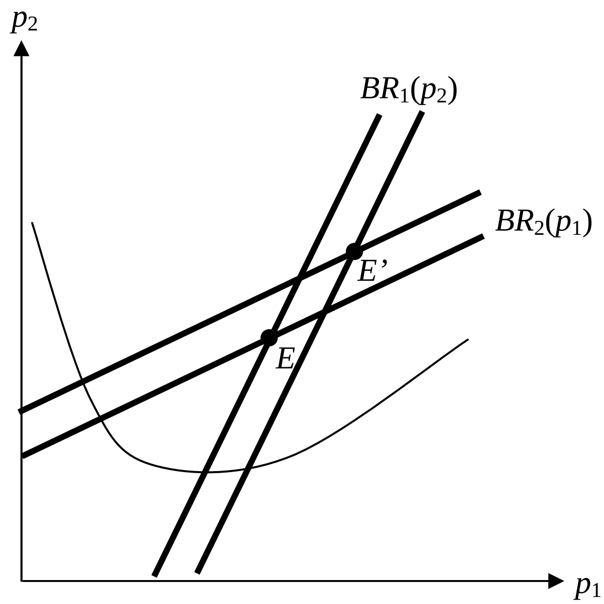
Amherst College
Economics 58
Fall 2010

Solutions to PS # 9

1. a. Firm 1 maximizes $p_1(1-p_1+bp_2)$ with respect to p_1, yielding the first-order condition $1-2p_1+bp_2=0$ and best-response function $p_1=(1+bp_2)/2$. Symmetrically, $p_2=(1+bp_1)/2$. Solving simultaneously, $p_1^*=p_2^*=1/(2-b)$.

b. $q_i^*=(1-2b)/(2-b)$. $\pi_i^*=1/(2-b)^2$.

c. An increase in *b* shifts the equilibrium from *E* to *E'*.



2. Inverse elasticity rule

a. Equation 15.2 can be rearranged as follows:

$$\frac{P-C'}{P}=\frac{-P'q_i}{P}=\frac{-dP/dq_i\cdot q_i}{P}=\frac{1}{|\varepsilon_{q_i,P}|},$$

where $\varepsilon_{q_i,P}$ is the elasticity of demand with respect to firm i's output. The second equality uses the fact that $P' = dP/dQ = dP/dq_i$. Using this same fact, we can also rearrange Equation 15.2 as

$$\frac{P-C'}{P} = \frac{-P'q_i}{P} = \frac{-dP/dQ \cdot q_i}{P} = \left(\frac{-dP/dQ \cdot Q}{P}\right)\left(\frac{q_i}{Q}\right) = \frac{s_i}{|\varepsilon_{Q,P}|}.$$

3. Competition on a circle

a. This is the indifference condition for a consumer located distance x from firm *i*: the generalized cost (price plus transportation cost) of buying from *i* equals the generalized cost of buying from the closest alternative firm.

b. Solving the displayed equation in part (a) of the statement of the problem for *x*, we obtain $x = (1/2n) + (p^* - p)/2t$. The firm's profit equals $(p-c)2x$. Substituting for x, taking the first-order condition with respect to p, and solving for p gives the best response $p = (p^* + c + t/n)/2$.

c. Setting $p = p^*$ and solving for p^* gives the specified answer. Equilibrium price is increasing in cost and the degree of differentiation, given by the transportation cost and the spacing between firms (depending on their numbers).

d. Substituting $p = p^* = c + t/n$ into the profit function gives the specified answer.

e. Setting $t/n^2 - K = 0$ and solving for n yields $n^* = \sqrt{t/K}$.

f. Total transportation costs equal the number of half-segments between firms, $2n$, times the transportation costs of consumers on the half segment, $\int_0^{1/2n} tx\,dx = t/8n^2$. Total fixed cost equal nF. The number of firms minimizing the sum of the two is $n^{**} = (1/2)\sqrt{t/K}$