Amherst College
Department of Economics
Economics 58
Fall 2010

Name__________________

Third Hour Test
Solutions

There are three questions on this one-hour examination. Each is of equal weight in grading of the examination.

1. Exponential demand and supply functions are especially easy to work with because they have constant elasticities.

a. Suppose that the demand for a product is given by $Q = ap^b$. Prove that the price elasticity of demand here is given by the parameter *b*.

$$e = \frac{\partial Q}{\partial p}\frac{p}{Q} = bap^{b-1}\frac{p}{ap^b} = b$$

b. Consider a market where demand is given by $Q_D = 16p^{-2.5}$ and supply by $Q_S = 2p^{0.5}$. Calculate the equilibrium price and quantity in this market.

$$16p^{-2.5} = 2p^{0.5} \Rightarrow 8 = p^3 \Rightarrow p = 2, Q = 2\sqrt{2}.$$

c. Suppose that the demand for this product increased by ten percent. Provide an intuitive answer for how much equilibrium price would rise.

Each 1% price rise reduces demand by 2.5% and increases supply by 0.5%. Hence to get a 10% reduction in excess demand requires a 10/3% increase in price.

d. Show how you would check your result mathematically. You do not need to make an explicit calculation, however.

$$1.1(16p^{-2.5}) = 2p^{0.5} \Rightarrow (1.1)^{1/3} p^* \Rightarrow (1.1)^{1/3} \approx 1.032$$

e. How would you change the elasticities in this problem to make the price rise described in part c larger?

If either demand or supply were less elastic, price rise would be greater.

2. Suppose that there are only two goods in an economy: x and y. The production possibility frontier for this economy is given by $x^2 + 2y^2 = 150$.

a. Suppose that consumers in this economy have preferences that can be represented by the aggregate utility function $U(x, y) = x + y$. Given this utility function, what is the only price ratio that can prevail in equilibrium? At this price ratio how much of each good will be produced? What will utility be? (Hint: First show that the slope of the production possibility frontier in this problem is given by $\frac{dy}{dx} = -\frac{x}{2y}$)

The only price ratio that can prevail is 1.0 because preferences imply the goods are perfect substitutes.

Differentiation of the ppf wrt x yields $2x + 4y\frac{dy}{dx} = 0$ or $\frac{dy}{dx} = \frac{-2x}{4y} = \frac{-x}{2y}$.

Set this equal to -1.0. Yields x = 2y so $6y^2 = 150, y = 5, x = 10, U = 15$.

b. Suppose now that the production of good x is monopolized. The x-monopoly behaves as if it faced a demand curve for its product with an elasticity of demand of $e_{x,p_x} = -2$. How will marginal revenue for this monopoly relate to its product price? What does this imply about the ratio mr_x / p_y in this economy? (Hint: The ratio p_x / p_y must continue to be the value described in part a because of the nature of utility in this situation)

$mr_x = p_x(1 + \frac{1}{e}) = 0.5p_x$. This is what producers of x respond to. So set

$$\frac{0.5p_x}{p_y} = 0.5 = \frac{x}{2y} \Rightarrow x = y$$

c. What point on this economy's production possibility frontier will be chosen by profit-maximizing firms in the monopolized equilibrium?

If $x = y,\ x = y = \sqrt{50} = 5\sqrt{2}, U = 10\sqrt{2} \approx 14.14$

d. What is the deadweight loss from monopolization in this economy?

DWL = 15 – 14.14 = 0.86 units of utility. This loss arises from producing the wrong combination of outputs, not from being inside the ppf.

3. This problem concerns taxation of a monopoly. All questions must be answered mathematically, not graphically. Throughout you are to assume that the monopoly produces at a constant marginal cost of *c*. You should also assume that the monopoly faces a demand curve characterized by a constant elasticity of demand.

a. Show that the profit-maximizing price for a monopoly is given by $p_m = c\left(\frac{e}{1+e}\right)$ where *e* is the (constant) elasticity of demand. This result should be used in parts c and d of this problem.

$$\pi(q) = pq - cq \Rightarrow \frac{d\pi}{dq} = p + q\left(\frac{dp}{dq}\right) - c = 0$$

$$p\left(1+\frac{1}{e}\right) = c \Rightarrow p_m = c\left(\frac{e}{e+1}\right) > c \text{ because } \left(\frac{e}{e+1}\right) > 1$$

Because monopoly must produce where demand is elastic (e < -1).

b. Show that the imposition of a proportional tax on the monopolist's profits will not cause the monopoly to change its desired level of output nor the price it charges.

After-tax profits are $(1-t)\pi(q)$. Maximizing this expression yields the same result as maximizing profits without the tax. So this tax does not change price.

c. Show that the imposition of a per-unit tax of *t* will cause the monopoly to raise its price by more than *t*. Explain this result intuitively.

Now marginal cost is *c* + *t* so $p_m^{'} = c\left(\frac{e}{e+1}\right) + t\left(\frac{e}{e+1}\right) > p_m + t$ because $\left(\frac{e}{e+1}\right) > 1.$

d. Show that the imposition of a proportional tax on the price charged by a monopolist [which thereby reduces the price received to $p_m(1-t)$] will cause price to rise by the same proportional amount (for a small *t*).

$$(1-t)p_m^{'} = c\left(\frac{e}{1+e}\right) = \frac{p_m}{1-t} \approx (1+t)p_m$$

e. By comparing parts b - d, what do you conclude about the most efficient ways to tax a monopoly? How would you rank b, c, and d? Explain your results intuitively.

Among taxes that collect the same revenue, the ones that raise price least would cause the least increase in deadweight loss. Hence the ranking here would be b, d, c.