Amherst College
Department of Economics
Economics 58
Fall 2007

Name: ________________________

First Hour Test

Solutions

There are three questions on this sixty-minute exam. Each is of equal weight in grading. Write your name neatly on the top of each page.

1. Suppose that a person's expenditure function is given by

$$E(P_1, P_2, \overline{U}) = \overline{U}\left[\frac{P_1 P_2}{P_1 + P_2}\right]$$

where (P_1 , P_2) are the prices for good 1 and good 2 respectively and $\overline{U}$ the fixed utility from consumption. Let *x* denote the quantity of good 1 consumed and *y* the quantity of good 2 consumed.

a. Derive the *indirect utility function,* $V(P_1, P_2, I)$, associated with this expenditure function (where *I* denotes income).

ANSWER: Just invert the expenditure function and solve for $\overline{U}$. At the utility maximizing consumption, the indirect utility for a given set of prices and income should match the "fixed" utility at which the expenditure function (for the given prices) matches the income at which the indirect utility function is evaluated.

$$V(P_1, P_2, I) = I\left[\frac{P_1 + P_2}{P_1 P_2}\right]$$

b. Suppose that P_1 = \$1 , P_2 = \$1 , and I = *\$100*. Assuming that this person is a utility maximizer, calculate the utility she gets from consuming goods 1 and 2 at the given prices and income.

ANSWER: Evaluate the indirect utility function obtained in part (a) at the given prices and income above.

$$V(1,1,100) = 100\left[\frac{1+1}{1\cdot 1}\right] = 200$$

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c. Given the prices and income in part (b), calculate the person's utility maximizing consumption of good 1. Denote this as x^*.

HINT: you may want to solve for the Hicksian demand function using *Shepard's Lemma* or for the Marshallian demand function using *Roy's Identity*, first.

ANSWER #1: You can use Shepard's Lemma to derive the Hicksian demand function and evaluate that at the given prices and the utility level solve in part (b)

$$\frac{\partial E}{\partial P_1} = \left[\frac{P_2^2}{(P_1 + P_2)^2}\right]\overline{U} \Rightarrow \left[\frac{1^2}{(1+1)^2}\right]\cdot 200 = 50$$

ANSWER #2: You can use Roy's Identity to derive the Marshallian demand function and evaluate that at the given prices and income

$$-\frac{\partial V / \partial P_1}{\partial V / \partial I} = -\left[\frac{-I / P_1^2}{(P_1 + P_2)/P_1 P_2}\right] = \left[\frac{P_2}{P_1(P_1 + P_2)}\right]\cdot I \Rightarrow \left[\frac{1}{1\cdot(1+1)}\right]\cdot 100 = 50$$

d. Suppose that the price of good 1 rose from P_1 = \$1 to P_1 = \$1.50 while the price of good 2 stayed fixed at P_2 = \$1. Calculate how much the person's income would have to increase in order for her to attain the same utility as before the price change (*compensating variation*).

ANSWER: There were many ways to show this one.

1. You could use the indirect utility function and solve for the income that gives you V = 200 (the original utility achieved) for the NEW prices (P_1 = \$1.50, P_2 = \$1) and subtract the original income (\$100) from it: \$120-\$100 = \$20
2. You could integrate the area to the left of Hicksian demand curve between the old and new prices (P_1 = \$1 to P_1 = \$1.50): $CV = \int_{1.00}^{1.50} x^c(P_1, 1, 200)\,dP_1 = \$20$
3. You could take the difference in the expenditure function between the old and new prices: $CV = E(1.50,1,200) - E(1,1,200) = \$120 - \$100 = \$20$

e. Explain why the compensating variation in part (d) is *less* than the additional income necessary to allow the person to consume the original utility levels of x^* and y^* (the utility maximizing amounts before the price change).

ANSWER: The increase in P_1 alters the relative prices of the two goods. Good 2 is now cheaper than good 1 and the consumer will substitute out of the expensive good (good 1) toward the cheaper good (good 2) in an effort to get more utility from her limited budget. Compensating variation (CV) accounts for this substitution but pure replacement behavior does not.

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2. A farmer will harvest the crops tomorrow. There is a 20 percent chance that tomorrow will be rainy and an 80 percent chance that the weather will be fair. The value of the harvest will be *W* if it is fair and $W - k$ if it rains.

a. If the farmer has a utility of wealth function given by $U(W) = \ln(W)$, write out his or her expected utility. Suppose $W = 100, k = 20$. What is this farmer's expected utility?

ANSWER $E[U] = 0.8\ln(100) + 0.2\ln(80) = 4.5605$

b. The farmer can purchase contingent contracts that will pay $1 if it rains tomorrow. What is the fair price for these contracts and how many will he or she buy? What will expected utility be after purchase of the contracts?

ANSWER The fair price for the contingent contract is $0.20. Because this person's utility function is concave, he or she will fully insure at this fair price. Hence he or she will purchase 20 contracts at a total of $0.20x20=$4. Wealth will therefore be 96 in both states of the world. $E[U] = \ln(96) = 4.5643$. Notice the increase in utility from the original position.

c. More generally, let *x* represent the number of contingent contracts purchased and *p* be the price of each contract. Write out expected utility for this farmer as a function of *W*, *k*, *x*, and *p*.

ANSWER $E[U] = 0.8\ln(W - px) + 0.2\ln(W - px + x - k)$**.** Notice that the cost of the contingent contracts is certain and therefore must be subtracted in both states.

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d. Without explicitly solving the expected utility maximization problem in part c, discuss how you would expect the optimal solution for x to depend on its three determinants. That is, what sign do $\frac{\partial x}{\partial k}, \frac{\partial x}{\partial p}, \frac{\partial x}{\partial W}$ have?

ANSWER

$\frac{\partial x}{\partial k} > 0$ -- a greater size of potential loss will cause this person to buy more contingent contracts.

$\frac{\partial x}{\partial p} \leq 0$ -- as the price of contingent contracts rises this person will buy fewer of them. The extent of this change will depend on how risk averse the person is.

$\frac{\partial x}{\partial W} < 0$ -- because this person exhibits diminishing absolute risk aversion, he or she will buy less protection as wealth increases.

e. What degree of constant relative risk aversion is illustrated in this problem? How would you expect the answers to part d to change if this farmer were more risk averse?

ANSWER Relative risk aversion is defined as $rr(W) = \frac{-WU''(W)}{U'(W)}$. In the log case this expression yields $rr(W) = \frac{-W(-1/W^2)}{1/W} = 1$. An increase in risk aversion would probably make this person more sensitive to k but less sensitive to p. The effect on this person's reactions to changes in W will depend on precisely how risk aversion is increased.

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3. This question relates to Borcherding and Silberberg's paper on "Shipping the Good Apples Out"

a. Explain in no more than five sentences and one equation what the point of this article is.

ANSWER The paper concerns how demanders react when a constant transport charge is added to the price of a premium grade and regular grade product. Such a charge reduces the relative price of the premium grade product because: $\frac{p_1+t}{p_2+t} < \frac{p_1}{p_2}$. In a two-good world such a reduction in relative price will definitely increase the demand for the premium good if we look only at substitution effects. B+S show that the result also holds when there are three goods providing that both the premium and regular goods have similar relationships to the third good.

b. Explain intuitively why the "Alchian-Allen proposition is clearly correct with two goods, but not obvious with three goods

ANSWER The theorem is clearly true with two goods because it is just a statement that the substitution effect is negative with convex indifference curves. It is potentially ambiguous with three goods because a change in relative price could also affect demand for a third good which could then affect the relative quantities of premium and regular goods that a person chooses.

c. The discussion below equation 5 makes a claim about the sign of the first term in brackets. What is the claim? Why is the claim true? How does the claim support the hypothesis of this paper?

ANSWER They wish to show that Equation 5 is positive – that an increase in *t* increases the ratio x_1/x_2. The first term in brackets is clearly positive because it is the product of two negative terms:
$e_{11} < 0$ (own substitution effect is negative), $e_{21} > 0$ (the goods are close substitutes)
and $\frac{1}{p_1} < \frac{1}{p_2}$ because $p_1 > p_2$.

d. The paper also claims that the second term in brackets in equation 5 “should be small”. Explain why this is so.

ANSWER The second term is small because $e_{23} \approx e_{13}$. That is, the premium and regular goods have roughly the same relationship to the third good.

e. Describe another possible application of the Alchian/Allen Theorem that differs from the transport cost case.

ANSWER There are many possibilities.

1. A constant per unit tax will reduce the relative price of a more expensive item – for example a gasoline tax per gallon will reduce the relative price of premium gasoline.

2. Having to hire a baby sitter will reduce the relative price of expensive restaurants or entertainment.

3. The time cost of playing golf reduces the relative price of expensive golf courses.

4. Storage costs for frozen foods reduces the relative price of more expensive items.