Physics 16 – Spring 2010 – Problem Set 2

Y&F questions

2.90. **IDENTIFY:** Both objects are in free-fall. Apply the constant acceleration equations to the motion of each person.

SET UP: Let $+y$ be downward, so $a_y = +9.80\ \text{m/s}^2$ for each object.

EXECUTE: **(a)** Find the time it takes the student to reach the ground: $y - y_0 = 180\ \text{m}$, $v_{0y} = 0$, $a_y = 9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(180\ \text{m})}{9.80\ \text{m/s}^2}} = 6.06\ \text{s}$. Superman must reach the ground in $6.06\ \text{s} - 5.00\ \text{s} = 1.06\ \text{s}$: $t = 1.06\ \text{s}$, $y - y_0 = 180\ \text{m}$, $a_y = +9.80\ \text{m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = \dfrac{y-y_0}{t} - \frac{1}{2}a_y t = \dfrac{180\ \text{m}}{1.06\ \text{s}} - \frac{1}{2}(9.80\ \text{m/s}^2)(1.06\ \text{s}) = 165\ \text{m/s}$. Superman must have initial speed $v_0 = 165\ \text{m/s}$.

(b) The graphs of y-t for Superman and for the student are sketched in Figure 2.90.

(c) The minimum height of the building is the height for which the student reaches the ground in 5.00 s, before Superman jumps. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\ \text{m/s}^2)(5.00\ \text{s})^2 = 122\ \text{m}$. The skyscraper must be at least 122 m high.

EVALUATE: $165\ \text{m/s} = 369\ \text{mi/h}$, so only Superman could jump downward with this initial speed.

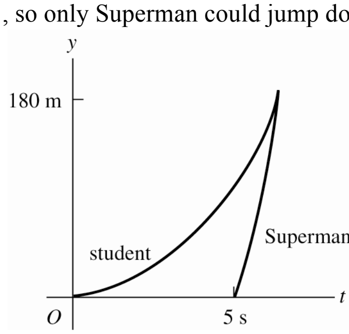


Figure 2.90

3.4. **IDENTIFY:** $\vec{v} = d\vec{r}/dt$. This vector will make a $45°$-angle with both axes when its x- and y-components are equal.

SET UP: $\dfrac{d(t^n)}{dt} = nt^{n-1}$.

EXECUTE: $\vec{v} = 2bt\hat{i} + 3ct^2\hat{j}$. $v_x = v_y$ gives $t = 2b/3c$.

EVALUATE: Both components of $\vec{v}$ change with t.

3.53. **IDENTIFY:** The cannister moves in projectile motion. Its initial velocity is horizontal. Apply constant acceleration equations for the x and y components of motion.

SET UP:

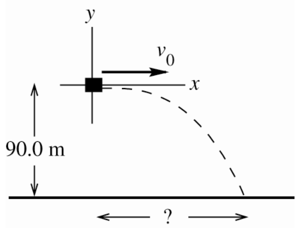


Figure 3.53

Take the origin of coordinates at the point where the canister is released. Take $+y$ to be upward. The initial velocity of the canister is the velocity of the plane, 64.0 m/s in the $+x$-direction.

Use the vertical motion to find the time of fall:

$t = ?$, $v_{0y} = 0$, $a_y = -9.80\ \text{m/s}^2$, $y - y_0 = -90.0\ \text{m}$ (When the canister reaches the ground it is 90.0 m <u>below</u> the origin.)

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$$

EXECUTE: Since $v_{0y} = 0$, $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(-90.0\ \text{m})}{-9.80\ \text{m/s}^2}} = 4.286\ \text{s}.$

SET UP: Then use the horizontal component of the motion to calculate how far the canister falls in this time:

$x - x_0 = ?$, $a_x - 0$, $v_{0x} = 64.0\ \text{m/s}$,

EXECUTE: $x - x_0 = v_0 t + \tfrac{1}{2}at^2 = (64.0\ \text{m/s})(4.286\ \text{s}) + 0 = 274\ \text{m}.$

EVALUATE: The time it takes the cannister to fall 90.0 m, starting from rest, is the time it travels horizontally at constant speed.

3.54. **IDENTIFY:** The equipment moves in projectile motion. The distance *D* is the horizontal range of the equipment plus the distance the ship moves while the equipment is in the air.

SET UP: For the motion of the equipment take $+x$ to be to the right and $+y$ to be upwards. Then $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$, $v_{0x} = v_0\cos\alpha_0 = 7.50\ \text{m/s}$ and $v_{0y} = v_0\sin\alpha_0 = 13.0\ \text{m/s}$. When the equipment lands in the front of the ship, $y - y_0 = -8.75\ \text{m}$.

EXECUTE: Use the vertical motion of the equipment to find its time in the air: $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = \dfrac{1}{9.80}\left(13.0 \pm \sqrt{(-13.0)^2 + 4(4.90)(8.75)}\right)\text{s}$. The positive root is $t = 3.21\ \text{s}$. The horizontal range of the equipment is $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = (7.50\ \text{m/s})(3.21\ \text{s}) = 24.1\ \text{m}$. In 3.21 s the ship moves a horizontal distance $(0.450\ \text{m/s})(3.21\ \text{s}) = 1.44\ \text{m}$, so $D = 24.1\ \text{m} + 1.44\ \text{m} = 25.5\ \text{m}$.

EVALUATE: The equation $R = \dfrac{v_0^2\sin 2\alpha_0}{g}$ from Example 3.8 can't be used because the starting and ending points of the projectile motion are at different heights.

3.56. **IDENTIFY:** The water moves in projectile motion.

SET UP: Let $x_0 = y_0 = 0$ and take $+y$ to be positive. $a_x = 0$, $a_y = -g$.

EXECUTE: The equations of motions are $y = (v_0 \sin\alpha)t - \tfrac{1}{2}gt^2$ and $x = (v_0\cos\alpha)t$. When the water goes in the tank for the *minimum* velocity, $y = 2D$ and $x = 6D$. When the water goes in the tank for the *maximum* velocity, $y = 2D$ and $x = 7D$. In both cases, $\sin\alpha = \cos\alpha = \sqrt{2}/2$.

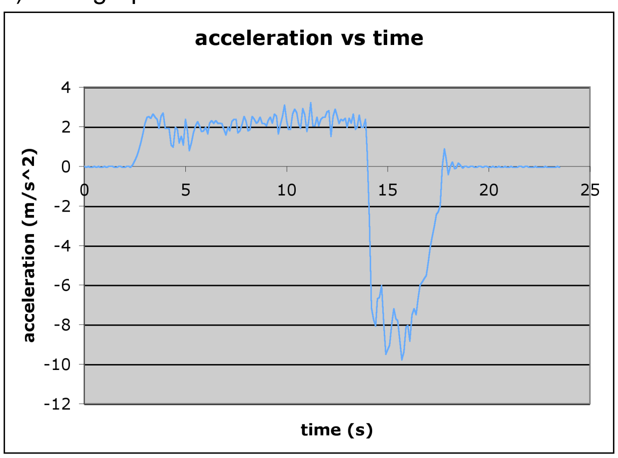
To reach the *minimum* distance: $6D = \frac{\sqrt{2}}{2} v_0 t$, and $2D = \frac{\sqrt{2}}{2} v_0 t - \frac{1}{2} g t^2$. Solving the first equation for t gives $t = \frac{6D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 6D - \frac{1}{2} g \left(\frac{6D\sqrt{2}}{v_0} \right)^2$. Solving this for v_0 gives $v_0 = 3\sqrt{gD}$.

To reach the *maximum* distance: $7D = \frac{\sqrt{2}}{2} v_0 t$, and $2D = \frac{\sqrt{2}}{2} v_0 t - \frac{1}{2} g t^2$. Solving the first equation for t gives $t = \frac{7D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 7D - \frac{1}{2} g \left(\frac{7D\sqrt{2}}{v_0} \right)^2$. Solving this for v_0 gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD}$, which, as expected, is larger than the previous result.

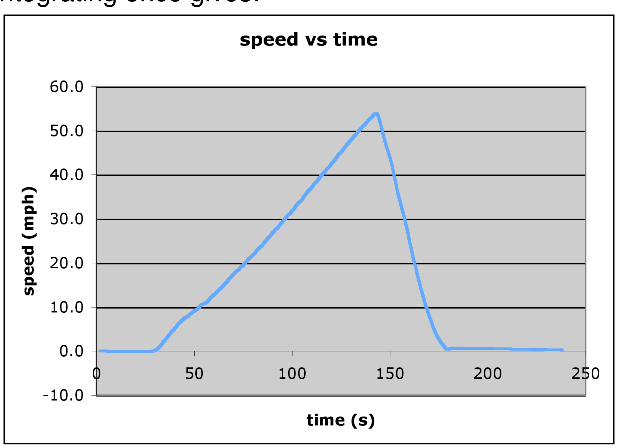
EVALUATE: A launch speed of $v_0 = \sqrt{6}\sqrt{gD} = 2.45\sqrt{gD}$ is required for a horizontal range of $6D$. The minimum speed required is greater than this, because the water must be at a height of at least $2D$ when it reaches the front of the tank.

Accelerometer question

(a) The graph of acceleration vs time looks like:

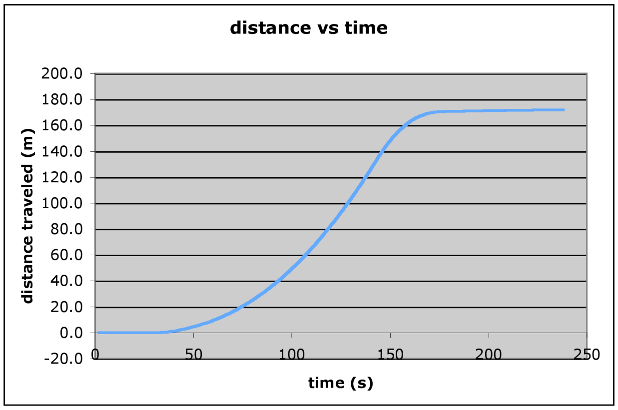


Integrating once gives:



(notice I've changed the units from m/s to mph)

Integrating again gives:



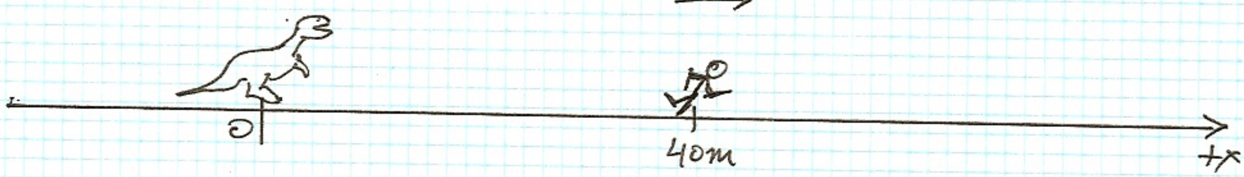
(b) According to this data, I got up to about 53 mph; by my speedometer it was a little over 50 mph. Note that the final speed is, according to the integral, about 0.2 mph, while I told you that I braked to a stop. This makes me think that the accuracy of the speed data is not much better than +/- 0.2 mph (0.1 m/s) by the time we've integrated for a while.

(c) According to this data, I went about 170 m.

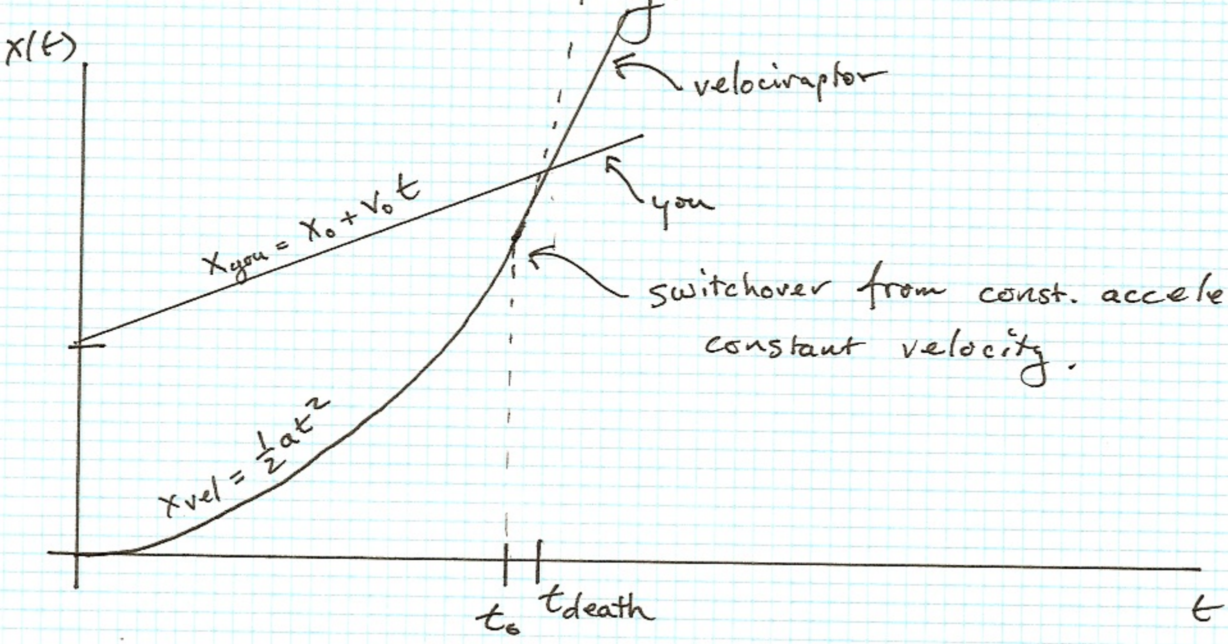
Velociraptor

$v_{max} = 25\,m/s$
$a = 4\,m/s^2$

$v_0 = 6\,m/s$



The velociraptor is accelerating uniformly (from rest), at least until it reaches its top speed of $25\,m/s$. You move at a constant $6\,m/s$. Sketching:



When $x_{you} = x_{vel.}$ you die; this occurs at t_{death}, but at this point we don't know whether it occurs before or after the switchover at t_0. During the constant acceleration phase, $v_{vel} = at$, so switchover happens when

$$v_{max} = a t_0 \rightarrow t_0 = \frac{v_{max}}{a} = \frac{25\,m/s}{4\,m/s^2} = \boxed{6.25\,s. = t_0}$$

At t_0,

$$x_{you} = x_0 + v_0 t_0 = 40m + 6\tfrac{m}{s} \times 6.25s = 77.5m$$

$$x_{vel} = \tfrac{1}{2} a t_0^2 = \tfrac{1}{2}\left(4\tfrac{m}{s^2}\right)(6.25s)^2 = 78.125m$$

You are already dead: i.e. $t_{death} < t_0$ and the whole process occurs <u>before</u> the velociraptor reaches v_{max}.

We only need to solve $x_{you} = x_{vel}$ for t_{death}:

$$x_0 + v_0 t_{death} = \frac{1}{2} a t_{death}^2$$

$$0 = \frac{1}{2} a t^2 - v_0 t - x_0 \quad : \text{ quadratic w/ sol}^{\underline{n}}$$

$$t = \frac{v_0 \pm \sqrt{v_0^2 - 4(\frac{1}{2}a)(-x_0)}}{2(\frac{1}{2}a)} = \frac{v_0 \pm \sqrt{v_0^2 + 2ax_0}}{a}$$

Only the + is physical, so

$$t_{death} = \frac{6\frac{m}{s} + \sqrt{(6\frac{m}{s})^2 + 2(4\,\text{m/s}^2)(40\text{m})}}{4\frac{m}{s^2}} = \boxed{6.22\ \text{s}}$$

During your last 6.22s of life you ran $v_0 t_{death}$ distance:

$$6\frac{m}{s} \times 6.22\text{s} = \boxed{37.3\ \text{m}}$$