Amherst College
Department of Economics
Economics 58
Fall 2005

Name_Solutions__

First Hour Test

There are three questions on this sixty-minute examination. Each is of equal weight in grading.

1. Suppose that a person's utility for *x* and *y* is given by $U(x,y) = x^{.5}y^{.5}$. It is easy to show that the Marshallian demand functions for these goods are given by
$x = \frac{.5I}{p_x}$, $y = \frac{.5I}{p_y}$.

a. Calculate the indirect utility function for this person.

$$U = x^{.5}y^{.5} = \left(\frac{.5I}{p_x}\right)^{.5}\left(\frac{.5I}{p_y}\right)^{.5} = \frac{.5I}{p_x^{.5}p_y^{.5}}$$

b. Calculate the expenditure function for this person. Show that the function is homogeneous of degree one in the prices.

$E = I = 2p_x^{.5}p_y^{.5}U$ if double both prices get $E' = 2(2p_x)^{.5}(2p_y)^{.5}U = 2E$ so the function is homogeneous of degree one in the prices.

c. Use the expenditure function together with Shepherd's Lemma to calculate the compensated demand for good *x* for this person.

$$x^c(p_x, p_y, U) = \frac{\partial E(p_x, p_y, U)}{\partial p_x} = p_x^{-0.5}p_y^{0.5}U$$

d. Use the Marshallian and compensated demand functions for good *x* to show that the own-price Slutsky equation in elasticity form holds in this case. That is, first write out the equation in general terms and then use the elasticities from the functions you have calculated. (Hint: The class example shows that exponents in demand functions are elasticities).

Using the notion that exponents are elasticities gives $e_{x,p_x} = -1, \quad e^c_{x,p_x} = -0.5, \quad e_{x,I} = 1$.

Slutsky equation is $e_{x,p_x} = e^c_{x,p_x} - s_x e_{x,I}$ Here $s_x = 0.5$. Plugging in the numbers gives $-1 = -0.5 - 0.5 \cdot 1 = -1$. So the various equations do obey the Slutsky Equation.

e. Write out the cross-price Slutsky equation in elasticity form. Show that it also holds in this case. Use your result to explain why p_y does not enter the Marshallian demand function for good *x*.
Cross-price Slutsky: $e_{x,p_y} = e^c_{x,p_y} - s_y e_{x,I}$. The various equations yield: $e_{x,p_y} = 0, \quad e^c_{x,p_y} = 0.5, \quad s_y = 0.5, \quad e_{x,I} = 1$ so $0 = 0.5 - 0.5 \cdot 1 = 0$. This shows that p_y does not enter the Marshallian demand function because substitution and income effects cancel out.

2. In class your favorite professor was heard to say “Income effects just screw things up.” Borcherding and Silberberg say much the same thing. That is why one usually uses compensated demand curves for most theoretical analysis.

a. Explain in words what an “income effect” is.
The income effect is the effect of a price change on quantity demanded that comes about because of the change in real purchasing power that the price change causes.

b. Explain how the income effect can cause a violation of the “law of demand” (that $\frac{\partial x_i}{\partial p_i} \le 0$) in the case of Giffen’s Paradox.

The Slutsky Equation says $\frac{\partial x}{\partial p_x} = \frac{\partial x^c}{\partial p_x} - x\frac{\partial x}{\partial I}$. Hence the law of demand can be violated if $\frac{\partial x}{\partial I} < 0$ and the entire income effect is sufficiently large.

c. Explain why Hicks chose to define substitutes and complements using compensated demand functions.

Income effects in cross-price derivatives are always negative (except for inferior goods). Hence they can make substitutes look like complements. It is also possible for *x* to be defined as a substitute for *y* and *y* to be a complement to *x* if gross cross-price effects are used.

d. Would Hicks' observation that "most" goods are substitutes be strengthened or weakened if he had considered income effects of price changes?

Because $\frac{\partial x}{\partial p_y} = \frac{\partial x^c}{\partial p_y} - y\frac{\partial x}{\partial I}$, for a normal good the income effect will always be negative.

Thus it will make it look like more goods are complements ($\frac{\partial x_i}{\partial p_j} < 0$) than is actually the case under the Hicks definition.

e. "The primary difference between Marshallian and Hicks notions of demand is whether one holds nominal or real income constant." Do you agree? Explain.

Agree – Marshall holds nominal income constant. Hicks holds utility constant – but utility can be regarded as synonymous with "real income" – it is in fact what we should mean by "real income".

3. This question relates to Borcherding and Silberberg's paper on "Shipping the Good Apples Out"

a. Explain in no more than five sentences and one equation what the point of this article is.

The paper concerns how demanders react when a constant transport charge is added to the price of a premium grade and regular grade product. Such a charge reduces the relative price of the premium grade product because: $\frac{p_1 + t}{p_2 + t} < \frac{p_1}{p_2}$. In a two-good world such a reduction in relative price will definitely increase the demand for the premium good if we look only at substitution effects. B+S show that the result also holds when there are three goods providing that both the premium and regular goods have similar relationships to the third good.

b. Explain intuitively why the "Alchian-Allen proposition is clearly correct with two goods, but not obvious with three goods

The theorem is clearly true with two goods because it is just a statement that the substitution effect is negative with convex indifference curves. It is potentially ambiguous with three goods because a change in relative price could also affect demand for a third good which could then affect the relative quantities of premium and regular goods that a person chooses.

c. The discussion below equation 5 makes a claim about the sign of the first term in brackets. What is the claim? Why is the claim true? How does the claim support the hypothesis of this paper?

They wish to show that Equation 5 is positive – that an increase in *t* increases the ratio x_1/x_2 . The first term in brackets is clearly positive because it is the product of two negative terms:
$e_{11} < 0$ (own substitution effect is negative), $e_{21} > 0$ (the goods are close substitutes) and $\frac{1}{p_1} < \frac{1}{p_2}$ because $p_1 > p_2$.

d. The paper also claims that the second term in brackets in equation 5 "should be small". Explain why this is so.

The second term is small because $e_{23} \approx e_{13}$. That is, the premium and regular goods have roughly the same relationship to the third good.

e. In *Murder at the Margin* a wealthy couple vacationing in the Bermuda claims that they ate at a low cost restaurant rather than a high cost one because they also had to pay a baby sitter in order to go out. How did the sleuth-economist know that their alibi was probably bogus?

The baby-sitting expense is a fixed addition to both types of meals. It therefore reduces the relative price of the expensive meal. Because the couple is wealthy we can assume that the income effects do not reverse the Alchian-Allen Theorem.