Maxwell's Equations – The Fundamental Laws of Electromagnetism

- Gauss' Law
  - The total electric flux through a closed surface is proportional to the charge enclosed:

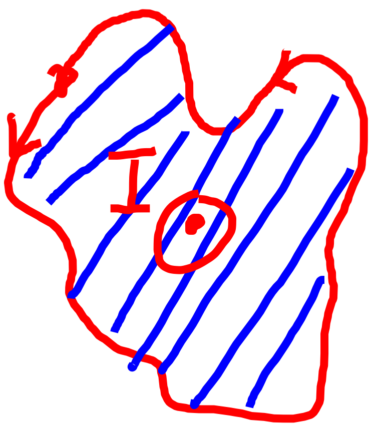
$$\oint \vec{\mathbf{E}} \bullet \hat{\mathbf{n}} dA = \frac{Q_{encl}}{\varepsilon_0}$$

- Gauss' Law for Magnetism
  - The total magnetic flux through a closed surface is zero.
  - There are no magnetic charges.

$$\oint \vec{\mathbf{B}} \bullet \hat{\mathbf{n}} dA = 0$$

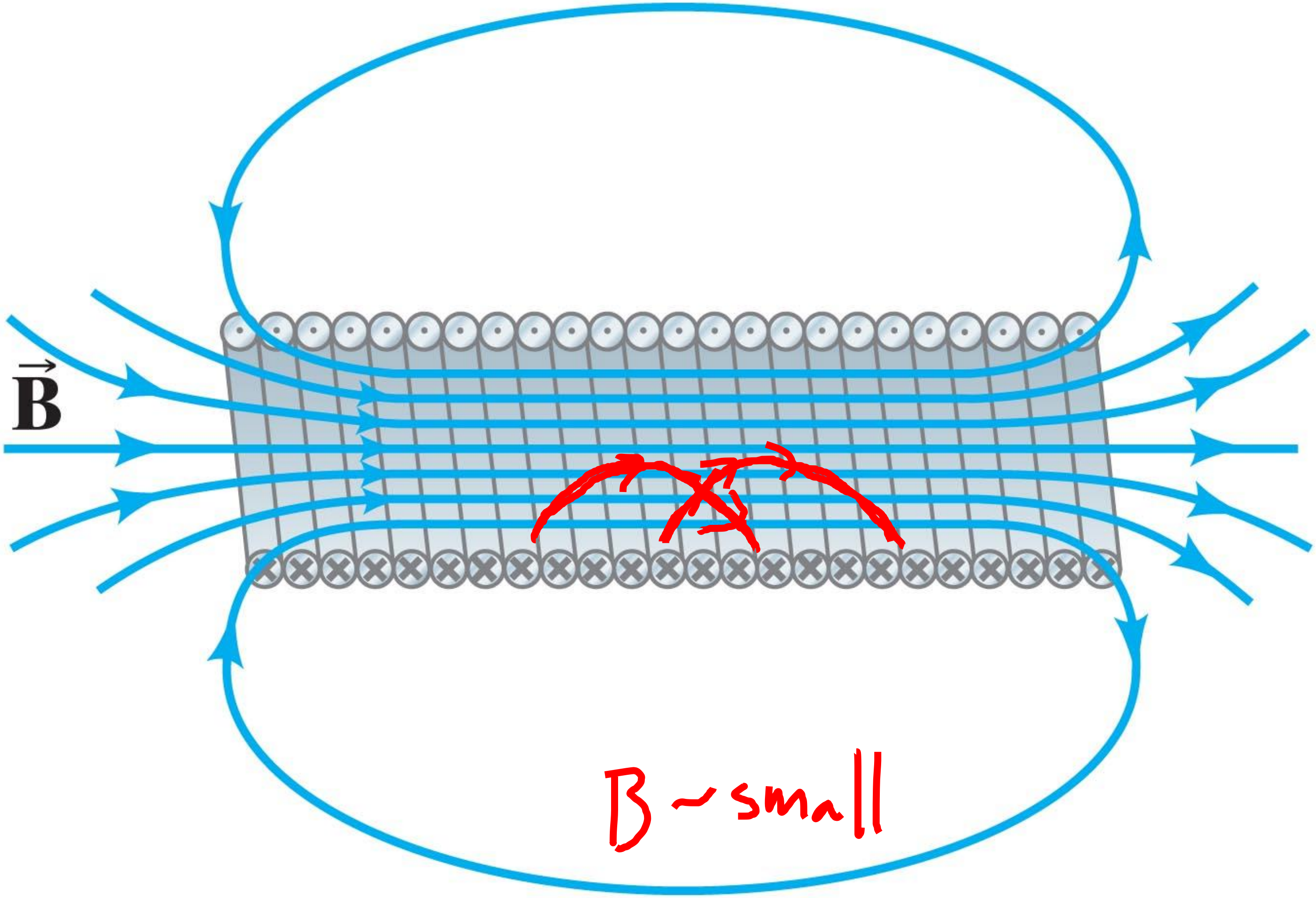
Maxwell's Equations – The Fundamental Laws of Electromagnetism

- Ampere's Law (valid for constant currents)
  - The integral of $\vec{\mathbf{B}} \bullet d\vec{l}$ is proportional to the current piercing a surface bounded by the curve.
  - This Law will need generalization to the case of non-constant currents (in a few weeks).



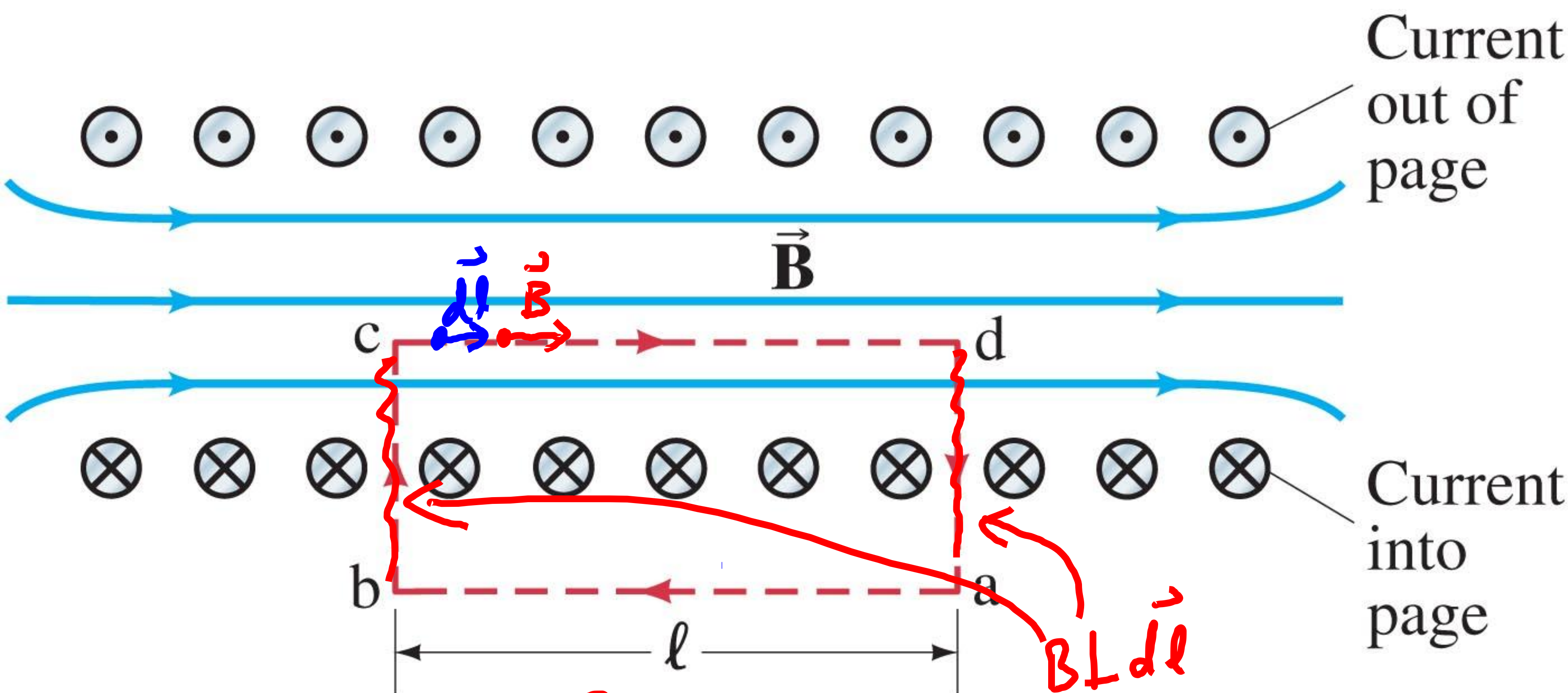
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Solenoid



(b)

Figure 28.15b



$$\oint \vec{B}\cdot d\vec{l} = \overbrace{\cancelto{0}{\int_a^b \vec{B}\cdot d\vec{l}}}^{B\approx 0} + \cancelto{0}{\int_b^c \vec{B}\cdot d\vec{l}} + \int_c^d \vec{B}\cdot d\vec{l} + \cancelto{0}{\int_d^a \vec{B}\cdot d\vec{l}}$$

$$= \int_c^d B\,dl = B\int_c^d dl = Bl = \mu_0 I_{encl} = \mu_0 N I$$

$$B = \mu_0 (N/l) I = \mu_0 n I$$

Figure 28.16

Maxwell's Equations – The Fundamental Laws of Electromagnetism

- Faraday's Law
  - When the magnetic flux through some loop (C) changes, it induces an emf around the loop proportional to the rate at which the flux changes.
  - If loop has *N* turns, the emf is *N* times larger.
  - Lenz' Law: the sign of the emf is such that an induced current opposes the change in flux.

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}\int \vec{\mathbf{B}}\bullet\hat{\mathbf{n}}\,dA$$

Ways in which magnetic flux can change:

- Magnitude of Field ($|B|$)
- Direction of Field $(\hat{\mathbf{B}})$
- Orientation of Loop $(\hat{\mathbf{n}})$
- Area of loop (A)
- Any combination of the above.

Faraday's Law and Motional emf

- Motional emf:

$$\vec{F}_E = \vec{F}_B$$

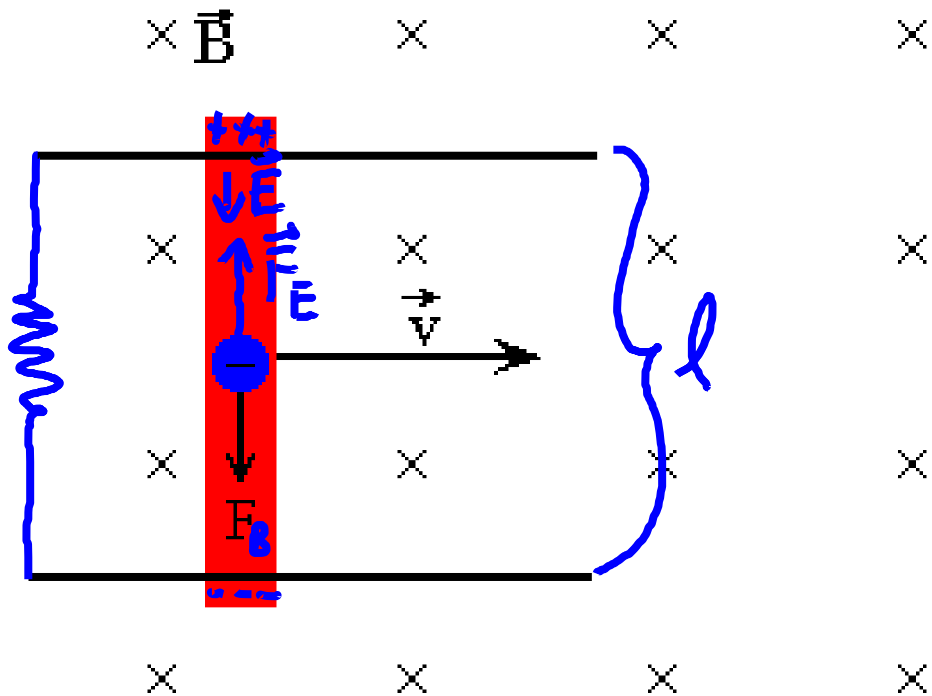
$$\cancel{q}E = \cancel{q}vB$$

$$\mathcal{E} = \Delta V = -\int \vec{E}\cdot d\vec{\ell}$$

$$= E\int d\ell$$

$$= E\ell$$

$$\mathcal{E} = vB\ell$$



Faraday's Law and Motional emf

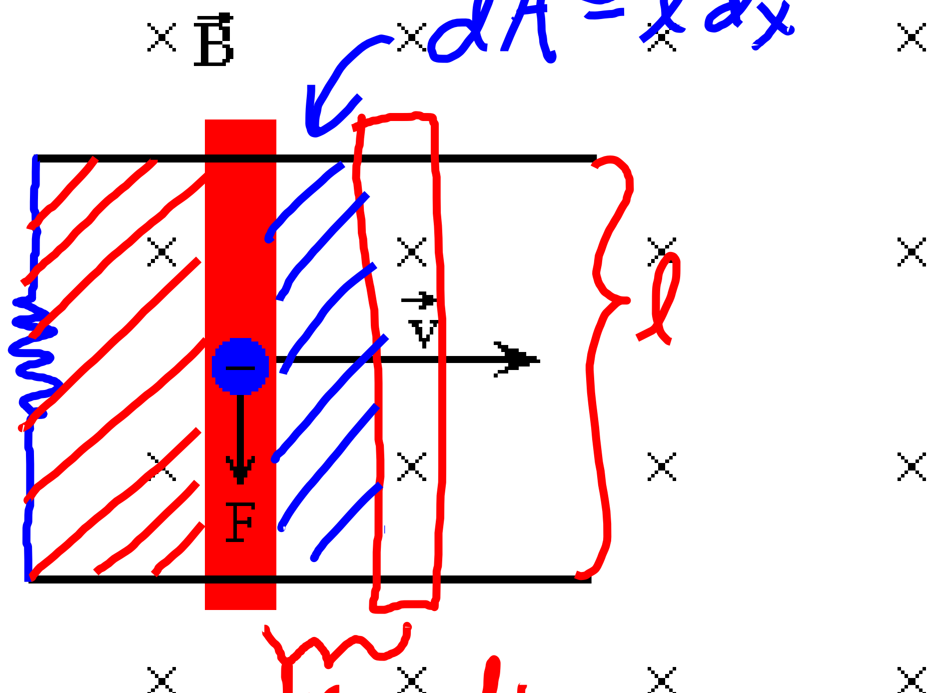
- Faraday's Law:

$dA = l\,dx$
$= l v\,dt$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = B\frac{dA}{dt}$$

$$\mathcal{E} = Blv$$

$dA = l\,dx$



$dx = v\,dt$