Math 211, Fall 2011: Midterm III Study Guide

The third midterm will take place on Friday 2 December at 1:00pm in our usual classroom, the Paino Lecture Hall. It will be 50 minutes long. You will not be allowed to use notes, books or calculators of any kind. The exam will cover sections 14.7 through 15.10 (with the exception of 15.5 and 15.6).

Format of the exam

The exam will have five questions, each will be worth 10 points. You should do as many of the questions as fully as possible. You will get partial credit if you have the right idea for a question, even if you do not get it completely correct. You should also explain your answers fully, to an amount of detail appropriate to the question. The rule of thumb is that you should explain the key steps needed to solve the problem, but do not have to explain minor steps that you use along the way. In the solutions to the practice exam problems, I'll try to indicate what is expected.

Techniques of integration

The only techniques of integration that you need to know for this exam are the following:

- integrals of power functions
- integrals of sine and cosine
- integrals of exponential functions
- integrals of sums, differences and constant multiples
- changes of variables (integration by substitution) in relatively simple cases

You will NOT need to know integration by parts, partial fractions, trig substitutions, integrals of powers of sine and cosine (except for relatively simple substitutions such as integrating $\sin x \cos^n x$).

Syllabus

Here is a summary of what you need to know and be able to do. Unless stated otherwise, you may use any fact or result from class or from the sections of the textbook we have covered. You may *not* use any results from later in the textbook. You should:

- maximum and minimum values (14.7)

- know what is meant by an absolute or local maximum or minimum for a function of two or three variables
- know that a local maximum or minimum value for a function f must occur at one of the following three types of points:
  - a critical point (where $\nabla f = \mathbf{0}$)
  - a point where f is not differentiable
  - a point on the boundary of the domain of f
- be able to use the Extreme Value Theorem to justify the existence of an absolute maximum or minimum in appropriate cases
- know what is meant by a saddle point for a function of two variables
- be able to use the second derivative test to classify the critical points of a function of two variables
- be able to solve word problems that involve finding the absolute maximum or minimum of a function of two variables

• Lagrange multipliers (14.8)

- know what is meant by a constrained max/min problem
- be able to use the Lagrange multiplier method to find constrained critical points for a function $f(x, y)$ subject to a constraint of the form $g(x, y) = c$
- be able to find the absolute maximum or minimum of a function of two variables subject to a constraint of the form $g(x, y) = c$
- be able to solve absolute max/min problems for a function of two variables using the Lagrange multiplier method to deal with points on the boundary of the domain

• double integrals in the xy-plane (15.1, 15.2, 15.3)

- know what is meant by the integral of a function of two variables over a region in the xy-plane, in terms of the volume of the solid region between the graph of the function and the xy-plane
- know that the integral of the constant function 1 is equal to the area of the region of integration
- be able to calculate integrals of functions of two variables from knowledge of the volumes of rectangular boxes, prisms with triangular cross-sections and cylinders
- know what is meant by a Type I or Type II region in the xy-plane
- be able to express a region in the xy-plane using inequalities to describe the range of each of the x and y variables
- be able to express the integral of a function of two variables over a region in the plane as an iterated integral (that is, either an x-integral followed by a y-integral, or vice versa) in which the limits of integration for the inside integration may depend on the other variable

  - be able to work out the region in the plane corresponding to an iterated integral in which the limits of integration for the inside integral may depend on the other variable
  - be able to change the order of integration in an iterated integral in which the limits of integration of integration for the inside integral may depend on the other variable
  - be able to use the above methods to evaluate a double integral of a function of two variables over a region in the xy-plane

- polar coordinates (15.4)
  - know what is meant by the polar coordinates of a point in the xy-plane and how these are related to the x and y coordinates
  - be able to describe a region in the xy-plane in terms of polar coordinates, including cases where the range of one coordinate may depend on the other
  - be able to express an integral of a function over a region in the xy-plane as an iterated integral using polar coordinates and thus calculate such integrals (not forgetting the extra factor of r)

- triple integrals (15.7)
  - be able to express solid regions in $\mathbb{R}^3$ using inequalities to describe the ranges of the x, y and z coordinates, where the range for one coordinate may depend on the other two coordinates
  - be able to express the integral of a function of three variables over a solid region in $\mathbb{R}^3$ as an iterated integral using x, y and z coordinates in any order, where the limits of one of the inside integrals may depend on the variables in the outer integrals
  - be able to calculate the integral of a function of three variables over a solid region in $\mathbb{R}^3$

- cylindrical coordinates (15.8)
  - know what is meant by the cylindrical coordinates of a point in $\mathbb{R}^3$ and how these are related to the x, y and z coordinates
  - be able to express a solid region in $\mathbb{R}^3$ in terms of cylindrical coordinates using inequalities to describe the ranges of the coordinates, including cases where the limits of one of the coordinates may depend on the other two
  - be able to calculate a triple integral using cylindrical coordinates

- spherical coordinates (15.9)
  - know what is meant by the spherical coordinates of a point in $\mathbb{R}^3$ and how these are related to the x, y and z coordinates
  - be able to express a solid region in $\mathbb{R}^3$ in terms of spherical coordinates using inequalities to describe the ranges of the coordinates
  - be able to calculate a triple integral using spherical coordinates

- changes of variables for double integrals (15.10)
  - know what is meant by the Jacobian for a change from x, y to new variables u, v in cases where either u, v are given in terms of x, y, or vice versa
  - be able to express a region in the xy-plane in terms of new variables u, v in cases where either u, v are given in terms of x, y, or vice versa
  - be able to calculate the integral of a function over a region in the xy-plane by changing to new variables whose relationship to x, y is given

This is not a complete list of what you might have to do on the test but it covers most of the ideas involved. In particular, you may have to combine several of these ideas or techniques, and you may to think to decide what to use to solve a problem.

Preparing for the test

The best way to prepare for the test is to do practice tests. This means you should sit down, without a textbook or your notes, and try to do as much as you can of the practice test in 50 minutes, as though it were the real thing. This will give you an idea of how well prepared you are, what topics you might need to review, and how you react under test conditions. This is especially important if you don't have much experience taking timed tests, or if you have had anxiety problems with tests in the past. The more practice you do, the better prepared you will be.

You should also go back over past homework problems, especially those for which the grader has written a comment or deducted points and make sure you understand the comment or why you lost points. If you can't work this out or have any other questions about the grading, please come and ask me about it. I'll have extra office hours on Wednesday and Thursday next week to help you prepare.

You should also just work through more practice problems. If you didn't do the practice problems assigned for the homeworks, now would be a good time to do those. If you did, you can make up some more problems on your own (which is also a good exercise to see if you understand the material) and try to solve them. You can always ask me if you are unsure of something.

Beyond that, please let me know how else I can help you prepare, and good luck!