Physics 16 Problem Set 7 Solutions

Y&F Problems

7.14. **IDENTIFY:** Only gravity does work, so apply Eq.(7.4). Use $\sum \vec{F} = m\vec{a}$ to calculate the tension.

SET UP: Let $y = 0$ at the bottom of the arc. Let point 1 be when the string makes a $45°$ angle with the vertical and point 2 be where the string is vertical. The rock moves in an arc of a circle, so it has radial acceleration $a_{\text{rad}} = v^2/r$

EXECUTE: **(a)** At the top of the swing, when the kinetic energy is zero, the potential energy (with respect to the bottom of the circular arc) is $mgl(1-\cos\theta)$, where l is the length of the string and θ is the angle the string makes with the vertical. At the bottom of the swing, this potential energy has become kinetic energy, so $mgl(1-\cos\theta) = \frac{1}{2}mv^2$, or

$v = \sqrt{2gl(1-\cos\theta)} = \sqrt{2(9.80\ \text{m/s}^2)(0.80\ \text{m})(1-\cos 45°)} = 2.1\ \text{m/s}$.

(b) At $45°$ from the vertical, the speed is zero, and there is no radial acceleration; the tension is equal to the radial component of the weight, or $mg\cos\theta = (0.12\ \text{kg})(9.80\ \text{m/s}^2)\cos 45° = 0.83\ \text{N}$.

(c) At the bottom of the circle, the tension is the sum of the weight and the mass times the radial acceleration,

$$mg + mv_2^2/l = mg(1 + 2(1-\cos 45°)) = 1.9\ \text{N}$$

EVALUATE: When the string passes through the vertical, the tension is greater than the weight because the acceleration is upward.

7.38. **IDENTIFY:** Apply Eq.(7.16).

SET UP: $\dfrac{dU}{dx}$ is the slope of the U versus x graph.

EXECUTE: **(a)** Considering only forces in the x-direction, $F_x = -\dfrac{dU}{dx}$ and so the force is zero when the slope of the U *vs* x graph is zero, at points b and d.

(b) Point b is at a potential minimum; to move it away from b would require an input of energy, so this point is stable.

(c) Moving away from point d involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so d is an unstable point.

EVALUATE: At point b, F_x is negative when the marble is displaced slightly to the right and F_x is positive when the marble is displaced slightly to the left, the force is a restoring force, and the equilibrium is stable. At point d, a small displacement in either direction produces a force directed away from d and the equilibrium is unstable.

7.42. **IDENTIFY:** Apply Eq.(7.14).

SET UP: Only the spring force and gravity do work, so $W_{\text{other}} = 0$. Let $y = 0$ at the horizontal surface.

EXECUTE: **(a)** Equating the potential energy stored in the spring to the block's kinetic energy,

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, or $v = \sqrt{\dfrac{k}{m}}x = \sqrt{\dfrac{400\ \text{N/m}}{2.00\ \text{kg}}}(0.220\ \text{m}) = 3.11\ \text{m/s}$.

(b) Using energy methods directly, the initial potential energy of the spring equals the final gravitational potential energy, $\frac{1}{2}kx^2 = mgL\sin\theta$, or

$$L = \frac{\frac{1}{2}kx^2}{mg\sin\theta} = \frac{\frac{1}{2}(400\ \text{N/m})(0.220\ \text{m})^2}{(2.00\ \text{kg})(9.80\ \text{m/s}^2)\sin 37.0°} = 0.821\ \text{m}.$$

EVALUATE: The total energy of the system is constant. Initially it is all elastic potential energy stored in the spring, then it is all kinetic energy and finally it is all gravitational potential energy.

7.63. **IDENTIFY** and **SET UP:** First apply $\sum \vec{F} = m\vec{a}$ to the skier.

Find the angle α where the normal force becomes zero, in terms of the speed v_2 at this point. Then apply the work-energy theorem to the motion of the skier to obtain another equation that relates v_2 and α. Solve these two equations for α.

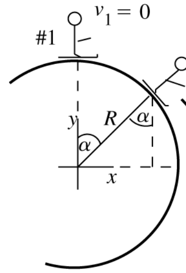

Figure 7.63a

Let point 2 be where the skier loses contact with the snowball, as sketched in Figure 7.63a Loses contact implies $n \to 0$.

$y_1 = R, \quad y_2 = R\cos\alpha$

First, analyze the forces on the skier when she is at point 2. The free-body diagram is given in Figure 7.63b. For this use coordinates that are in the tangential and radial directions. The skier moves in an arc of a circle, so her acceleration is $a_{\text{rad}} = v^2/R$, directed in towards the center of the snowball.

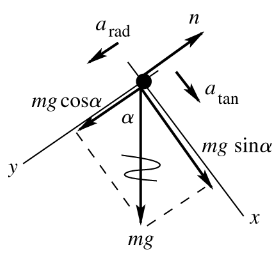

Figure 7.63b

EXECUTE: $\sum F_y = ma_y$

$mg\cos\alpha - n = mv_2^2/R$

But $n = 0$ so $mg\cos\alpha = mv_2^2/R$

$v_2^2 = Rg\cos\alpha$

Now use conservation of energy to get another equation relating v_2 to α:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the skier is gravity, so $W_{\text{other}} = 0$.

$K_1 = 0, \quad K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1 = mgR, \quad U_2 = mgy_2 = mgR\cos\alpha$

Then $mgR = \frac{1}{2}mv_2^2 + mgR\cos\alpha$

$v_2^2 = 2gR(1 - \cos\alpha)$

Combine this with the $\sum F_y = ma_y$ equation:

$Rg\cos\alpha = 2gR(1 - \cos\alpha)$

$\cos\alpha = 2 - 2\cos\alpha$

$3\cos\alpha = 2$ so $\cos\alpha = 2/3$ and $\alpha = 48.2°$

EVALUATE: She speeds up and her a_{rad} increases as she loses gravitational potential energy. She loses contact when she is going so fast that the radially inward component of her weight isn't large enough to keep her in the circular path. Note that α where she loses contact does not depend on her mass or on the radius of the snowball.

7.75. **(a) IDENTIFY** and **SET UP:** Apply $K_A + U_A + W_{\text{other}} = K_B + U_B$ to the motion from A to B.

EXECUTE: $K_A = 0, \quad K_B = \frac{1}{2}mv_B^2$

$U_A = 0, \quad U_B = U_{\text{el},B} = \frac{1}{2}kx_B^2$, where $x_B = 0.25$ m

$W_{\text{other}} = W_F = Fx_B$

Thus $Fx_B = \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$. (The work done by F goes partly to the potential energy of the stretched spring and partly to the kinetic energy of the block.)

$Fx_B = (20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ and $\frac{1}{2}kx_B^2 = \frac{1}{2}(40.0\text{ N/m})(0.25\text{ m})^2 = 1.25\text{ J}$

Thus $5.0\text{ J} = \frac{1}{2}mv_B^2 + 1.25\text{ J}$ and $v_B = \sqrt{\dfrac{2(3.75\text{ J})}{0.500\text{ kg}}} = 3.87\text{ m/s}$

(b) IDENTIFY: Apply Eq.(7.15) to the motion of the block. Let point C be where the block is closest to the wall. When the block is at point C the spring is compressed an amount $|x_C|$, so the block is $0.60\text{ m} - |x_C|$ from the wall, and the distance between B and C is $x_B + |x_C|$.

SET UP: The motion from A to B to C is described in Figure 7.75.

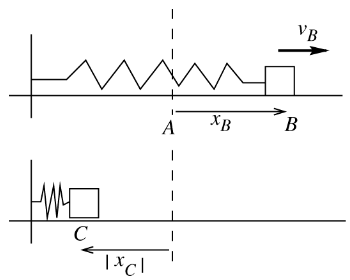

Figure 7.75

$K_B + U_B + W_{\text{other}} = K_C + U_C$

EXECUTE: $W_{\text{other}} = 0$

$K_B = \frac{1}{2}mv_B^2 = 5.0\text{ J} - 1.25\text{ J} = 3.75\text{ J}$ (from part (a))

$U_B = \frac{1}{2}kx_B^2 = 1.25\text{ J}$

$K_C = 0$ (instantaneously at rest at point closest to wall)

$U_C = \frac{1}{2}k|x_C|^2$

Thus $3.75\text{ J} + 1.25\text{ J} = \frac{1}{2}k|x_C|^2$

$$|x_C| = \sqrt{\frac{2(5.0\text{ J})}{40.0\text{ N/m}}} = 0.50\text{ m}$$

The distance of the block from the wall is $0.60\text{ m} - 0.50\text{ m} = 0.10\text{ m}$.

EVALUATE: The work $(20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ done by F puts 5.0 J of mechanical energy into the system. No mechanical energy is taken away by friction, so the total energy at points B and C is 5.0 J.