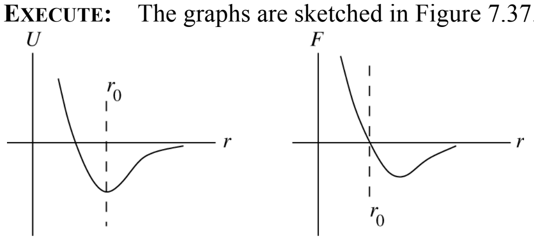
Physics 16 Problem Set 7 Solutions

Y&F Problems

7.37. **IDENTIFY** and **SET UP:** Use Eq.(7.17) to calculate the force from U. At equilibrium $F = 0$.

(a) EXECUTE: The graphs are sketched in Figure 7.37.



$$U = \frac{a}{r^{12}} - \frac{b}{r^6}$$

$$F = -\frac{dU}{dr} = +\frac{12a}{r^{13}} - \frac{6b}{r^7}$$

Figure 7.37

(b) At equilibrium $F = 0$, so $\frac{dU}{dr} = 0$

$$F = 0 \text{ implies } \frac{+12a}{r^{13}} - \frac{6b}{r^7} = 0$$

$6br^6 = 12a$; solution is the equilibrium distance $r_0 = (2a/b)^{1/6}$

U is a minimum at this r; the equilibrium is stable.

(c) At $r = (2a/b)^{1/6}$, $U = a/r^{12} - b/r^6 = a(b/2a)^2 - b(b/2a) = -b^2/4a$.

At $r \to \infty$, $U = 0$. The energy that must be added is $-\Delta U = b^2/4a$.

(d) $r_0 = (2a/b)^{1/6} = 1.13\times10^{-10}$ m gives that

$2a/b = 2.082\times10^{-60}\ \text{m}^6$ and $b/4a = 2.402\times10^{59}\ \text{m}^{-6}$

$b^2/4a = b(b/4a) = 1.54\times10^{-18}$ J

$b(2.402\times10^{59}\ \text{m}^{-6}) = 1.54\times10^{-18}$ J and $b = 6.41\times10^{-78}\ \text{J}\cdot\text{m}^6$.

Then $2a/b = 2.082\times10^{-60}\ \text{m}^6$ gives $a = (b/2)(2.082\times10^{-60}\ \text{m}^6) =$

$\frac{1}{2}(6.41\times10^{-78}\ \text{J}\cdot\text{m}^6)(2.082\times10^{-60}\ \text{m}^6) = 6.67\times10^{-138}\ \text{J}\cdot\text{m}^{12}$

EVALUATE: As the graphs in part (a) show, $F(r)$ is the slope of $U(r)$ at each r. $U(r)$ has a minimum where $F = 0$.

7.38. **IDENTIFY:** Apply Eq.(7.16).

SET UP: $\frac{dU}{dx}$ is the slope of the U versus x graph.

EXECUTE: **(a)** Considering only forces in the x-direction, $F_x = -\frac{dU}{dx}$ and so the force is zero when the slope of the U *vs* x graph is zero, at points b and d.

(b) Point b is at a potential minimum; to move it away from b would require an input of energy, so this point is stable.

(c) Moving away from point d involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so d is an unstable point.

EVALUATE: At point b, F_x is negative when the marble is displaced slightly to the right and F_x is positive when the marble is displaced slightly to the left, the force is a restoring force, and the equilibrium is stable. At point d, a small displacement in either direction produces a force directed away from d and the equilibrium is unstable.

7.42. IDENTIFY: Apply Eq.(7.14).

SET UP: Only the spring force and gravity do work, so $W_{\text{other}} = 0$. Let $y = 0$ at the horizontal surface.

EXECUTE: **(a)** Equating the potential energy stored in the spring to the block's kinetic energy,

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2, \text{ or } v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400 \text{ N/m}}{2.00 \text{ kg}}}(0.220 \text{ m}) = 3.11 \text{ m/s}.$$

(b) Using energy methods directly, the initial potential energy of the spring equals the final gravitational potential energy, $\tfrac{1}{2}kx^2 = mgL\sin\theta$, or

$$L = \frac{\tfrac{1}{2}kx^2}{mg\sin\theta} = \frac{\tfrac{1}{2}(400 \text{ N/m})(0.220 \text{ m})^2}{(2.00 \text{ kg})(9.80 \text{ m/s}^2)\sin 37.0°} = 0.821 \text{ m}.$$

EVALUATE: The total energy of the system is constant. Initially it is all elastic potential energy stored in the spring, then it is all kinetic energy and finally it is all gravitational potential energy.

7.46. IDENTIFY: Apply Eq.(7.14) to relate h and v_B. Apply $\sum \vec{F} = m\vec{a}$ at point B to find the minimum speed required at B for the car not to fall off the track.

SET UP: At B, $a = v_B^2/R$, downward. The minimum speed is when $n \to 0$ and $mg = mv_B^2/R$. The minimum speed required is $v_B = \sqrt{gR}$. $K_1 = 0$ and $W_{\text{other}} = 0$.

EXECUTE: **(a)** Eq.(7.14) applied to points A and B gives $U_A - U_B = \tfrac{1}{2}mv_B^2$. The speed at the top must be at least $\sqrt{gR}$. Thus, $mg(h-2R) > \frac{1}{2}mgR$, or $h > \frac{5}{2}R$.

(b) Apply Eq.(7.14) to points A and C. $U_A - U_C = (2.50)Rmg = K_C$, so

$$v_C = \sqrt{(5.00)gR} = \sqrt{(5.00)(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 31.3 \text{ m/s}.$$

The radial acceleration is $a_{\text{rad}} = \frac{v_C^2}{R} = 49.0 \text{ m/s}^2$. The tangential direction is down, the normal force at point C is horizontal, there is no friction, so the only downward force is gravity, and $a_{\tan} = g = 9.80 \text{ m/s}^2$.

EVALUATE: If $h > \tfrac{5}{2}R$, then the downward acceleration at B due to the circular motion is greater than g and the track must exert a downward normal force n. n increases as h increases and hence v_B increases.

7.46. IDENTIFY: Apply Eq.(7.14) to relate h and v_B. Apply $\sum \vec{F} = m\vec{a}$ at point B to find the minimum speed required at B for the car not to fall off the track.

SET UP: At B, $a = v_B^2/R$, downward. The minimum speed is when $n \to 0$ and $mg = mv_B^2/R$. The minimum speed required is $v_B = \sqrt{gR}$. $K_1 = 0$ and $W_{\text{other}} = 0$.

EXECUTE: **(a)** Eq.(7.14) applied to points A and B gives $U_A - U_B = \tfrac{1}{2}mv_B^2$. The speed at the top must be at least $\sqrt{gR}$. Thus, $mg(h-2R) > \frac{1}{2}mgR$, or $h > \frac{5}{2}R$.

(b) Apply Eq.(7.14) to points A and C. $U_A - U_C = (2.50)Rmg = K_C$, so

$$v_C = \sqrt{(5.00)gR} = \sqrt{(5.00)(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 31.3 \text{ m/s}.$$

The radial acceleration is $a_{\text{rad}} = \frac{v_C^2}{R} = 49.0 \text{ m/s}^2$. The tangential direction is down, the normal force at point C is horizontal, there is no friction, so the only downward force is gravity, and $a_{\tan} = g = 9.80 \text{ m/s}^2$.

EVALUATE: If $h > \tfrac{5}{2}R$, then the downward acceleration at B due to the circular motion is greater than g and the track must exert a downward normal force n. n increases as h increases and hence v_B increases.

7.63. **IDENTIFY** and **SET UP:** First apply $\sum \vec{F} = m\vec{a}$ to the skier.

Find the angle α where the normal force becomes zero, in terms of the speed v_2 at this point. Then apply the work-energy theorem to the motion of the skier to obtain another equation that relates v_2 and α. Solve these two equations for α.

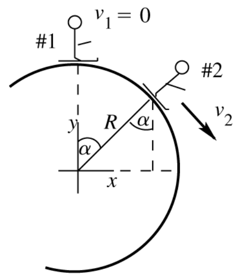


Figure 7.63a

Let point 2 be where the skier loses contact with the snowball, as sketched in Figure 7.63a Loses contact implies $n \to 0$.

$y_1 = R, \quad y_2 = R\cos\alpha$

First, analyze the forces on the skier when she is at point 2. The free-body diagram is given in Figure 7.63b. For this use coordinates that are in the tangential and radial directions. The skier moves in an arc of a circle, so her acceleration is $a_{\text{rad}} = v^2/R$, directed in towards the center of the snowball.

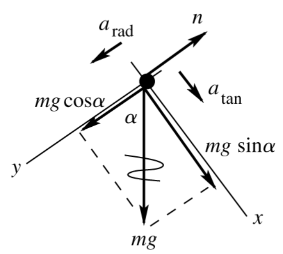


Figure 7.63b

EXECUTE: $\sum F_y = ma_y$

$mg\cos\alpha - n = mv_2^2/R$

But $n = 0$ so $mg\cos\alpha = mv_2^2/R$

$v_2^2 = Rg\cos\alpha$

Now use conservation of energy to get another equation relating v_2 to α:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the skier is gravity, so $W_{\text{other}} = 0$.

$K_1 = 0, \quad K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1 = mgR, \; U_2 = mgy_2 = mgR\cos\alpha$

Then $mgR = \frac{1}{2}mv_2^2 + mgR\cos\alpha$

$v_2^2 = 2gR(1 - \cos\alpha)$

Combine this with the $\sum F_y = ma_y$ equation:

$Rg\cos\alpha = 2gR(1 - \cos\alpha)$

$\cos\alpha = 2 - 2\cos\alpha$

$3\cos\alpha = 2$ so $\cos\alpha = 2/3$ and $\alpha = 48.2°$

EVALUATE: She speeds up and her a_{rad} increases as she loses gravitational potential energy. She loses contact when she is going so fast that the radially inward component of her weight isn't large enough to keep her in the circular path. Note that α where she loses contact does not depend on her mass or on the radius of the snowball.

7.65. **IDENTIFY** and **SET UP:**

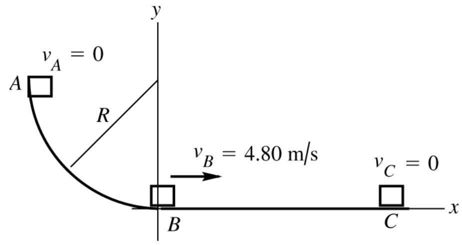


Figure 7.65

$y_A = R$

$y_B = y_C = 0$

(a) Apply conservation of energy to the motion from B to C:

$K_B + U_B + W_{\text{other}} = K_C + U_C$. The motion is described in Figure 7.65.

EXECUTE: The only force that does work on the package during this part of the motion is friction, so

$W_{\text{other}} = W_f = f_k(\cos\phi)s = \mu_k mg(\cos 180°)s = -\mu_k mgs$

$K_B = \frac{1}{2}mv_B^2, \quad K_C = 0$

$U_B = 0, \quad U_C = 0$

Thus $K_B + W_f = 0$

$\frac{1}{2}mv_B^2 - \mu_k mgs = 0$

$$\mu_k = \frac{\mu_B^2}{2gs} = \frac{(4.80\text{ m/s})^2}{2(9.80\text{ m/s}^2)(3.00\text{ m})} = 0.392$$

EVALUATE: The negative friction work takes away all the kinetic energy.

(b) IDENTIFY and **SET UP:** Apply conservation of energy to the motion from A to B:

$$K_A + U_A + W_{\text{other}} = K_B + U_B$$

EXECUTE: Work is done by gravity and by friction, so $W_{\text{other}} = W_f$.

$K_A = 0, \quad K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(4.80\text{ m/s})^2 = 2.304\text{ J}$

$U_A = mgy_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(1.60\text{ m}) = 3.136\text{ J}, \quad U_B = 0$

Thus $U_A + W_f = K_B$

$W_f = K_B - U_A = 2.304\text{ J} - 3.136\text{ J} = -0.83\text{ J}$

EVALUATE: W_f is negative as expected; the friction force does negative work since it is directed opposite to the displacement.

7.73. **IDENTIFY:** Apply Eq.(7.15) to the motion of the block.

SET UP: The motion from A to B is described in Figure 7.73.

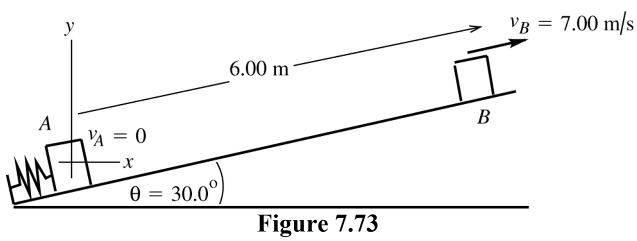


Figure 7.73

The normal force is $n = mg\cos\theta$, so $f_k = \mu_k n = \mu_k mg\cos\theta$.

$y_A = 0; \quad y_B = (60.0\text{ m})\sin 30.0° = 3.00\text{ m}$

$K_A + U_A + W_{\text{other}} = K_B + U_B$

EXECUTE: Work is done by gravity, by the spring force, and by friction, so $W_{\text{other}} = W_f$ and

$U = U_{\text{el}} + U_{\text{grav}}$

$K_A = 0, \quad K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(1.50\text{ kg})(7.00\text{ m/s})^2 = 36.75\text{ J}$

$U_A = U_{\text{el},A} + U_{\text{grav},A} = U_{\text{el},A}$, since $U_{\text{grav},A} = 0$

$U_B = U_{\text{el},B} + U_{\text{grav},B} = 0 + mgy_B = (1.50\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m}) = 44.1\text{ J}$

$W_{\text{other}} = W_f = (f_k \cos\phi)s = \mu_k mg\cos\theta(\cos180°)s = -\mu_k mg\cos\theta s$

$W_{\text{other}} = -(0.50)(1.50\text{ kg})(9.80\text{ m/s}^2)(\cos30.0°)(6.00\text{ m}) = -38.19\text{ J}$

Thus $U_{\text{el},A} - 38.19\text{ J} = 36.75\text{ J} + 44.10\text{ J}$

$U_{\text{el},A} = 38.19\text{ J} + 36.75\text{ J} + 44.10\text{ J} = 119\text{ J}$

EVALUATE: U_{el} must always be positive. Part of the energy initially stored in the spring was taken away by friction work; the rest went partly into kinetic energy and partly into an increase in gravitational potential energy.

7.75. **(a) IDENTIFY** and **SET UP:** Apply $K_A + U_A + W_{\text{other}} = K_B + U_B$ to the motion from *A* to *B*.

EXECUTE: $K_A = 0$, $K_B = \frac{1}{2}mv_B^2$

$U_A = 0$, $U_B = U_{\text{el},B} = \frac{1}{2}kx_B^2$, where $x_B = 0.25\text{ m}$

$W_{\text{other}} = W_F = Fx_B$

Thus $Fx_B = \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$. (The work done by *F* goes partly to the potential energy of the stretched spring and partly to the kinetic energy of the block.)

$Fx_B = (20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ and $\frac{1}{2}kx_B^2 = \frac{1}{2}(40.0\text{ N/m})(0.25\text{ m})^2 = 1.25\text{ J}$

Thus $5.0\text{ J} = \frac{1}{2}mv_B^2 + 1.25\text{ J}$ and $v_B = \sqrt{\dfrac{2(3.75\text{ J})}{0.500\text{ kg}}} = 3.87\text{ m/s}$

(b) IDENTIFY: Apply Eq.(7.15) to the motion of the block. Let point *C* be where the block is closest to the wall. When the block is at point *C* the spring is compressed an amount $|x_C|$, so the block is $0.60\text{ m} - |x_C|$ from the wall, and the distance between *B* and *C* is $x_B + |x_C|$.

SET UP: The motion from *A* to *B* to *C* is described in Figure 7.75.

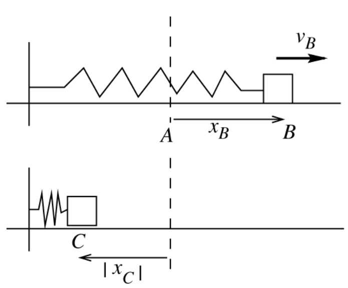


Figure 7.75

$K_B + U_B + W_{\text{other}} = K_C + U_C$

EXECUTE: $W_{\text{other}} = 0$

$K_B = \frac{1}{2}mv_B^2 = 5.0\text{ J} - 1.25\text{ J} = 3.75\text{ J}$ (from part (a))

$U_B = \frac{1}{2}kx_B^2 = 1.25\text{ J}$

$K_C = 0$ (instantaneously at rest at point closest to wall)

$U_C = \frac{1}{2}k|x_C|^2$

Thus $3.75\text{ J} + 1.25\text{ J} = \frac{1}{2}k|x_C|^2$

$|x_C| = \sqrt{\dfrac{2(5.0\text{ J})}{40.0\text{ N/m}}} = 0.50\text{ m}$

The distance of the block from the wall is $0.60\text{ m} - 0.50\text{ m} = 0.10\text{ m}$.

EVALUATE: The work $(20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ done by *F* puts 5.0 J of mechanical energy into the system. No mechanical energy is taken away by friction, so the total energy at points *B* and *C* is 5.0 J.