Faraday's and Maxwell-Ampere Laws

- A changing magnetic flux produces a curly electric field:

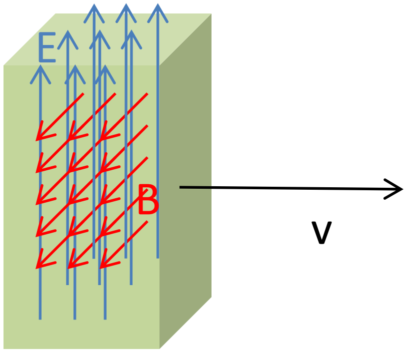
$$\oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{l}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \bullet \hat{\mathbf{n}}\, dA$$

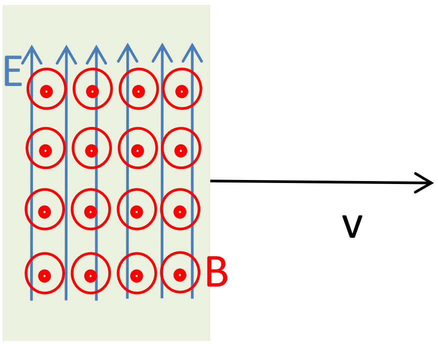
- A changing electric flux produces a (curly) magnetic field:

$$\oint \vec{\mathbf{B}} \bullet d\vec{\mathbf{l}} = \mu_0 \left(I_{encl} + \varepsilon_0 \frac{d}{dt}\int \vec{\mathbf{E}} \bullet \hat{\mathbf{n}}\, dA \right)$$

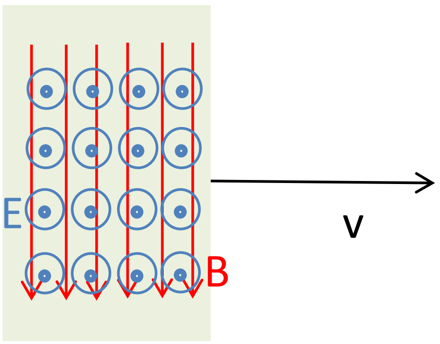
A moving "slab" of E and B fields

- Thin region in which there are uniform electric and magnetic fields.
  - No charges or currents in the vicinity of the slab.
  - Outside the slab, both fields are zero.
  - Inside the slab, E and B are perpendicular to each other.
  - Slab moves in a direction perpendicular to both fields.





front view

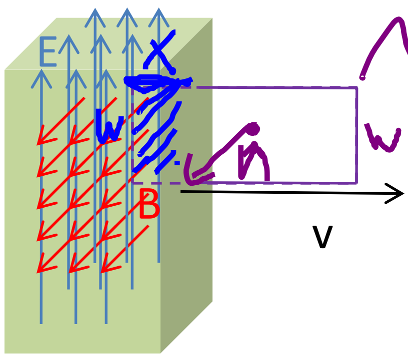


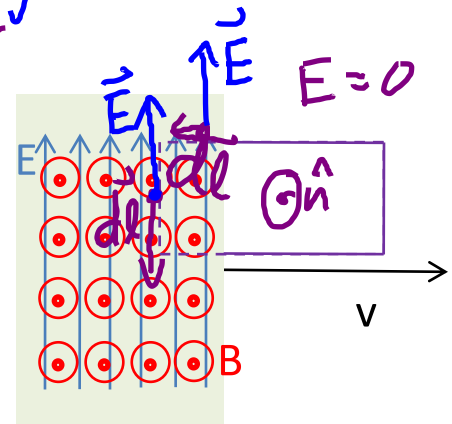
top view

Does Slab obey Faraday's Law?

$$\oint_C \vec{E}\cdot d\vec{l} = \int_{\text{left edge}} \vec{E}\cdot d\vec{l} = -\int E\,dl = -E\int dl = -Ew$$

$$\Phi_B = Bwx$$





front view

$$\frac{d\Phi_B}{dt} = Bw\frac{dx}{dt} = Bwv$$

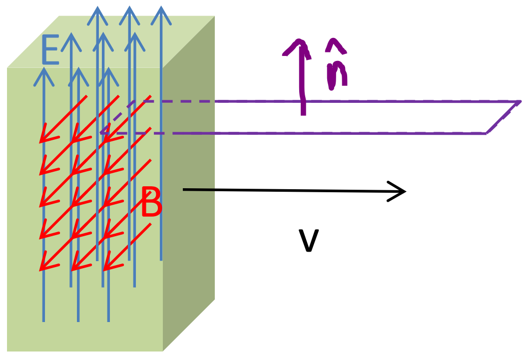
$$\oint_C \vec{\mathbf{E}}\bullet d\vec{\mathbf{l}} = -\frac{d}{dt}\int_S \vec{\mathbf{B}}\bullet\hat{\mathbf{n}}\,dA$$

$$+E\cancel{w} = +B\cancel{w}v$$

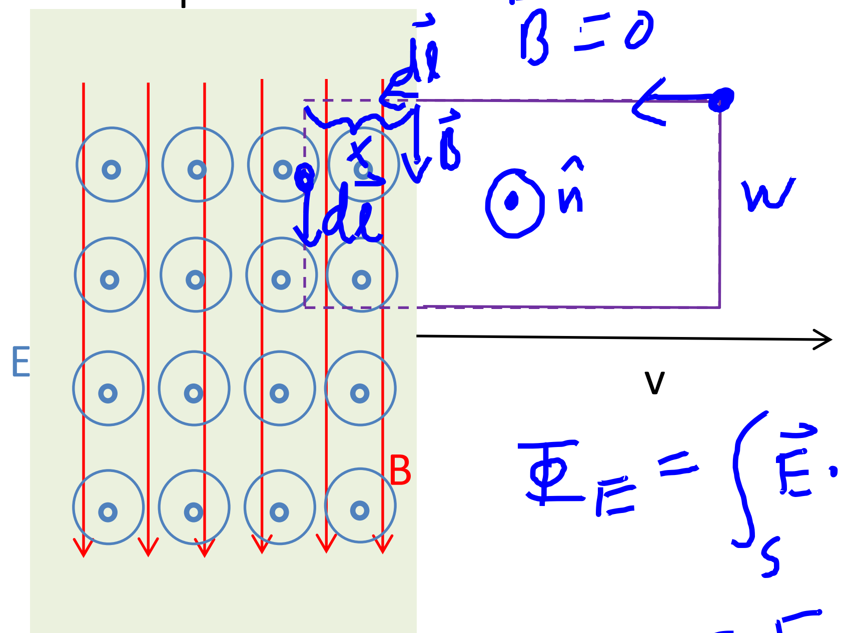
$$\boxed{E = Bv}$$

Does Slab obey Maxwell-Ampere Law?

$$\int_c \vec{B}\cdot d\vec{\ell} = \int_{\text{left edge}} \vec{B}\cdot d\vec{\ell} = \int B\,d\ell = B\int d\ell = Bw$$



top view



$E = 0$
$B = 0$

$$\Phi_E = \int_S \vec{E}\cdot\hat{n}\,dA$$

$$= E\int dA = Ewx$$

$$\frac{dI_E}{dt} = Ew\frac{dx}{dt} = Ewv$$

$$\oint_c \vec{\mathbf{B}}\bullet d\vec{\mathbf{l}} = \mu_0\left(\cancel{I_{encl}}^{\,0} + \varepsilon_0 \frac{d}{dt}\int \vec{\mathbf{E}}\bullet\hat{\mathbf{n}}\,dA\right)$$

$$B\cancel{w} = \mu_0\epsilon_0 E\cancel{w}v$$

Constraints imposed by Faraday's and Maxwell-Ampere Laws

$$\boxed{E = Bv}$$

$$B = \mu_0 \varepsilon_0 E v = \mu_0 \varepsilon_0 B v^2$$

$$1 = \mu_0 \varepsilon_0 v^2$$

$$v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \frac{m}{s}$$

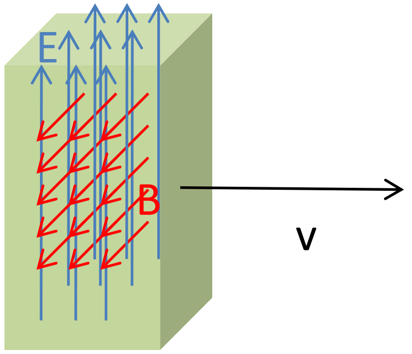
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \quad \mu_0 = 4\pi \times 10^{-7} \frac{Ns^2}{C^2}$$

$$c = 3.00 \times 10^8 \frac{m}{s}$$

Properties of EM Waves

- Move at the speed of light (in vacuum): $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$
- Magnitudes of fields related by $E = Bc$.
- $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other.
- Wave moves in direction of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$, i.e. perpendicular to both fields.



Dipole Antenna

- To create electromagnetic waves, charges must be accelerating.
- Recall that in the steady state, the fields are constant.

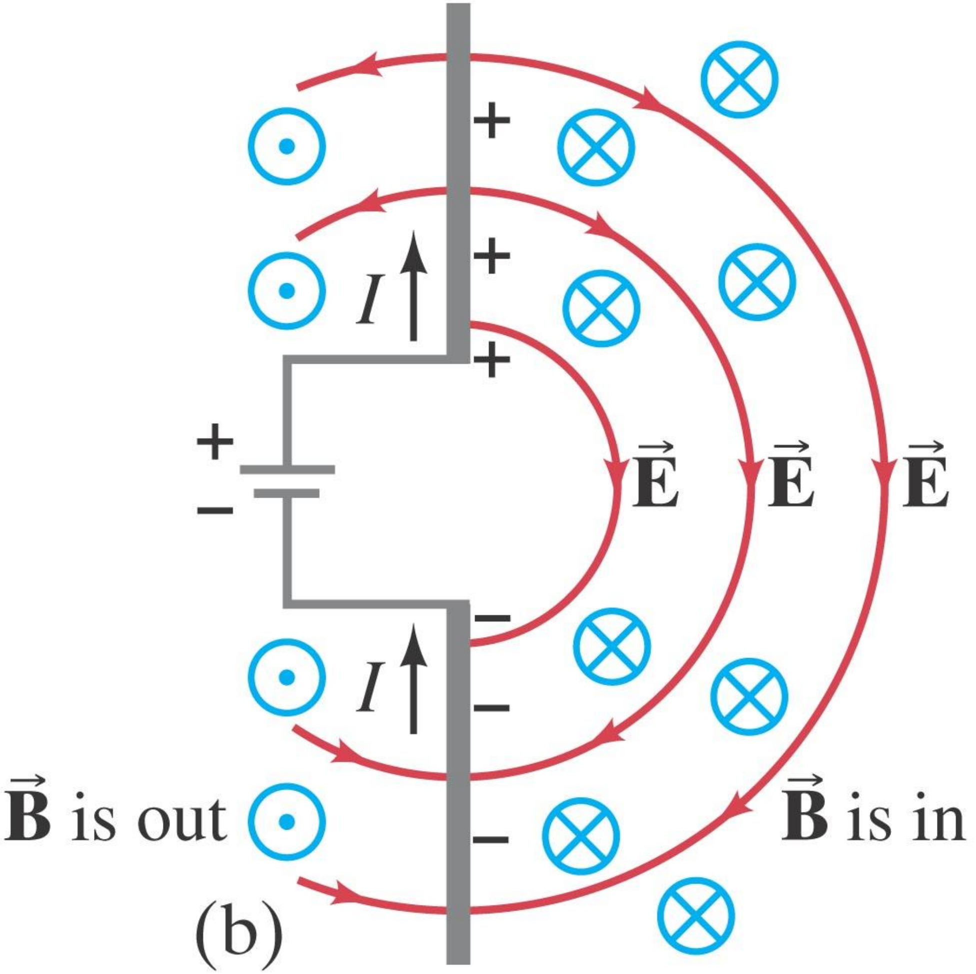
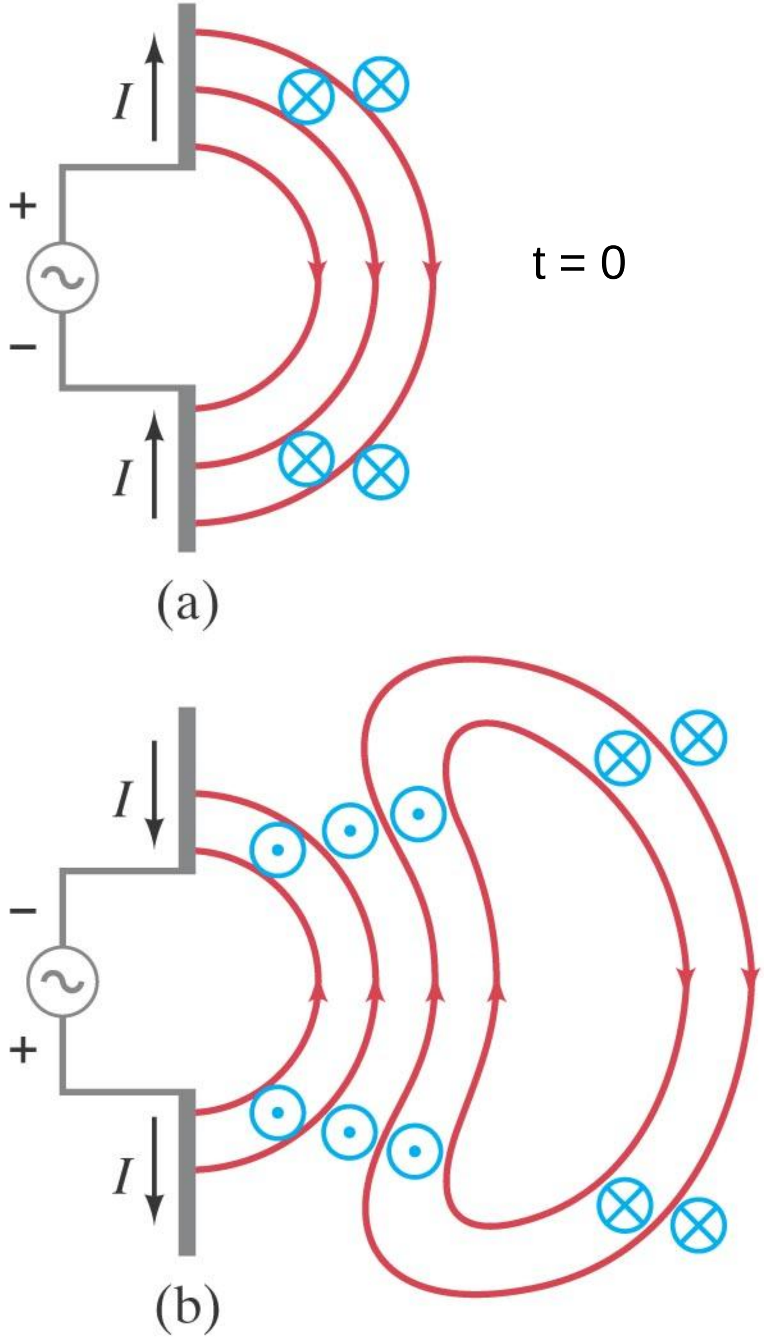


Figure 31.6b



t = 0

Later time

Figure 31.7

Plane waves

- Far from the antenna, the EM waves are well described as *plane waves* – the fields are uniform within planes perpendicular to the direction of propagation.
- Sinusoidal waves – fields are sinusoidal functions of position and time.
  - E.g. wave moving along x axis:

$$\vec{\mathbf{E}} = E_0 \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)\hat{\mathbf{j}} = E_0 \sin(kx - \omega t)\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}} = B_0 \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)\hat{\mathbf{k}} = B_0 \sin(kx - \omega t)\hat{\mathbf{k}}$$

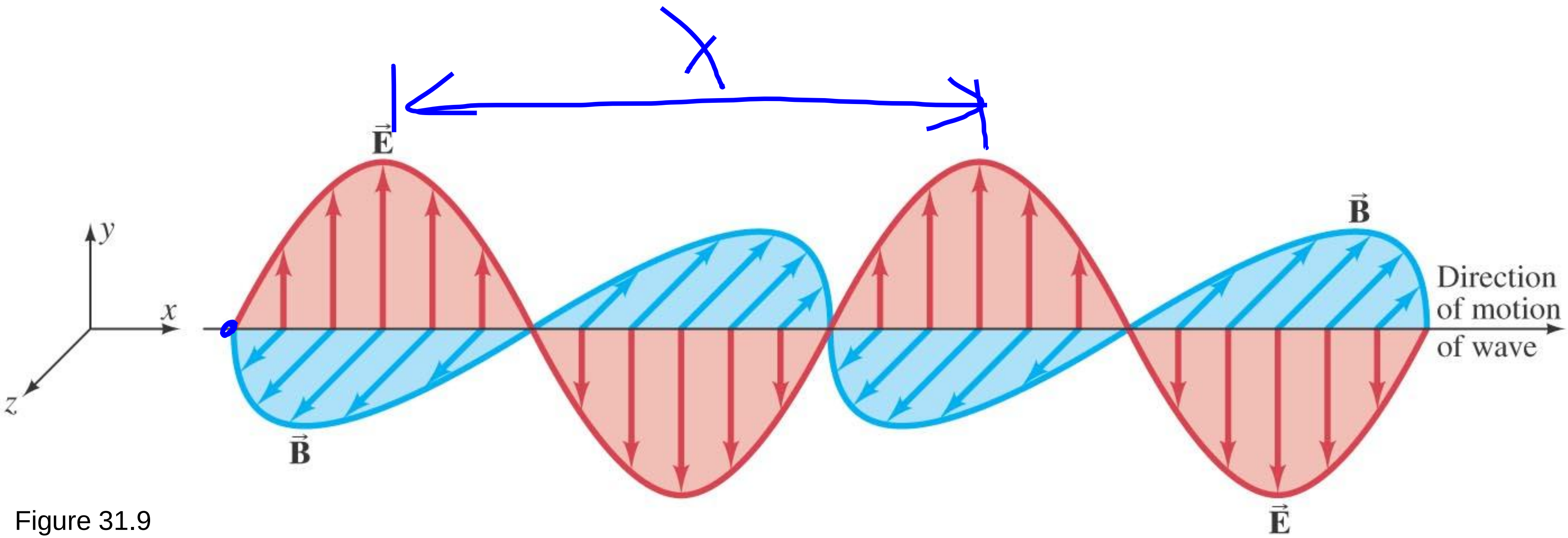


Figure 31.9

$$\vec{\mathbf{E}} = E_0 \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)\hat{\mathbf{j}} = E_0 \sin(kx - \omega t)\hat{\mathbf{j}}$$

$$c = \frac{\lambda}{T} = \lambda f$$

$$\vec{\mathbf{B}} = B_0 \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)\hat{\mathbf{k}} = B_0 \sin(kx - \omega t)\hat{\mathbf{k}}$$

Energy density in EM Waves

Electric Field Energy density

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2$$

Magnetic Field Energy density

$$u_B = \tfrac{1}{2}\frac{B^2}{\mu_0}$$

$$E = Bc$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\frac{u_B}{u_E} = \frac{\cancel{\tfrac{1}{2}} B^2/\mu_0}{\cancel{\tfrac{1}{2}}\epsilon_0 E^2} = \frac{B^2}{\mu_0\epsilon_0 E^2} = \frac{\cancel{B^2}}{\cancel{\mu_0}\cancel{\epsilon_0}\cancel{B^2}\cancel{c^2}}$$

$$= 1$$

Half of the energy in an EM wave is in the electric field and half is in the magnetic field.