Math 13 Fall 2008: Final Exam

Instructions: There are 10 questions on this exam of which you must do 8. Each problem is scored out of 8 points for a total of 64 points. You may not use any outside materials (eg. notes or calculators). You have 3 hours to complete this exam. Remember to fully justify your answers. Mark clearly in your blue book which problems are to be graded.

Problem 1. A particle moves through space with position $r(t) = \left\langle t, t^2, \frac{4}{3}t^{3/2} \right\rangle$.

(1) Find the symmetric and parametric equations for the tangent line at $\left(1, 1, \frac{4}{3}\right)$.
(2) Find the curvature at $\left(1, 1, \frac{4}{3}\right)$.

Problem 2. For the two planes

$$S_1 : 4x + y - z = 4 \quad \text{and} \quad S_2 : 2x + 2y + z = 3.$$

(1) Find the line of intersection of S_1 and S_2.
(2) Find the angle between S_1 and S_2.

Problem 3. Determine the following two limits (if they exist).

(1) $\lim_{(x,y)\to(0,0)} \frac{5x^2y}{x^2+y^2}$.
(2) $\lim_{(x,y)\to(0,0)} \frac{5x^2y}{x^4+y^2}$.

Problem 4. The plane $8x - 8y - 2z = C$ is tangent to the surface $z = x^2 + 2y^2$ at a certain point (x_0, y_0, z_0). Find (x_0, y_0, z_0) and the constant C.

Problem 5. Classify the critical points for $f(x, y) = x^4 + y^4 - 4xy + 1$.

Problem 6. Find the point(s) on the surface $2x^2+3y^2+6z^2 = 36$ the sum of whose coordinates is maximal.

Problem 7. Find the moment of inertia about the x-axis for the triangle bounded by $3x+4y = 24$, $x = 0$, and $y = 0$ and having constant density 1.

Problem 8. Find the volume bounded by $y = x^2 + 2z^2$ and $y = 4 - x^2$.

Problem 9. Evaluate

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{3/2} dz dy dx.$$

Problem 10. A closed path C is traversed counter-clockwise beginning at $(-1, -1)$ and moving to $(2, 2)$ along the curve $y = x^2 - 2$ then returning to the point $(-1, -1)$ along the curve $y = x$. Find

$$\int_C -xy dx + \frac{x^2}{2} dy$$

in two ways.

(1) Parameterizing the curves and computing the line integral directly.
(2) Using Green's Theorem.