

Name:

Math 29 – Probability

**Practice** Second Midterm Exam 2

Instructions:

1. Show all work. You may receive partial credit for partially completed problems.
2. You may use calculators and a one-sided sheet of reference notes. You may not use any other references or any texts.
3. You may not discuss the exam with anyone but me.
4. Suggestion: Read all questions before beginning and complete the ones you know best first. Point values per problem are displayed below if that helps you allocate your time among problems. Problem 4 is spread over 2 pages for space (you may not need that much).
5. You need to demonstrate that you can solve all integrals that do not have a (DO NOT SOLVE) statement. I.E. write out some work showing how you solved the integration, including if necessary integration by parts.
6. Good luck!

Problem	1	2	3	4	Total
Points Earned					
Possible Points					50

1. a. Let  $X$  be a random variable with moment generating function given by  $M_X(t) = e^{5t+2t^2}$

Let  $Y$  be  $(X-5)/2$ . Find the moment generating function of  $Y$ ,  $E(Y)$ ,  $V(Y)$ , and identify the distribution of  $Y$ .

b. Let  $W$  be a RV with a Chisquare(6) distribution and  $V$  be a RV with a Chisquare(8) distribution. Assume  $W$  is independent of  $V$ . Determine the distribution of  $U=W+V$  with some work/an argument for why your answer is correct.

2. Suppose stress is applied to a 20-inch long steel bar which is clamped in a fixed position at each end until the bar snaps. Let  $Y$  = the fraction of the 20-inches from the left end at which the bar snaps (i.e. distance from left end at snap/20).

a. In one sentence, explain why it would not make sense to model  $Y$  using a normal distribution.

b. Suppose  $Y$  has a beta distribution with  $E(Y)=1/2$  and  $V(Y)=1/20$ . Determine what the distribution of  $Y$  is with specific numeric values for the parameters.

c. Set up an integral (but DO NOT SOLVE) for the probability that the break occurs in the middle 50% of the bar (i.e. a break between 5 and 15 inches from the left end).

d. Find an interval that will contain the value of  $Y$  with a probability of at least  $\frac{3}{4}$ , with some supporting work. Your interval may not be the entire range of values of  $Y$ .

3. A gasoline service station along a toll road has 2 islands – one self-service and one full-service. Each island has 2 available pumps/hoses. Let  $X$  denote the number of hoses in use on the self-service island and let  $Y$  denote the number of hoses in use on the full-service island. The joint probability distribution of  $X$  and  $Y$  is given in the table ( $X$ =rows,  $Y$ =columns).

$X/Y$	0	1	2
0	.10	.04	.02
1	.08	.20	.06
2	.06	.14	.30

a. What is  $F(1,1)$ ?

b. Find the marginal distribution of  $X$  and report it in a table format.

c. What is the conditional distribution of  $Y$  given that two hoses are in use at the self-service island? Report in a table format.

d. Compute  $E(X)$ .

e. Compute  $E(X|Y=1)$ .

4. GPAs of graduating seniors at a small college are distributed as a continuous random variable  $X$  with pdf given by  $f(x) = k(1 - (x - 3)^2)$ ,  $2 \leq x \leq 4$ , and 0, otherwise.

a. What value of  $k$  makes this a valid pdf?

b. What is the probability a GPA is within .25 of 3?

4 c. Someone decides to convert all the grades back to percentages to compare them. For this school, the GPA to percentage conversion is given by  $Y=10(X-2)+75$ . What are the mean and variance of  $Y$ ?