Electric Potential

- Electric Potential energy:

$$\Delta U_{elec} = -\int_a^b \vec{\mathbf{F}}_{elec} \bullet \mathbf{d}\vec{\mathbf{l}}$$

- Electric Potential:

$$\Delta V = -\int_a^b \vec{\mathbf{E}} \bullet \mathbf{d}\vec{\mathbf{l}}$$

Field is the (negative of) the *Gradient* of Potential

$$\mathbf{F} = -\nabla U \quad \Rightarrow \quad F_x = -\frac{dU}{dx}, \quad F_y = -\frac{dU}{dy}, \quad F_z = -\frac{dU}{dz}$$

$$\mathbf{E} = -\nabla V \quad \Rightarrow \quad E_x = -\frac{dV}{dx}, \quad E_y = -\frac{dV}{dy}, \quad E_z = -\frac{dV}{dz}$$

In what direction can you move relative to an electric field so that the electric potential does not change?

1) parallel to the electric field
2) perpendicular to the electric field
3) Some other direction.
4) The answer depends on the symmetry of the situation.

Electric field of single point charge

$$\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$$

Electric potential of single point charge

$$\Delta V = -\int_a^b \vec{\mathbf{E}} \bullet \mathbf{d}\vec{\mathbf{l}}$$

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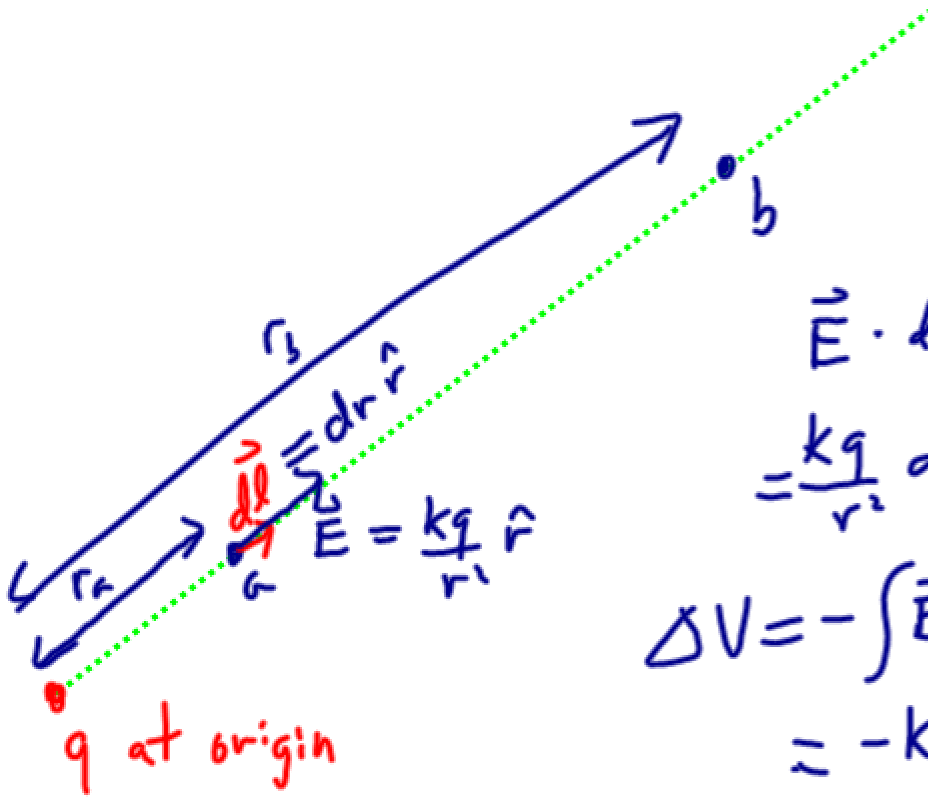


$$\vec{E}\cdot d\vec{l}$$

$$= \frac{kq}{r^2}\, dr\, \hat{r}\cdot\hat{r} \rightarrow 1$$

$$\Delta V = -\int \vec{E}\cdot d\vec{l}$$

$$= -kq\int_{r_a}^{r_b} \frac{dr}{r^2}$$

$$V_b - V_a = kq\left(\frac{1}{r}\right)\Big|_{r_a}^{r_b}$$

Electric potential of single point charge

$$\Delta V = -\int_a^b \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}}$$

$$\vec{\mathbf{E}} = \frac{kq}{r^2}\hat{\mathbf{r}}$$

$$\Delta V = -\int_a^b \frac{kq}{r^2}\hat{\mathbf{r}} \bullet d\vec{\mathbf{l}}$$

$$\Delta V = V_b - V_a = \frac{kq}{r_b} - \frac{kq}{r_a}$$

$$V = \frac{kq}{r} + \underbrace{const.}_{0 \text{ by convention}}$$

Potential for Multiple Charges

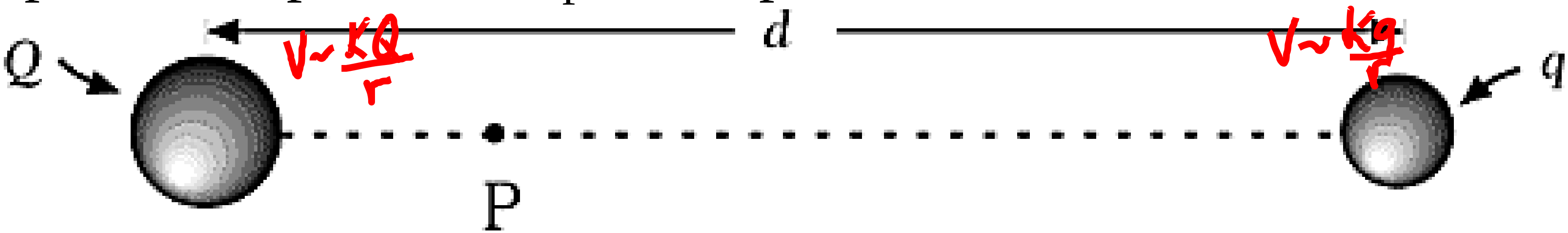
$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 + \ldots$$

$$\Delta V = -\int_a^b \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}}$$

$$= -\int_a^b \vec{\mathbf{E}}_1 \bullet d\vec{\mathbf{l}} - \int_a^b \vec{\mathbf{E}}_2 \bullet d\vec{\mathbf{l}} - \int_a^b \vec{\mathbf{E}}_3 \bullet d\vec{\mathbf{l}} - \ldots$$

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots$$

Charges Q and q ($Q \neq q$), separated by a distance d, produce a potential $V_P = 0$ at point P. This means that



1) no force is acting on a test charge placed at point P.

2) Q and q must have the same sign.

3) the electric field must be zero at point P.

4) the net work in bringing Q to distance d from q is zero.

5) the net work needed to bring a charge from infinity to point P is zero.

Finding the Electric Field due to Continuous Charge Distributions

- When possible, use symmetry to eliminate one or more component of the electric field.
- Define a (linear, areal or volume) density to relate small spatial regions to small bits of charge.
- Calculate the electric field due to each small charge bit: $dE = \frac{k\,dq}{r^2}$.
- Sum up (integrate) the electric fields to find the total field.

Finding the Potential due to Continuous Charge Distributions

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- Define a (linear, areal or volume) density to relate small spatial regions to small bits of charge.
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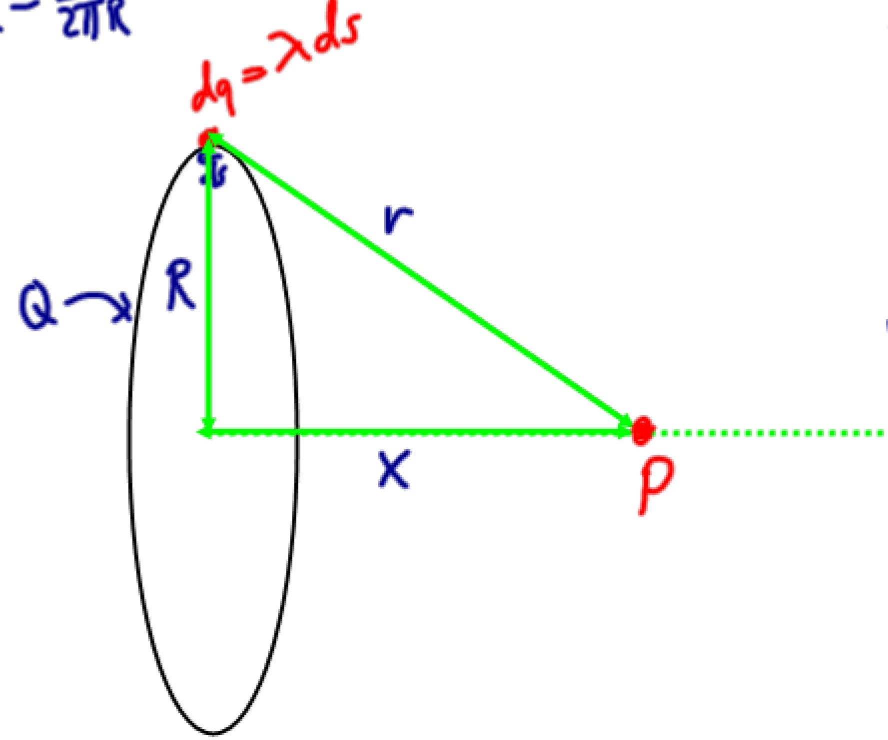
$$\lambda = \frac{Q}{2\pi R}$$



$$dV = \frac{k\,dq}{r}$$

$$= \frac{k\lambda\,ds}{\sqrt{x^2+R^2}}$$

$$V = \frac{k\lambda}{\sqrt{x^2+R^2}} \underbrace{\int ds}_{2\pi R}$$

$$V = \frac{k(\lambda 2\pi R)}{\sqrt{x^2+R^2}}$$

$$V = \frac{kQ}{\sqrt{x^2+R^2}}$$

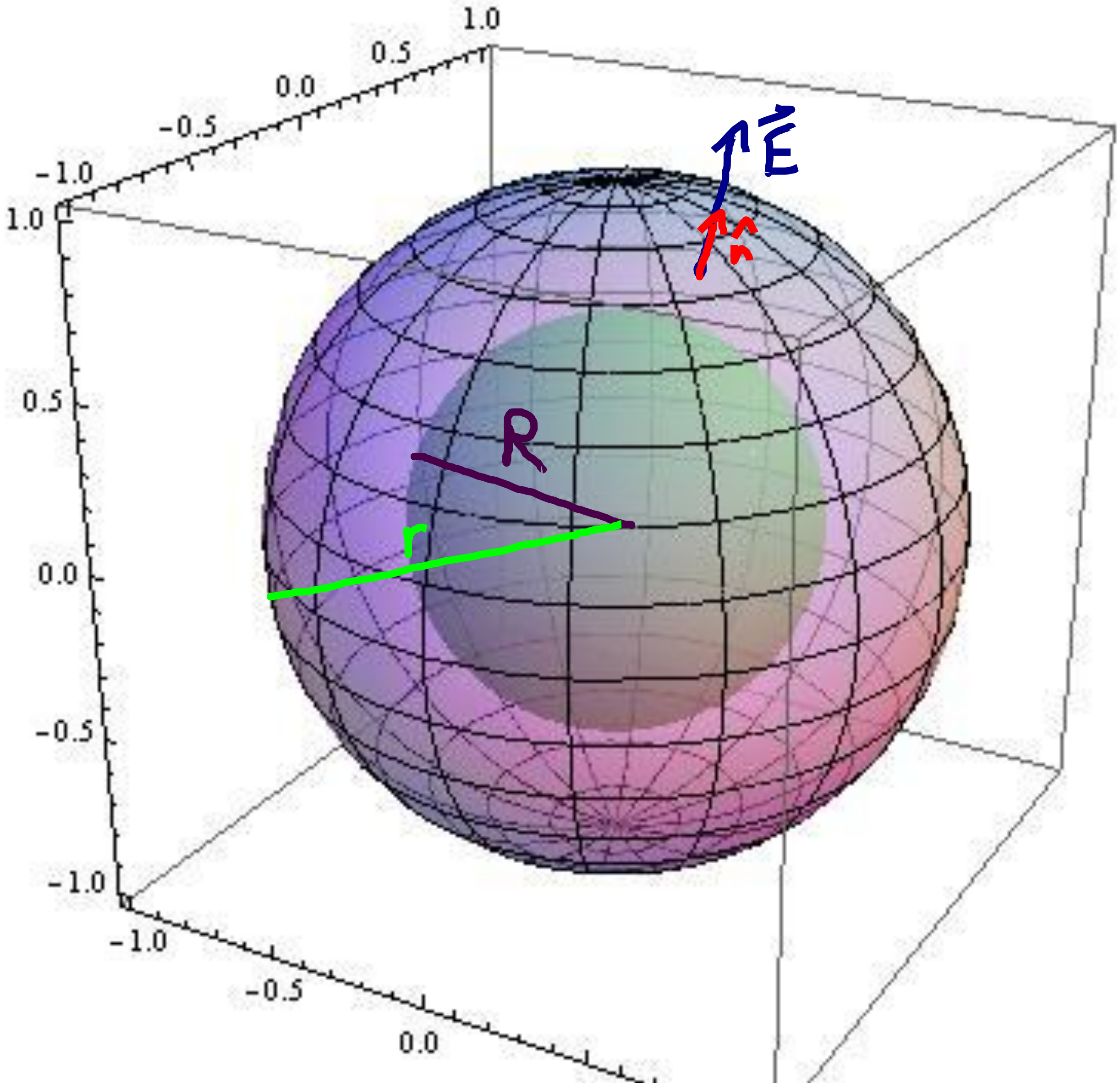
$$V = \frac{kQ}{\sqrt{x^2+R^2}} = kQ\left(x^2+R^2\right)^{-1/2}$$

$$E_x = -\frac{dV}{dx} = +kQ\left(+\tfrac{1}{2}\right)(2x)\left(x^2+R^2\right)^{-3/2}$$

$$= \frac{kQx}{\left(x^2+R^2\right)^{3/2}}$$

$$E_y = \frac{-dV}{dy} = 0$$

Spherical shell of charge



$$\vec{\mathbf{E}}_{out} = \frac{kQ}{r^2}\hat{\mathbf{r}} \quad (r > R)$$

$$\vec{\mathbf{E}}_{in} = 0 \quad (r < R)$$

$$V_{out} = \frac{kQ}{r} \quad (r > R)$$

$$V_{in} = \frac{kQ}{R} \quad [\text{a constant}]$$

$$(r < R)$$