**Astronomy 11 Spring 2009**

**Fourth Problem set**

**Due at the beginning of class Friday, May 8**

*Throughout this problem set – and all the others – let’s do calculations to only three figure accuracy: never write down a result with more than three significant figures.*

***If you would like this set returned to you, attach a stamped, self-addressed envelope.***

(1) A Message

Below is a possible message we might send to an exterrestrial civilization. In it, ‘L’ stands for a long burst of emission and ‘S’ for a short one.

In class, we got started translating the message. Here is the translation, as far as we got:

1< 2 < 3 < 4 < 5 < 6 < 7 < 8 < 9

LSLLSLLLSLLLLSLLLLLSLLLLLLSLLLLLLLSLLLLLLLLSLLLLLLLLLSSSSSSS

9 > 8 > 7 > 6 > 5 > 4 > 3 > 2 >

LLLLLLLLLSSLLLLLLLLSSLLLLLLLSSLLLLLLSSLLLLLSSLLLLSSLLLSSLLSS

1

LSSSSSSSLSLLSSSSSSSLSLLLLLSSSSSSSLLSLLLSSSSSSSLLSLLLLLSSSSSSSL

LLSLLLLLSSSSSSSLLLLSLLLLLSSSSSSSLLLSSLSSSSSSSLLLLLSSLSSSSSSSL

LLSSLLSSSSSSSLLLLLSSLLSSSSSSSLLLLLSSLLLSSSSSSSLLLLLSSLLLLSSSS

SSSLSSSLSSSSSSSLLSSSLLSSSSSSSLLLSSSLLLSSSSSSSLLLLSSSLLLLSSSSSS

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LLLLLLLLSSSSSSSLLLLLSSSSSLSSSLLLLSSSSSSSLLLLLSSSSSLLSSSLLLSSS

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SLLLSSSSSSLLSSSSSSLSSSSSSSLLLSSSSLLLLLSSSSSSLLLLSSSSLLLLLSSSSS

SSLLLSSSSLLLLLSSSSSSLLSSSSLLLLLSSSSSSS

* Continue the process, and translate the rest of the message. As I have, write your translation on this page, with each segment just above the relevant part of the message.

(2) A VERY Big Engineering Project

In class we discussed the possibility that our technological civilization might survive a very long time. If we look back on the progress of technology over the past decade, we realize how hard it is to predict what might be possible if we have millions of years before us. But let us try to imagine a very ambitious engineering project. In what follows, we will let our imagination run wild – but we will be very careful to stick to the laws of nature as we know them. Our philosophy will be that if something does not actually violate a law of nature, then it is possible.

The project we envision will be one designed to give humanity more living room, and more solar energy. We will do this by taking apart the entire planet Earth, and first assembling it into a long thin stip of length d, width h and thickness t:

t

h

d

And then bending it into a ring entirely surrounding the sun:

length d all the way around

h

Radius of ring R = 1 Astronomical Unit

(I’m not good enough at drawing to indicate the thickness t, but you get the point.)

We will live on the inside surface of the ring. If we make the radius of the ring equal to1 Astronomical Unit, the temperature on this inside surface will pretty much be the same as that of our Earth, which is what we want.

We will have built an artificial world – just like the one that appears in the game “halo.” Because the ring has a far greater surface area than the Earth, it will give us more room to live, so that the ring will support a far greater population than our poor old Earth does right now. Furthermore, remember that half of our Earth is always in darkness, and lots of the Earth is oriented at a slant angle to the incoming sunlight, making it an inefficient collector of solar energy. But notice that in our new artificial world, the entire inside surface of the ring faces the sun, making the ring a perfect collector of solar power. So humanity will have a far greater source of energy than it does now.

Our goal in this problem is to explore various aspects of this project.

First, let us ask how thick the ring can be. Let us suppose that, once we have built it, we will have used up all the matter in the entire Earth. Let us also suppose that the ring will be made of the same sort of matter – namely, rock – as the Earth. In that case, the volume of matter making up the ring must equal the volume of the Earth.

* Suppose we make the ring’s width h equal to 100 miles. Given that the volume of such a ring is just that of the strip we began with (before bending it into a ring), find the ring’s thickness ‘t.’ Express your answer in miles to get a feel for it.
* Find the surface area of the inside surface of the ring.
* Find R, defined to be the ratio between this area and the surface area of our Earth.

The present population of the Earth is about 6.7 billion. Suppose the number of people per acre in this new world is the same as now: then R is also the ratio between the new world’s population and the present population.

* How many people can this world support? Express your answer in billions of people to get a feel for it.

Now let us think about “artificial gravity,” which will come from the rotation of the ring. If we set the ring rotating with velocity V, then everything on its inside surface will be moving with velocity V in a circle of radius R about the Sun. So everything will experience a centripetal acceleration

Acent = V2 / R

If we set this equal to the acceleration of gravity we experience right now, which is 980 cm / sec2, then everyone living on the inside surface of the ring will experience an “artificial gravity” which feels like the real gravity we experience.

* Find the velocity V which we must give the ring in order for this to be so. Express it in millions of miles per hour to get a feel for it.

Now let us ask how much energy will be required to bring the ring up to this speed.

* Remembering the formula for kinetic energy of a body of mass M moving with velocity V [K.E. = (1/2) M V2] calculate the energy required.

Where are we going to get all that energy? Let us propose that we will get it from solar power. Before taking apart the Earth, we will accumulate the required energy by coating the whole Earth with solar power collectors, and trapping every bit of solar energy that falls upon it. In the second problem set you calculated how much energy falls upon the Earth per second: recall that the answer you got was 1.74 X 10  24  ergs/sec.

* At this rate, how long would it take to accumulate the required amount of energy? Express your answer in billions of years to get a feel for it.

Your answer shows that this is going to take a long time. Let us try to imagine a faster way. It involves a two-step process. First we will completely surround the Sun with a gigantic spherical solar collector, trapping all the energy the Sun emits. Then we will use this accumulated energy to set our artificial world spinning.

* At this far greater rate of trapping solar power, how long would it take to accumulate the required amount of energy? Express your answer in years to get a feel for it.

You may feel that this is all crazy. I myself do not agree: if we have millions of years to do things, the most amazing things are possible. And here’s another point: this may have been done already – not by us, but by some alien civilization. Perhaps we should search for exterrestrial civilizations, not by looking for their signals, but by looking for such rings.

(3) A “Timing” Probability

In class we discussed the “timing probability:” the probability that the communicating phase of an alien civilization coincides with now. Here’s a diagram:

T star

Tcommunicating

Star forms the ability to Now the ability to Star ‘dies’

communicate communicate

begins ends

Although I have indicated a particular time for “now” in this diagram, in reality “now” can fall at any time. The probability that “now” falls at some point within the communicating phase of the alien civilization is

Ptiming = Tcommunicating / T star

Here’s a problem that uses exactly the same logic:

Mary regularly goes to bed at midnight and gets up at 8 in the morning. But she is an insomniac. Every night she wakes up for half an hour. But the times at which she awakens are random: sometimes she awakens after only a few minutes of sleep, at other times just before she finally climbs out of bed, and at other times she is awake in the middle of the night.

* What is the probability that she is awake at 3:15 AM?
* Suppose you yourself awaken at random times during the night – but only for an instant, after which you fall right back to sleep again. What is the probability that, the moment you awaken, she is awake?
* How many nights pass before there is a pretty good chance that (a) Mary is awake at 3:15 AM, and (b) you find her awake when you awaken?
* Would your answers to the above questions be different if, when you awakened, you did not fall to sleep immediately, but rather stayed awake for a while?

For each question, carefully explain your reasoning: a quick and glib answer will get no credit whatsoever.

(4) Geosynchronous Satellites

If you use a satellite dish for TV reception, that dish is pointing at a geosynchronous communications satellite. These satellites appear to hover motionlessly above the Earth. Of course they do not: if they really were motionless, they would simply fall back down. In reality, they are orbiting about the Earth once every 24 hours.

* In class we developed a formula for the velocity of a satellite in a circular orbit of radius R about an object of mass M: Vorbit = √GM/R. Use it to find a formula for the length of time it takes a satellite to orbit once.
* Let’s design an orbit of a spacecraft circling about Jupiter which will put it perpetually over the same place on Jupiter – a “Jupiter-o-synchronous orbit.” How far must such satellite lie from the planet’s center? To get an intuitive feel for your answer, express it in miles. *[Note* “siderereal rotation period” is what you want from Appendix F. And remember, *there’s an error* in Appendix F: the mass of the Earth is actually 5.974 X 10 27 grams, not 5.974 X 10 24 grams]

[The answer you will get to the last part is about twice the diameter of Jupiter. This gives you a good feeling for the scale of these things.]

(5) Cepheid Variable Star

In class we figured out how the luminosity of a Cepheid variable star can be determined from the period of its variation. The results we obtained are given by the graph on the next page. Use this graph to answer the following questions:

* Suppose there is a Cepheid with a period of 8 days and a parallax of 0.02 seconds of arc. (1) How far away is this Cepheid? Give your answer in light years. (2) What flux do we receive from it?
* Suppose we discover a second Cepheid also with a period of 8 days – but its apparent brightness (its flux) is three million times fainter that that from the first. (1) How far away is it? Give your answer in light years. (2) What parallax would this star have? (3) The finest telescopes we have are capable of detecting a parallax as small as 0.002 seconds of arc. Would these be capable of detecting the parallax of this second Cepheid?

