

Chapter 8 Review with Applications for Math 30: Hypothesis Testing

1. A manufacturer claims that at least 70 percent of the public prefers their product over the competition. A consumer group is suspicious of the 70 percent claim and wishes to investigate the related hypotheses: $H_0 : p \leq .7$ vs. $H_1 : p > .7$. Identify a UMP test of these hypotheses with $\alpha_0 < .1$ for a random sample of size 20. Specify the exact level α_0 for your test. How do you know that a UMP test exists?

2. A researcher is only able to draw ONE observation in order to evaluate the following hypotheses:
 $H_0 : f(x) = f_0(x)$ vs. $H_1 : f(x) = f_1(x)$ where

$$f_0(x) = \frac{1}{2}, -1 \leq x \leq 1; \text{ 0, o.w. and } f_1(x) = \frac{3}{2}x^2, -1 \leq x \leq 1; \text{ 0, o.w.}$$

Identify a test procedure where $2\alpha(\delta) + \beta(\delta)$ is a minimum. Express the minimum value of $2\alpha(\delta) + \beta(\delta)$ in terms of integrals that could be solved. (Do not actually solve unless you have time after finishing problem 3.) Recall that $\alpha(\delta)$ is the probability of rejecting the null hypothesis when the null is true (probability of a Type I Error) and $\beta(\delta)$ is the probability of not rejecting the null when the alternative is true (probability of a Type II Error).

3. A pharmaceutical manufacturer obtains a component for a drug from two different suppliers. However, there is some concern over the level of impurities in the component. Using the sample information below, determine via hypothesis tests whether there is a difference in the mean level of impurities between the two suppliers as well as if there is a difference in the variability of the level of impurities. You may perform the hypothesis tests in whichever order makes the most sense to you. Which supplier would you choose? Assume each suppliers population of impurity levels is normally distributed with unknown mean and unknown variance, and those variances may not be equal. Use a .05 level test for both tests, but note this raises the overall level when both tests are considered together.

Supplier A: $\bar{X} = 1.89, \sigma_1'^2 = .273, m = 10$
Supplier B: $\bar{Y} = 1.85, \sigma_2'^2 = .094, n = 10$