Math 17 – Spring 2011 Name:______SOLUTIONS___________
Monday, May 9, 2011
9:00 AM – 11:00 AM

Final Exam

Please complete the following problems. Be sure to ask me if you have any questions or anything is unclear. Partial credit will be given, so **please be sure to show all of your work. Please use complete sentences to answer ALL questions.**

- Note that this exam is worth 150 points. Point values are listed next to each question.
- Please ask if you don't understand a question.
- Be sure to answer ALL parts of each question.
- Unless specifically asked, you need not check the needed assumptions.
- Any students wishing to discuss their final exam or grade should contact me at jstratton@amherst.edu or at jeffreystratton@gmail.com. As a visiting instructor, I'm not sure when my Amherst email is deleted.
- The *Z*-Table and *t*-Table are included at the end of the test.
- Some cumulative binomial probabilities are tabulated at the end of the test as well.

Good luck!! Thanks for your hard work this semester.

DATA SOURCES:

- The data were provided by Victoria Whitman, a realtor in Eugene, in 2005.
- Pardoe, I. (2006). Applied Regression Modeling: A Business Approach. Hoboken, NJ: Wiley.

Data Scenario

Most of the questions in this exam are related to this data scenario. Any general questions will be flagged with the words "General Question."

THE STORY:
We have a data set containing information on 76 single-family homes in Eugene, Oregon during 2005. At the time the data were collected, the data submitter was preparing to place his house on the market and it was important to come up with a reasonable asking price. Whereas realtors use experience and local knowledge to subjectively value a house based on its characteristics (size, amenities, location, etc.) and the prices of similar houses nearby, statistical analysis provides an alternative that more objectively models local house prices using these same data. Better still, realtor experience can help guide the modeling process to fine-tune a final predictive model.

VARIABLE DESCRIPTIONS:

- Price = sale price (thousands of dollars)
- Size = floor size (thousands of square feet)
- Bath = number of bathrooms (with half-bathrooms counting as 0.1)
- Bed = number of bedrooms (between 2 and 6)
- Year = year built
- Garage = garage size (0, 1, 2, or 3 cars)
- Status = act (active listing), pen (pending sale), or sld (sold)
- Active = indicator for active listing (reference: pending or sold)
- Elem = nearest elementary school (edgewood, edison, harris, adams, crest, or parker)

A snapshot of the first 20 observations is given here:

HouseData

	id	Price	Size	Bath	Bed	year	Garage	Status	Active	Elementary
1	1	388.00	2.180	3.0	4	1940	0	sld	0	edison
2	2	450.00	2.054	3.0	4	1957	2	sld	0	edison
3	3	386.00	2.112	2.0	4	1955	2	sld	0	edison
4	4	350.00	1.442	1.0	2	1956	1	act	1	adams
5	5	155.50	1.800	2.0	4	1994	1	sld	0	adams
6	6	220.00	1.965	2.0	3	1940	1	sld	0	adams
7	7	239.50	1.800	1.1	4	1958	1	act	1	parker
8	8	207.00	2.254	2.0	4	1961	2	sld	0	parker
9	9	269.90	1.922	2.1	4	1965	2	act	1	parker
10	10	238.80	1.920	2.1	3	1968	2	sld	0	parker
11	11	359.90	2.200	2.0	3	1970	2	act	1	edge
12	12	249.70	1.868	2.0	4	1965	2	sld	0	edge
13	13	265.00	1.875	2.1	3	1979	2	sld	0	edge
14	14	349.00	2.000	2.0	3	1997	2	pen	0	edge
15	15	319.00	1.855	2.0	4	1925	2	sld	0	harris
16	16	339.00	1.928	3.0	3	1972	2	act	1	edge
17	17	283.00	1.980	3.0	4	1971	2	act	1	edge
18	18	275.00	1.528	2.1	3	1975	2	act	1	edge
19	19	299.90	1.882	2.1	3	1976	2	act	1	edge
20	20	277.00	1.440	2.0	3	1948	2	act	1	edison

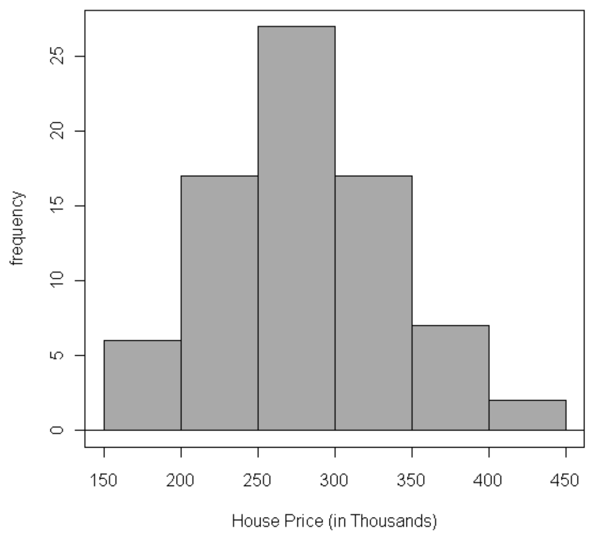
1. (14 pts) **General Question.** Several scenarios are listed below on the left. A number of statistical procedures, distribution, and measures that we've covered are listed on the right. For each scenario, list the statistical method you should use. Not all of the procedures listed will be used.

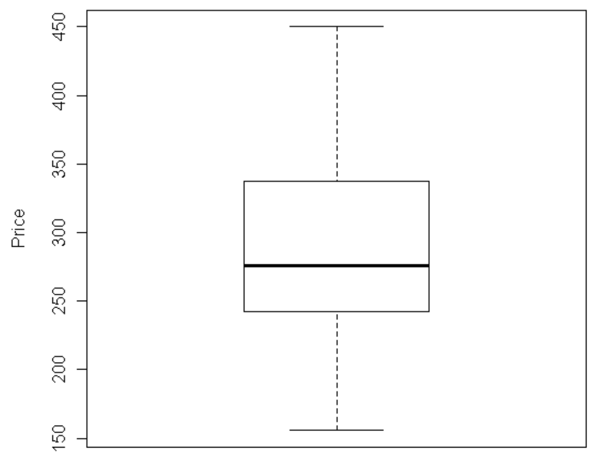
- You wish test a hypothesis about a mean using a small sample size. What procedure should you use? __r____
- You'd like to look for outliers in your data. The distribution is skewed, though, so you've opted to use the five number summary. What procedure will help you identify possible outliers? __ e___
- Which measure can be used to find the relative standing of two observations from different distributions of data? __ f___
- You are taking a sample of 50 people and measuring their average height. Suppose the individual heights have a mean of 56 inches and a standard deviation of 5 inches. What distribution would help you find the probability that the average is greater than 70 inches tall? __ k___
- You have a quantitative variable, and want to visually picture the data in such a way that the original data values are preserved. What graph will do this? __h___
- You'd like to find the mean income of all Amherst residents, using a sample of 35 people. What procedure should you used to produce a 95% confidence interval? __ m___
- You'd like to see if there is a difference in the mean incoming SAT score of incoming freshmen among all of the five colleges __ v___

a) Bar Graph
b) Pie Chart
c) Mean and Standard deviation
d) Five Number Summary
e) 1.5 IQR rule
f) Z-score
g) Histogram
h) Stem-and-leaf Plot
i) Boxplot
j) Binomial Distribution
k) Normal Distribution
l) Uniform Distribution
m) One-sample t-Interval for a mean
n) Two-sample t-Interval for a mean
o) One-sample z-Interval for a proportion
p) Two-sample z-Interval for a proportion
q) Matched pairs t-Test
r) One-sample t-Test for a mean
s) Two-sample t-Test for a mean
t) One-sample z-Test for a proportion
u) Two-sample z-Test for a proportion
v) ANOVA

2. (8 pts) A histogram and boxplot of house price is shown below. Give a brief description of the price variable based on these graphs.

The price variable is quite symmetric. It is centered at about $275,000, with a spread from $150,000 to $450,000. There do not appear to be any outliers.





3. (6 pts) Some summary statistics of PRICE are given below. Is the mean or the median a better estimate of the center of this variable? Please explain why.

```
> numSummary(HouseData[,"Price"], statistics=c("mean", "sd", "quantiles"),
+   quantiles=c(0,.25,.5,.75,1))
     mean        sd     0%     25% 50%     75% 100%  n
 285.7954 60.33269 155.5 242.75 276 336.75  450 76
```

As the mean is larger than the median, price is skewed to the right. However, it is not skewed by much. The histogram showed price was quite symmetric, so the mean is a better estimate of the center of this variable. The mean is more commonly used when the data allow it.

4. (10 pts) **General Question.** Explain how the central limit theorem of statistics has been useful throughout this course.

The central limit theorem has been used throughout this course to allow us to use normal distribution probabilities to make inferences for both means and proportions.

5. (8 pts) Suppose that the distribution of PRICE is truly Normal with a mean of μ = \$300,000 and a standard deviation of σ = \$70,000. Find the probability that a randomly selected house has a price of over \$450,000. Find the probability that the *mean* of 8 houses is over \$450,000.

$$P(X > 450) = P\left(Z > \frac{450 - 300}{70}\right) = P(Z > 2.14) = 1 - 0.9838 = 0.0162$$

The sampling distribution of $\bar{x}$ is normal, with a mean of $\mu_{\bar{x}} = \mu = 300$ and a standard deviation of $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{70}{\sqrt{8}} = 24.7487$.

$$P(\bar{X} > 450) = P\left(Z > \frac{450 - 300}{24.7487}\right) = P(Z > 6.06) \approx 0$$

There is a 1.62% probability that a single house costs more than \$450,000, but there is practically no chance that the average of eight houses exceeds \$450,000.

6. (8 pts) Compute and interpret the 90% confidence interval for the mean price of homes in Eugene, Oregon.

$$\bar{X} \pm t^*_{n-1}\frac{s}{\sqrt{n}} = 285.7594 \pm 1.665\frac{60.3327}{\sqrt{76}} = 285.7594 \pm 11.5229 = (274.24\,, 297.28)$$

We are 90% confident that the true mean price of homes in Eugene, OR, is between \$274,000 and \$297,000.

7. (8 pts) A pilot study in Eugene estimated the proportion of pending sales to be 0.20. Is our sample large enough to estimate the percentage of homes with sales pending to within 10 percent with a 95% confidence interval? Please justify your answer.

$$ME = Z^* \sqrt{\frac{pq}{n}}$$

$$0.10 = 1.96 \sqrt{\frac{0.20(0.80)}{n}}$$

$$n = \frac{1.96^2(0.20)(0.80)}{0.10^2}$$

$$n = 61.47$$

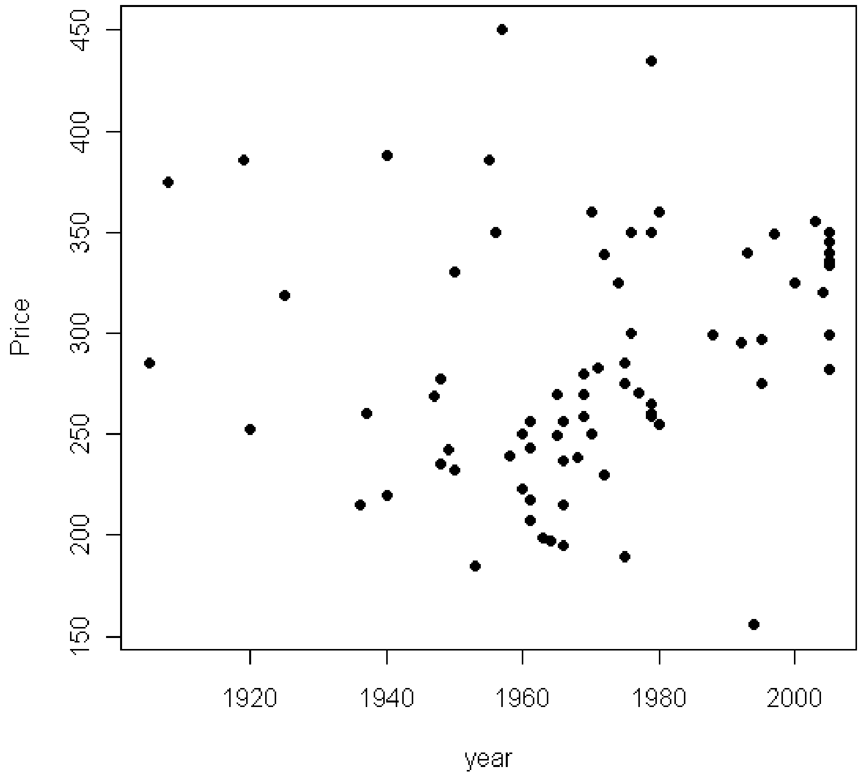
Yes our sample is large enough. We sampled 76 houses, but only needed 62.

8. (8 pts) We find that 13 of the 76 houses have sales pending. Find and interpret the 98% confidence interval for the true proportion of houses in Eugene with sales pending.

$$\hat{p} \pm Z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.1711 \pm 2.326 \sqrt{\frac{0.1711(1-0.1711)}{76}} = 0.1711 \pm 0.1005 = (0.0706\,, 0.2716)$$

We are 98% confident that the true proportion of houses in Eugene with sales pending is between 7.06% and 27.16%.

9. (6 pts) A scatterplot of PRICE versus YEAR is given below. Use it to describe the association between these two variables. What do you think would be the value of the correlation if you were to compute it?



There doesn't seem to be much association at all between the year and the price. I'd guess that the correlation would be something like 0.24.

10. (8 pts) Summary statistics of the PRICE and SIZE variables are given below. We'll use linear regression to try to predict the price of a house based on its size.

	$\bar{x}$	s
SIZE (square feet)	1,970	212.4
PRICE (dollars)	$285,795	$60,333
r = 0.2014		

Typo!

Write down the equation of the least squares regression line. Do you think it will work well in explaining the sales price of homes in Eugene? Explain.

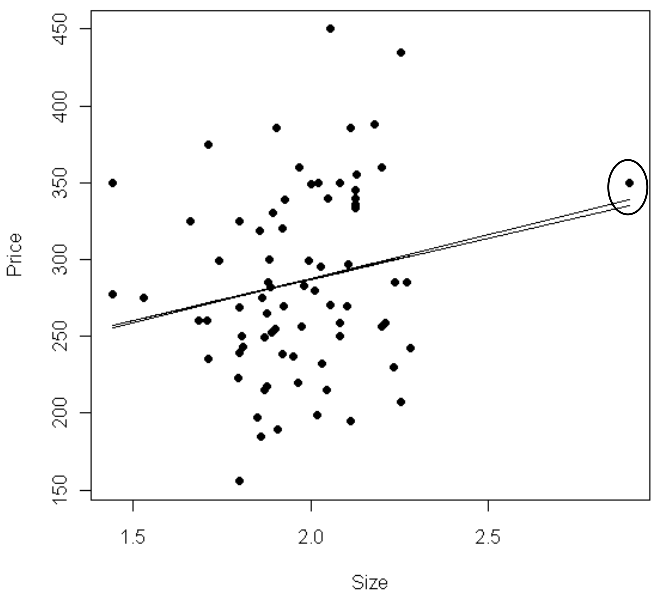
$$b_1 = r\frac{s_y}{s_x} = 0.2014\left(\frac{60333}{212.4}\right) = 57.208$$
$$b_0 = \bar{Y} - b_1\bar{X} = 285795 - 57.208(1970) = 173095.24$$

The regression line is:

$$\widehat{Price} = 173095.24 + 57.208(Size)$$

This line doesn't work very well. First, there isn't much of a linear relationship on the scatterplot. Also the $R^2 = 0.2014^2 = 0.04056$. Only 4.06% of the variation in house price is explained by the linear regression with house size.

11. (8 pts) A plot of the PRICE versus SIZE data is given below, along with two least squares regression lines. One line is for all of the data and the other is after removing the circled data point on the right-hand side of the plot. What would we call such a point?



This is a point with high leverage that is not very influential.

12. (8 pts) The sample average house size is 1.97 thousand square feet, with a sample standard deviation of 212 square feet. Conduct a statistical hypothesis test to see if the mean house size is less than 2,000 square feet.

We are testing: $H_0: \mu = 2000\ ft^2$
$H_A: \mu < 2000\ ft^2$

The test statistic is: $t = \frac{X - \mu_0}{\sigma / \sqrt{n}} = \frac{1970 - 2000}{212 / \sqrt{76}} = -1.2337$

From row 75 of the table, 1.2337 is off the table to the left. This means that the *p*-value is greater than 0.10.

We do not have evidence to reject the null hypothesis. We have no evidence that the mean house size is less than 2,000 square feet.

13. (10 pts) Describe how you might study the relationship between price and the elementary school closest to the house. What numerical/graphical summaries might you use? What inference procedure(s) might you use?

Elementary school is a categorical random variable, while price is numeric. We could do some frequencies or a bar graph/pie chart of the schools. Since there are more than two schools, We might consider trying an ANOVA test to compare the mean price amongst all schools.

The remaining questions do not pertain to the housing price data scenario.

14. (8 pts) The American Freshman Survey found that 3 out of 10 college freshmen describe themselves politically as liberal of "far left." Suppose a random sample of 17 college freshmen is selected.

    What is the probability that at least 12 of them describe themselves as "far left" politically? How many students would we expect to describe themselves as “far left” politically?

    We have a binomial random variable.

    $$P(X \geq 12) = 1 - P(X < 12) = 1 - P(X \leq 11) = 1 - 0.999344 = 0.000656$$

    We expect to see $np = 17(0.30) = 5.1$ describe themselves as “far left” politically.

15. (8 pts) What assumption(s) persist throughout ALL of the statistical inferences methods we’ve learned in this class?

    All of the methods we’ve learned require us to have taken a simple random sample and independent observations.

16. (8 pts) For a space vehicle to gain reentry into the earth's atmosphere, the reentry propulsion system must work properly. One component of the system operates successfully only 60% of the time. To increase the reliability of the system, four of these components are installed in such a way that the system will operate successfully if at least one component is working. What is the probability that the system will fail? (Assume that the components operate independently.)

To fail, we need all four components to fail. They are independent, so

$$P(failure) = P(1^{st} \text{ fails}) \times P(2^{nd} \text{ fails}) \times P(3^{rd} \text{ fails}) \times P(4^{th} \text{ fails}) = 0.40^4 = 0.0256$$

There is a 2.56% probability that the system will fail.

17. (8 pts) Data on age and yearly income was collected from a sample of Massachusetts residents and is displayed in the following two-way table:

		Income			
		<$25k	$25k – $70k	> 70k	Total
Age (years)	< 25	952	1,050	53	2,055
	25 – 45	456	2,055	1,570	4,081
	> 45	54	952	1,008	2,014
	Total	1,462	4,057	2,631	8,150

Find the probability that a randomly chosen Massachusetts resident is less than 25 years old.

Find the probability that a randomly chosen Massachusetts resident is over 45, given that they make less than $70,000 per year.

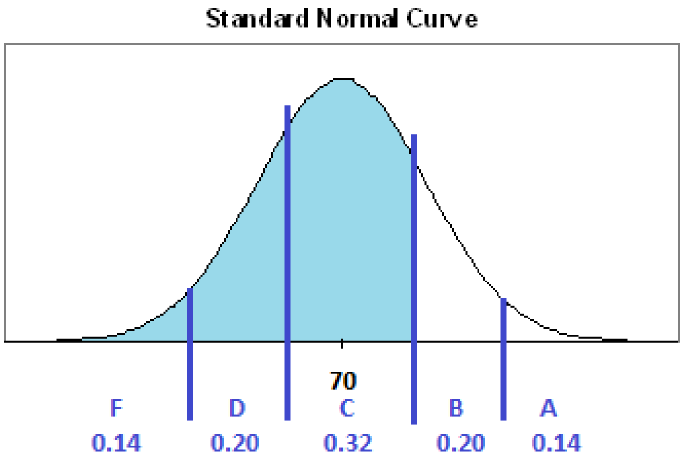
$$P(\text{less than 25}) = \frac{2055}{8150} = 0.2521$$

$$P(\text{over 45}|\text{less than 70k}) = \frac{P(\text{over 45 and less than 70k})}{P(\text{less than 70k})} = \frac{\frac{54+952}{8150}}{\frac{1462+4057}{8150}} = \frac{1006}{5519}$$

$$= 0.1823$$

There is a 25.21% probability of a randomly chosen Massachusetts resident being less than 25 years old. There is an 18.23% chance that a randomly chosen Massachusetts resident is over 45, given that they make less than $70k per year.

18. (8 pts) A college professor teaches statistics each spring to a large class of first year students. He uses standardized exams that he knows from past experience produce bell-shaped distributions with a mean of 70 and a standard deviation of 14. His philosophy of grading is to impose standards that, in the long run, will yield 14% A's, 20% B's, 32% C's, 20% D's and 14% F's. Where should the cut-off be between the B's and the C's? (Round your answer to a whole number. Don't worry…..I don't use this scheme! ☺)



We need the point on this normal curve with $0.14 + 0.20 + 0.32 = 0.66$ to the left of it. Looking in the Z-table, we find that the closest we get is $Z = 0.41$, corresponding to 0.6591. The cutoff score is then:

$$Z = \frac{X - \mu}{\sigma}$$

$$0.41 = \frac{X - 70}{14}$$

$$X = 0.41(14) + 70 = 75.74$$

You'd need to score a 75.74 to move from a C to a B.

Cumulative Probability Distribution
***n* = 17 and *p* = 0.3**

0	0.002326
1	0.019275
2	0.077385
3	0.201907
4	0.388690
5	0.596819
6	0.775215
7	0.895360
8	0.959723
9	0.987307
10	0.996765
11	0.999344
12	0.999897
13	0.999988
14	0.999999
15	1.000000
16	1.000000
17	1.000000

Cumulative Probability Distribution
***n* = 17 and *p* = 0.7**

0	0.000000
1	0.000000
2	0.000001
3	0.000012
4	0.000103
5	0.000656
6	0.003235
7	0.012693
8	0.040277
9	0.104640
10	0.224785
11	0.403181
12	0.611310
13	0.798093
14	0.922615
15	0.980725
16	0.997674
17	1.000000

Cumulative Probability Distribution
***n* = 10 and *p* = 0.3**

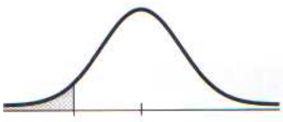
0	0.028248
1	0.149308
2	0.382783
3	0.649611
4	0.849732
5	0.952651
6	0.989408
7	0.998410
8	0.999856
9	0.999994
10	1.000000

Cumulative Probability Distribution
***n* = 10 and *p* = 0.7**

0	0.000006
1	0.000144
2	0.001590
3	0.010592
4	0.047349
5	0.150268
6	0.350389
7	0.617217
8	0.850692
9	0.971752
10	1.000000

Table Z

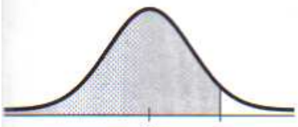
Areas under the standard Normal curve



Second decimal place in z										
0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
									0.0000†	−3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	−3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	−3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	−3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	−3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	−3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	−3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	−3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	−3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	−2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	−2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	−2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	−2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	−2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	−1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	−1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	−1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	−1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	−1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

†For $z \leq -3.90$, the areas are 0.0000 to four decimal places.

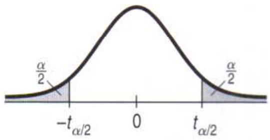
Table *Z* (cont.)
Areas under the standard Normal curve



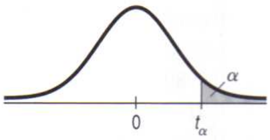
	Second decimal place in z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000[†]									

[†]For $z \geq 3.90$, the areas are 1.0000 to four decimal places.

Table T
Values of t_α



Two tails



One tail

Two-tail probability	0.20	0.10	0.05	0.02	0.01	
One-tail probability	0.10	0.05	0.025	0.01	0.005	
df						df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
32	1.309	1.694	2.037	2.449	2.738	32
35	1.306	1.690	2.030	2.438	2.725	35
40	1.303	1.684	2.021	2.423	2.704	40
45	1.301	1.679	2.014	2.412	2.690	45
50	1.299	1.676	2.009	2.403	2.678	50
60	1.296	1.671	2.000	2.390	2.660	60
75	1.293	1.665	1.992	2.377	2.643	75
100	1.290	1.660	1.984	2.364	2.626	100
120	1.289	1.658	1.980	2.358	2.617	120
140	1.288	1.656	1.977	2.353	2.611	140
180	1.286	1.653	1.973	2.347	2.603	180
250	1.285	1.651	1.969	2.341	2.596	250
400	1.284	1.649	1.966	2.336	2.588	400
1000	1.282	1.646	1.962	2.330	2.581	1000
∞	1.282	1.645	1.960	2.326	2.576	∞
Confidence levels	80%	90%	95%	98%	99%	