Problem Session 3 for Math 29: M and M's (Mgfs and Markov Chains) and More

1. Eating Habits.
Three of your friends have very different preferences for restaurants. Assuming that they only remember the most recent restaurant and decide where to go next based on that with fixed transition probabilities, we would be dealing with a Markov chain with stationary transition probabilities for say, 3 states (restaurants). Here are the transition matrices for your three friends (apologies for the size but you get the idea):

$$P_1 = \begin{bmatrix} .5 & .5 & 0 \\ 0 & .5 & .5 \\ .5 & 0 & .5 \end{bmatrix}, \; P_2 = \begin{bmatrix} 0 & .5 & .5 \\ .5 & 0 & .5 \\ .5 & .5 & 0 \end{bmatrix}, \; P_3 = \begin{bmatrix} (1/3) & (1/3) & (1/3) \\ (1/2) & (1/6) & (1/3) \\ (7/12) & (1/3) & (1/12) \end{bmatrix}$$

Describe briefly in words, the preferences of your 3 friends. What is special (if anything) about the preferences of Friend 2? Do any of these transition matrices have absorbing states?

What is the probability you go to the third restaurant for your next time out if your last time out was at the second restaurant if you are with Friend 2?

Assuming you start at the three restaurants with equal probability, and focusing just on Friend 3, find the probability your second night out is at restaurant 2.

Finally, find the probability your third night out is at restaurant 3 given the first night out was at restaurant 1 (you can do this without P_3^2) assuming you are still with Friend 3.

Set up equations to solve (do not solve now, just set them up) for the stationary distribution of restaurant choices if you are with Friend 3. (If you decide to solve later, I have a solution you can check against).

2. Phone Calls in the Shower.
Suppose you get phone calls in the evening and that you can treat the number you get per hour as Poisson with λ=2 per hour.
a. If you take a 10 minute shower, what is the probability the phone rings during that time?
b. What is the longest shower you can take if you want the probability of no calls during that time to be at least .5?

3. Jury Duty.

A large club has to select a jury of 6 members to oversee an internal proceeding against another member. Out of all club members, 20 are considered eligible jurors due to eligibility constraints. Of the 20, 8 are women and 12 are men. Jury members are supposed to be chosen at random from all eligible jurors.

a. What distribution does X = number of women that end up on the jury have?

b. If only one woman ends up on the jury, i.e. X=1, do you doubt the randomness of the jury selection? (Think about relevant probabilities - including more extreme probabilities).

c. If jury selection is random, what are the mean and variance of the number of women selected for jury duty?

4.The length of time (hours) it takes students to complete an exam is a random variable with density given by $f(x) = cx^2 + x, 0 \leq x \leq 1$.
a. Find the value of c to make this a valid density function.

b. What is the expected amount of time it takes the students to complete the exam?

c. Find the CDF. Use the cdf to determine the fraction of students who can complete the exam in 30 minutes.

5. a. Derive the mgf for a Poisson RV.
b. What distribution does a RV have if its mgf is $M_X(t) = (.6e^t + .4)^3$?
c. What distribution does a RV have if its mgf is $M_X(t) = e^{6(e^t-1)}$?
d. What distribution does a RV have if its mgf is $M_X(t) = \frac{.2e^t}{1-.8e^t}$?