

Math 13 Fall 2008: Exam 2

Name:

Instructions: There are 5 questions on this exam of which you must do 4. Each problem is scored out of 8 points for a total of 32 points. You may not use any outside materials(eg. notes or calculators). You have 50 minutes to complete this exam. Remember to fully justify your answers.

Score:

Circle below the 4 problems you wish to be graded. Otherwise, I will grade the first 4 completed problems

1 2 3 4 5

Problem 1. Determine the following limits or show that they do not exist.

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 y^4}{(x^2 + y^4)^3}$$

(b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2x^4 y}{x^4 + y^4}$$

Problem 2. Let $w = f(x, y, z) = x^3 + y^3 - z$.

(a) Find the rate of change of f at the point $(1, 1, 2)$ along the line $\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-2}{2}$ in the direction of decreasing x .

(b) Find $\frac{\partial w}{\partial t}$ for

$$x(s, t) = \sin(st)$$

$$y(s, t) = \ln(st)$$

$$z(s, t) = se^t.$$

Problem 3. Classify the critical points of $f(x, y) = x^4 + y^3 - 32x - 27y - 1$.

Problem 4. Find the maximum and minimum for $3x^2 + 4xy$ for points on the disk $x^2 + y^2 \leq 14$.

Problem 5. Let $a < 0$. Consider the surface $xyz = a$.

- (a) Find the tangent plane to the surface at (x_0, y_0, z_0) .
- (b) The tangent plane forms a tetrahedron (pyramid with a triangular base) in the first octant. Show that the volume of this tetrahedron is $\frac{9}{2}a$. (Hint: the volume of a tetrahedron is given by $\frac{1}{3}(\text{area of the base triangle})(\text{height})$.)