Amherst College
Department of Economics
Economics 58
Fall 2006

Name_Solutions___________

Third Hour Test

There are three questions on this 50-minute examination. Each is of equal weight in grading of the examination.

1. Suppose that the demand for a product is given by $Q = 100 - 2P$ and the supply by $Q = -75 + 5P$.

a. What are the equilibrium price and quantity in this market?

$100 - 2P = -75 + 5P \quad 7P = 175 \quad P = 25 \quad Q = 50$

b. For a linear demand or supply curve, the elasticity at any point, P^*, Q^* can be calculated at $e = \frac{\partial Q}{\partial P} \cdot \frac{P^*}{Q^*}$. Use this fact to compute the elasticity of supply and demand for these curves <u>at the equilibrium point</u>.

$e^S = 5 \cdot \frac{25}{50} = 2.5 \quad e^D = -2 \cdot \frac{25}{50} = -1$

c. Suppose the supply curve in this problem shifted to $Q = -82 + 5P$. Show how the elasticities in part b can be used to estimate the percentage increase in price in this case. (Hint 1: What is the percentage reduction is quantity supplied at the initial equilibrium? Hint 2: You can check whether your final answer is right by re-computing the equilibrium price using the new supply curve – you must deal with the elasticities to answer the question, however)

Parallel left shift in S is 7 units at every price. At equilibrium of 50 this is a 14% reduction.

With the elasticities from part b, each 1% increase in price will reduce D – S by 3.5%. Hence, to close the 14% supply reduction will take a price rise of $\frac{14}{3.5} = 4\%$. A 4% price rise here is an increase from 25 to 26.

Equating S and D yields $7P = 182 \quad P = 26 \quad Q = 48$

d. Returning to the supply-demand equilibrium from part a, suppose the government instituted a tax of 3.5 per unit on this item. What would the new equilibrium in this market be?

Now $P^D = P^S + 3.5 \quad -75 + 5P^S = 100 - 2(P^S + 3.5) = 93 - 2P^S$

Hence, $7P^S = 168 \quad P^S = 24 \quad P^D = 27.5 \quad Q = 45$

e. How is the relative burden of this tax shared between producers and consumers? Does this allocation seem in accord with the elasticity estimates from part b? In general, who pays the producer's share of this tax?

Consumers Pay $(27.5 - 25)45 = 112.5$
Producers Pay $(25 - 24)45 = 45$
Total tax collections $3.5 \cdot 45 = 157.5$

Consumers pay most of the tax because they are the least elastic responders. The producers share of the tax will be paid by whatever input is giving the supply curve its upward slope.

2. Suppose that there are only two goods in an economy: *x* and *y*. The production functions for these goods are $x = \sqrt{l_x} \quad y = \sqrt{0.5 l_y}$. Total labor in the economy is constrained by $l_x + l_y = 150$. Given this situation the production possibility frontier for this economy is given by $x^2 + 2y^2 = 150$.

a. Suppose that consumers in this economy have preferences that can be represented by the aggregate utility function $U(x, y) = x + y$. Given this utility function, what is the only price ratio that can prevail in equilibrium? At this price ratio how much of each good will be produced? What will utility be?

The goods here are perfect substitutes – only a price ratio of 1:1 can prevail in equilibrium.

The RPT here can be calculated as $2xdx + 4ydy = 0 \quad \frac{dy}{dx} = -\frac{2x}{4y} = -\frac{x}{2y}$.

Setting $\frac{x}{2y} = \frac{1}{1} \quad x = 2y \quad 6y^2 = 150 \quad y = 5 \quad x = 10 \quad U = 15$

b. Suppose now that the production of good x is monopolized. The x-monopoly behaves as if it faced a demand curve for its product with an elasticity of demand of $e_{x,p_x} = -2$. How will marginal revenue for this monopoly relate to its product price? What does this imply about the ratio mr_x / p_y in this economy? (Hint: The ratio p_x / p_y must continue to be the value described in part a because of the nature of utility in this situation)

$$mr_x = p_x(1+1/e) = 0.5p_x \quad \frac{mr_x}{p_y} = \frac{0.5p_x}{p_y} = 0.5$$

c. What point on this economy's production possibility frontier will be chosen by profit-maximizing firms in the monopolized equilibrium?

Setting $RPT = \frac{mr_x}{p_y}$ (as required if the monopolist is to maximize profits) yields:

$$\frac{x}{2y} = 0.5 \quad x = y \quad 3x^2 = 150 \quad x = \sqrt{50} = y \quad U = 2\sqrt{50} = 10\sqrt{2} = 14.1$$

d. What is the deadweight loss from monopolization in this economy?

Deadweight loss is the loss in utility from monopolization which here is $15 - 14.1 = 0.9$

e. Another way to see the inefficiency of the monopoly equilibrium in this situation involves looking at the marginal productivities of labor in the two industries. What should the relationship between these marginal productivities be for efficiency? What is the actual relationship between them in the monopolized situation?

Values of marginal products of labor should be equal in the two markets. Initially this is the case: $l_x = 100 \quad l_y = 50 \quad VMP_x = p_x \frac{1}{2\sqrt{l_x}} = \frac{1}{20} = VMP_y = p_y \frac{.5}{2\sqrt{0.5l_y}} = \frac{1}{20}$

With Monopoly these are not equal; $l_x = 50 \quad l_y = 100 \quad VMP_x = \frac{1}{4\sqrt{2}} \neq VMP_y = \frac{1}{8\sqrt{2}}$.

Note with Monopoly $MRP_x = VMP_y$

3. The following questions are based on “Monopolistic Competition and Optimum Product Diversity” by A. K. Dixit and J. E. Stiglitz.

a. What is the primary conclusion from the Chamberlin model of imperfect competition that the authors of this paper are challenging?

Chamberlin hypothesized that monopolistically competitive markets would have “too much” product diversity. D+S show that is not the case once one takes account of the utility from diversity correctly.

b. The authors claim that market equilibrium in the model they have specified is given by combining equations 15 and 16. Explain intuitively what each of these equations mean and why they jointly describe the market equilibrium.

Equation 15 is just a statement of the profit-maximization rule that $MC = MR = P(1+1/e)$.

Equation 16 is the zero profit condition required with free entry. Hence the two equations fully describe the Chamberlin equilibrium in this model.

c. On page 300 at the start of section C the authors say “with economies of scale, the first best or unconstrained optimum requires pricing below average cost…” . Explain why there are economies of scale in this paper and why an optimum allocation would require pricing below average cost.

The discussion at the top on page 300 implies that the cost function here is $Cost = a + cx \quad a > 0$. In this case $mc = c \quad ac = \frac{a}{x} + c$. Hence marginal cost is constant, average cost is falling. This is one way of describing increasing returns.

With $mc < ac$ any attempt at marginal cost pricing will require $p = mc < ac$

d. After starting Section C with the quote mentioned above, the authors proceed to calculate what they term a “constrained optimum”. What do they mean by a constrained optimum and what do they show about such an optimum.

A constrained optimum is one is which each firm must earn zero profits. There are no subsidies. The authors show that the constrained optimum is precisely the Chamberlin equilibrium. That is, the Chamberlin outcome is the best that can be achieved, subject to the constraint that no firm is subsidized.

e. What conclusion do the authors reach when they come to consider an “unconstrained optimum”?

When subsidies are allowed, the authors show that the optimum has more firms than in the Chamberlin equilibrium. They do this by showing that each firm must produce less in the unconstrained equilibrium, but that utility must be higher. Hence (indirectly) there must be more firms. This contradicts Chamberlin’s conclusion.

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