Amherst College
Department of Economics
Economics 58
Fall 2008

Name__SOLUTIONS____

First Examination

There are three questions on this one-hour examination. The first is worth 40 points and the second and third are worth 30 points each.

1. (40 points) In class we looked in detail at the behavioral consequences of a simple utility function of the form $U(x,y) = x^{\alpha} y^{1-\alpha}$. This problem takes the dual approach to studying this function.

a. Set up the expenditure minimization problem for a person with this utility function. Then solve this constrained problem and use your solution to show that the expenditure function for this person is $E(p_x, p_y, U) = K^{-1} U p_x^{\alpha} p_y^{1-\alpha}$ where $K = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. (Hint: use the first order conditions to solve for *x* and *y*, then substitute into the utility function).

The expenditure minimization problem is:

$$\text{Minimize } E = p_x x + p_y y \quad s.t. \quad \overline{U} = U(x,y)$$

The Lagrangian and first order conditions are:

$$L = p_x x + p_y y + \lambda(\overline{U} - x^{\alpha} y^{1-\alpha})$$

$$\frac{\partial L}{\partial x} = p_x - \lambda\alpha(y/x)^{1-\alpha} = 0 \qquad \Rightarrow \quad \frac{p_x}{p_y} = \frac{\alpha}{1-\alpha}\frac{y}{x} \Rightarrow p_x x = \frac{\alpha}{1-\alpha} p_y y$$

$$\frac{\partial L}{\partial y} = p_y - \lambda(1-\alpha)(y/x)^{-\alpha} = 0$$

$$\frac{\partial L}{\partial \lambda} = \overline{U} - x^{\alpha} y^{1-\alpha} = 0$$

Since

$$E = p_x x + p_y y = \frac{\alpha}{1-\alpha} p_y y + p_y y = \frac{1}{1-\alpha} p_y y \quad \Rightarrow y = \frac{(1-\alpha)E}{p_y} \; and \; x = \frac{\alpha E}{p_x} \Rightarrow \overline{U} = KEp_x^{-\alpha} p_y^{-(1-\alpha}$$

$$E = K^{-1}\overline{U} p_x^{\alpha} p_y^{(1-\alpha)}$$

b. Use the envelope theorem to calculate the Hicksian demand function for good *x*. Describe intuitively why, in this case, this demand function must contain the variable p_y.

$x^c = \frac{\partial E}{\partial p_x} = \alpha K^{-1}\overline{U}p_x^{\alpha-1}p_y^{1-\alpha}$. This must contain the price of *y* because any change in the price ratio of the two goods will lead to a substitution effect reflected in a move along the indifference curve.

c. What is the Hicksian price elasticity of demand implied by the function calculated in part B? Use this value together with the value implied by the Marshallian demand function for good *x* (i.e. $x = \alpha I / p_x$) to show that the own-price Slutsky Equation holds here in elasticity form.

Remember – Exponents are elasticities. $e_{x,p_x} = -1, e_{x^c,p_x} = \alpha - 1, s_x = \alpha, e_{x,I} = 1$.

The Slutsky Equation is $e_{x,p_x} = e_{x^c,p_x} - s_x e_{x,I}$ which here is $-1 = \alpha - 1 - \alpha(1)$.

d. Without doing any math, describe how you would go about deriving the Marshallian demand function given above from parts a and b of this problem.

Solve for the indirect utility function from the expenditure function. Then use Roy's Identity which shows $x = \frac{-\partial V/\partial p_x}{\partial V/\partial I}$.

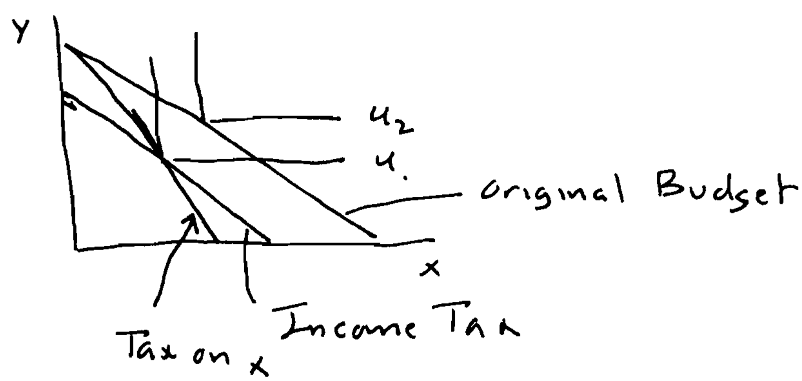
2 (30 points) This problem concerns the excess burden of a tax.

a. Define "the lump-sum principle of taxation" and how it relates to the excess burden notion.

The lump sum principle states that a tax on a specific item will cause a greater loss in consumer surplus than will a tax on income that raises the same revenue. The extra loss of consumer surplus is called the "excess burden" of the single good tax.

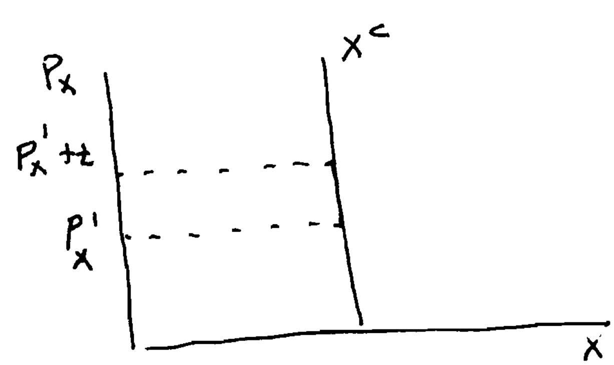
b. Use an indifference curve diagram to show that if an individual consumes only two goods in fixed proportions there is no excess burden from taxing only one of the goods (say good x) rather than taxing income.

Since indifference curves here are L-shaped, the two taxes result in the same choices.



c. Use the Hicksian demand curve for good x to make the same argument as in part b.

The Hicksian demand curve shows only substitution effects, which do not exist when the goods are consumed in fixed proportions. Hence the Hicksian demand curve is vertical and there is no excess burden triangle from taxing this good.



3. (30 points) This problem involves the paper "Sources of Bias and Solutions to Bian in the Consumer Price Index" by Jerry Hausman.

a. On page 27 of the paper Hausman shows how to approximate the total consumer surplus associated with any good by the equation $CS = (0.5p_n q_n)/\alpha_n$ where α_n is the absolute value of the price elasticity of demand for the good. Use a linear demand curve to show why this approximation holds.

Using Hausman's graph, $\alpha_n = \frac{-\Delta q}{\Delta p}\frac{p_n}{q_n} = \frac{-q_n}{p^* - p_n}\frac{p_n}{q_n} = \frac{p_n}{p^* - p_n}$.

$CS = 0.5(p^* - p_n)q_n = 0.5p_n q_n / \alpha_n$.

b. Throughout the appendix to the Hausman article, the author differentiates between "first order" and "second order" sources of bias. What does he mean by these terms? Why is the familiar "substitution bias" only a second order bias?

These refer to the first and second terms in the Taylor Expansion of the expenditure function. The first order terms involve $\partial E/\partial p_x = x^c$, so they involve changes in price times quantities. The second order terms involve $\partial^2 E/\partial p_x^2 = \partial x^c/\partial p_x$ so they involve changes in price times changes in quantities.

c. Hausman uses Figure 3 to describe the biases introduced by quality improvements. Explain what he is trying to do here. In your explanation you may also wish to refer to his mathematical treatment in Equations 6 and 7 in the appendix.

A higher quality shifts the demand curve outward. So any price index must account for the increase in consumer surplus under this new demand curve, no matter what happens to the price of the good. Hausman's math is devoted to showing that a lower bound for this added consumer surplus is given by half the increase in quantity demanded times $p^* - p_n$. That is, by the area of a triangle with a base given by the expansion in quantity demanded and height given by the gap between the virtual and actual price.