Bowl	1	2	3
White	1	2	3
Red	4	3	2

1. Three bowls are labeled 1, 2, and 3, respectively. Bowl *i* contains *i* white and 5-*i* red balls. In an experiment, a bowl is randomly selected from the set of three bowls. Then, 3 balls are randomly selected without replacement from the contents of the selected bowl.

a. Given that bowl 1 was NOT selected, what is the probability of drawing exactly 2 red balls?

$$P(2 \text{ red} \mid \text{Not Bowl 1}) = \frac{1}{2} \frac{\binom{3}{2}\binom{2}{1}}{\binom{5}{3}} + \frac{1}{2} \frac{\binom{2}{2}\binom{3}{1}}{\binom{5}{3}}$$

$$= \frac{1}{20}\left(6 + 3\right) = \frac{9}{20}$$

b. What is the probability that exactly 2 red balls are drawn?

$$P(\text{exactly 2 red}) = \frac{1}{3}\left(\frac{1}{10}\right)\left(\binom{4}{2}\binom{1}{1} + \binom{3}{2}\binom{2}{1} + \binom{2}{2}\binom{3}{1}\right)$$

$$= \frac{1}{30}\left(6 + 6 + 3\right) = \frac{1}{2}$$

c. Given that exactly 2 red balls were drawn, what is the probability that bowl 3 was selected?

$$P\left(\text{Bowl 3} \mid \text{exactly 2 red}\right) = \frac{P(\text{Bowl 3 and 2 red})}{P(2 \text{ red})}$$

$$= \frac{P(2 \text{ red} \mid 3^{rd} \text{ bowl}) \, P(\text{Bowl 3})}{P(2 \text{ red})} = \frac{\binom{2}{2}\binom{3}{1}\left(\frac{1}{3}\right)\left(\frac{1}{10}\right)}{\frac{1}{2}}$$

$$= \frac{3 \cdot \frac{1}{3}}{\frac{10}{2}} = \frac{1}{5}$$