Chapter 16 Problems (Sections 16.1-16.4):
7, 9, 13 and Additional problems below

Additional:
1. Suppose the proportion $\theta$ of defective items in a large manufactured lot is unknown.
   a. Suppose the prior distribution on $\theta$ is a uniform distribution on (0,1). When eight items are
      selected at random from the lot, it is found that exactly 3 of them are defective. Determine the
      posterior distribution of $\theta$.
   b. Now suppose the prior distribution on $\theta$ is Beta(2,200). If 100 items are selected at random and
      three are found to be defective, what is the posterior distribution of $\theta$?

2. Suppose that the number of defects in a 1200 foot roll of magnetic recording tape has a Poisson
   distribution with mean $\theta$, which is unknown.
   a. Suppose the prior distribution on $\theta$ is a Gamma* distribution (3,1). Suppose five rolls (1200
      feet each) are selected at random and the numbers of defects found are: 2,2,6,0, and 3. What is
      the posterior distribution of $\theta$?
   b. What is the Bayes estimate for $\theta$? (Give notation and numerical value for this scenario.)

3. Suppose that the time in minutes required to serve a customer at a certain facility has an
   exponential* distribution with unknown parameter $\lambda$. Suppose the prior distribution on $\lambda$ is a
   Gamma* distribution with mean 2 and standard deviation 1.
   a. If $X$ is Gamma*($\alpha$, $\beta$), what is $E(X)$? What is $V(X)$?
   b. What are the parameters of the Gamma* distribution used here as the prior?
   c. If the average time required to serve a random sample of 20 customers is found to be 3.8 minutes,
      what is the posterior distribution of $\theta$?
   d. What is the Bayes estimate of $\theta$? (Give notation and numerical value for this scenario.)

4. Suppose we are sampling from a normal distribution with unknown mean $\mu$ and precision $\tau$.
   Suppose we sample n=11 observations, and obtain a sample mean of 7.2 and $s^2_{11} = \sum (x_i - \bar{x})^2 = 20.3$. Suppose we assume a normal-gamma* prior for $\mu$ and $\tau$ with prior hyperparameters $\alpha_0 = 2$, $\beta_0 = 1$, $\mu_0 = 3.5$, and $\lambda_0 = 2$.
   a. Find the posterior hyperparameters.
   b. Find an interval that contains 95 percent of the posterior distribution of $\mu$ (i.e. a 95 percent CI
      for $\mu$ based on the posterior distribution).
   c. Find an interval that contains 95 percent of the prior distribution of $\mu$ (i.e. a 95 percent CI for
      $\mu$ based on the prior distribution). Compare your intervals in b and c with a sentence or two.