[
b = 8.67 \times 10^{-4}
\]
method 2

\[ \frac{N}{r^3} \text{ against } r_{\text{max}} \]

<table>
<thead>
<tr>
<th>dist D</th>
<th>N</th>
<th>( N / D^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>4.90E+09</td>
<td>4.9</td>
</tr>
<tr>
<td>2000</td>
<td>4.60E+10</td>
<td>5.75</td>
</tr>
<tr>
<td>3000</td>
<td>1.60E+11</td>
<td>5.92592593</td>
</tr>
<tr>
<td>4000</td>
<td>4.80E+11</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Excel fits a trend line with:

\[ y = 0.0008x + 4.025 \]

\[ b = 0.0008 = 8 \times 10^{-4} \]

These agree pretty well with the first method. But notice that they do not agree exactly. If we wanted very accurate answers, we would have to think about why, and which method for the most accurate answer.
we know \( a = \frac{4\pi}{3} \) so \( n = \frac{3}{4\pi} a \)

\[
\begin{align*}
& b = \frac{\pi}{2} \quad c = \pi \\
& n = \pi \quad s = n = b/4
\end{align*}
\]

[ I'll use \( a^3 = 6 \times 10^{26} \text{km}^3 \) from Excel ]

I get \( a = 0.76 \text{AU} \)

\[
\frac{(\text{millen light years})^3}{(\text{millen light years})^3} = a^3
\]

\[
\begin{align*}
& n = 2.55 \times 10^{-4} \\
& t = 1.26 \times 10^5
\end{align*}
\]

\[
\begin{align*}
& t = n/5 = \frac{0.76}{2.55 \times 10^{-4}} \\
& t = 3.76 \text{ millen years} = 3.76 \text{ billion yrs}
\end{align*}
\]
Test the expansion formula for cube roots

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$x = 1 + \varepsilon$ (exact)</th>
<th>$(x)^{\frac{1}{3}} 0.333$ (approx)</th>
<th>$1 + \frac{\varepsilon}{3}$ (approx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1.260</td>
<td>1.333</td>
</tr>
<tr>
<td>0.9</td>
<td>1.9</td>
<td>1.239</td>
<td>1.300</td>
</tr>
<tr>
<td>0.8</td>
<td>1.8</td>
<td>1.216</td>
<td>1.267</td>
</tr>
<tr>
<td>0.7</td>
<td>1.7</td>
<td>1.193</td>
<td>1.233</td>
</tr>
<tr>
<td>0.6</td>
<td>1.6</td>
<td>1.170</td>
<td>1.200</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>1.145</td>
<td>1.167</td>
</tr>
<tr>
<td>0.4</td>
<td>1.4</td>
<td>1.119</td>
<td>1.133</td>
</tr>
<tr>
<td>0.3</td>
<td>1.3</td>
<td>1.091</td>
<td>1.100</td>
</tr>
<tr>
<td>0.2</td>
<td>1.2</td>
<td>1.063</td>
<td>1.067</td>
</tr>
<tr>
<td>0.1</td>
<td>1.1</td>
<td>1.032</td>
<td>1.033</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.9</td>
<td>0.965</td>
<td>0.967</td>
</tr>
<tr>
<td>-0.2</td>
<td>0.8</td>
<td>0.928</td>
<td>0.933</td>
</tr>
<tr>
<td>-0.3</td>
<td>0.7</td>
<td>0.888</td>
<td>0.900</td>
</tr>
<tr>
<td>-0.4</td>
<td>0.6</td>
<td>0.843</td>
<td>0.867</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.5</td>
<td>0.794</td>
<td>0.833</td>
</tr>
<tr>
<td>-0.6</td>
<td>0.4</td>
<td>0.737</td>
<td>0.800</td>
</tr>
<tr>
<td>-0.7</td>
<td>0.3</td>
<td>0.669</td>
<td>0.767</td>
</tr>
<tr>
<td>-0.8</td>
<td>0.2</td>
<td>0.585</td>
<td>0.733</td>
</tr>
<tr>
<td>-0.9</td>
<td>0.1</td>
<td>0.464</td>
<td>0.700</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>0.000</td>
<td>0.667</td>
</tr>
</tbody>
</table>

Approximate to 2 figures for $|\varepsilon| \leq 0.1$ or so.
we know \( F = \frac{L}{4\pi R^2} \) so

\[
L = 4\pi R^2 F \quad \text{and} \quad R = 5.7 \text{ light years}
\]

\[
R = 5.37 \times 10^{19} \text{ cm}
\]

so

<table>
<thead>
<tr>
<th>#1</th>
<th>( 6.14 \times 10^{35} \text{ erg/s} )</th>
<th>( L/4\pi R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>#2</td>
<td>( 9.87 \times 10^{35} \text{ erg/s} )</td>
<td>160</td>
</tr>
<tr>
<td>#3</td>
<td>( 1.83 \times 10^{36} \text{ erg/s} )</td>
<td>258</td>
</tr>
</tbody>
</table>

\( \dot{L} \) the T-day cepheid must have a luminosity of about 385 \( L_{\odot} \), or

\[
1.47 \times 10^{36} \text{ erg/s}. \text{ It's distance is}
\]

\[
R = \left( \frac{L}{4\pi F} \right)^{1/2} = 4.25 \times 10^{24} \text{ cm}
\]

\[
= 4.47 \text{ million light years}
\]

\[
= 6.5 \times 10^{-15} \text{ erg/cm}^2 \text{- sec}
\]