Math 13, Section 01, Spring 2009

Final Exam, Tuesday, May 12, 2009

Instructions: No calculators, cell phones, iPods, etc.
No books or notes except for **one** 8.5×11" sheet of notes.
Express yourself clearly and legibly. Explain your reasoning and show your work.
Little or no credit may be awarded if you fail to justify your answers.
There are eight problems, totalling 200 points.

WRITE LEGIBLY. **NO CALCULATORS.**

1. **(15 points)** Let S be the plane that contains the points $(0, 0, 1)$, $(4, 2, 0)$, and $(1, -3, 2)$. Find an equation for the line through the point $(1, 0, -2)$ that is perpendicular to S.

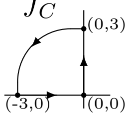
2. **(20 points)** Let $f(x,y) = \begin{cases} \dfrac{2x^3 + 3xy - 3y^2}{x^2 + 2y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

(a). Prove that f is **not** continuous at $(0, 0)$.
(b). Compute the directional derivative $D_{\vec{u}} f(0, 0)$, where $\vec{u} = \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$.

3. **(25 points)** Find and classify (as local minimum, local maximum, or saddle point) every critical point of the function $f(x, y) = xy^2 - 6x^2 - 3y^2 + 7$.

4. **(25 points)** Find the maximum and minimum values of the function $f(x, y) = x^2 y$ subject to the constraint $x^2 + y^2 = 9$.

5. **(30 points)** Let C be the path in the xy-plane that begins at $(0, 3)$, runs (counterclockwise) through the second quadrant along the arc of the circle of radius 3 centered at the origin to the point $(-3, 0)$, then moves right along the x-axis to the origin, and finally moves up the y-axis to return to the starting point $(0, 3)$. Compute $\displaystyle\int_C 6x^2 y\, dx + (2x^3 - xy)\, dy$.



6. **(30 points)** Let S denote the sphere in $\mathbb{R}^3$ of radius 2 centered at the origin, oriented outward, and let $\vec{F}(x, y, z) = \langle y^2 z, yz^2, x^2 e^y \rangle$. Compute $\displaystyle\iint_S \vec{F} \cdot d\vec{S}$.

7. **(25 points)** For each of the following vector fields, either find a potential function (i.e., a function that it is the gradient of) or prove that the vector field is not conservative.
(a). $\vec{F}(x, y) = \langle x^2 - \cos(2y), y^3 + 2x \sin(2y) \rangle$.
(b). $\vec{G}(x, y, z) = \langle 2xy - x^2, z^3, 3yz^2 \rangle$.

8. **(30 points)** Let C be the curve that lies in the surface $z = x^3 - xy + 2$ directly above the boundary of the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$. Compute $\displaystyle\int_C \sin(x^2)\, dx + yz\, dy - y^2\, dz$, if C is oriented **clockwise** when viewed from above.

(Bonus problems on back)

BONUS A. (2 points) Let R be region in the first quadrant of the xy-plane bounded to the upper right by $y = 4/x$, to the lower right by $y = x$, to the lower left by $y = 1/x$, and to the upper left by $y = 9x$. Use the transformation

$$x = \frac{\sqrt{u}}{v}, \quad y = v\sqrt{u}$$

to compute $\iint_R \cos(\pi xy)\, dy\, dx$.

BONUS B. (2 points) Let C be the curve in $\mathbb{R}^3$ that starts at the point $(0, 1, 1)$, goes to the point $(1, 0, 0)$ via $\vec{r}(t) = \langle t, 1 - t^2, 1 - t\rangle$ for $0 \le t \le 1$; and then goes from $(1, 0, 0)$ to the point $(0, 1, 0)$ along the arc of the parabola $y = 1 - x^2$ in the xy-plane.
Compute $\int_C \cos(e^x)\, dx + (z + 5y^4z^2)\, dy + (y^2 + 2y^5z)\, dz$.

BONUS C. (1 point) Name the largest battle (in terms of numbers of participants and casualties) in the history of human warfare.

BONUS D. (1 point) About two weeks ago, a US senator changed political parties. What is that senator's name?